Trishna's

Pearson SUCCESS COMPANION

Pearson IIT Foundation Series

Mathematics

CLASS 9

> Provides student-friendly content, application based problems and hints and solutions to master the art of problem solving

> Uses a graded approach to generate, build and retain interest in concepts and their applications

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CLASS

9

Pearson IIT Foundation Series

Mathematics

Seventh Edition

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Brief Contents

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Preface

Pearson IIT Foundation Series has evolved into a trusted resource for students who aspire to be a part of the elite undergraduate institutions of India. As a result, it has become one of the best-selling series, providing authentic and class-tested content for effective preparation—strong foundation, and better scoring.

The structure of the content is not only student-friendly but also designed in such a manner that it motivates students to go beyond the usual school curriculum, and acts as a source of higher learning to strengthen the fundamental concepts of Physics, Chemistry, and Mathematics.

The core objective of the series is to be a one-stop solution for students preparing for various competitive examinations. Irrespective of the field of study that the student may choose to take up later, it is important to understand that Mathematics and Science form the basis for most modern-day activities. Hence, utmost effort has been made to develop student interest in these basic blocks through real-life examples and application-based problems. Ultimately, the aim is to ingrain the art of problem-solving in the mind of the reader.

To ensure high level of accuracy and practicality, this series has been authored by a team of highly qualified teachers with a rich experience, and are actively involved in grooming young minds. That said, we believe that there is always scope for doing things better and hence invite you to provide us with your feedback and suggestions on how this series can be improved further.

Chapter Insights

REMEMBER

Before beginning this chapter, you should be able to:

- Represent numbers on number lines
- Obtain LCM and HCF of numbers
- Apply operations on integers, fractions, decimals, rational and irrational numbers

Remember section will help them to memorize and review the previous learning on a particular topic

Key points will help the students to identify the essential points in a chapter

KEY IDEAS

After completing this chapter, you should be able to:

- Define the numbers and represent them on a number line
- State the properties of numbers
- Understand the basic concepts of radicals and the laws of

QUADRATIC EQUATION

The equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$ is known as a quadratic equation, or an equation of the second degree.

Example: $2x^2 + 3x + 5 = 0$, $x^2 - 5 = 0$ and $3x^2 - 4x + \sqrt{5} = 0$ are some quadratic equations.

Solutions or Roots of a Quadratic Equation

The values of x for which the equation $ax^2 + bx + c = 0$ is satisfied are called the roots of the quadratic equation. A quadratic equation cannot have more than two roots.

Text: Concepts are explained in a well structured and lucid manner

Note boxes are some add-on information of related topics

Note If the loan is fully repaid, the date on which it is repaid is not counted for calculation of interest. If the loan is partially repaid, the day of repayment is also counted for calculating the interest.

EXAMPLE 1.11

Rationalize the denominator of $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}$.

SOLUTION

$$\frac{2+\sqrt{5}}{2-\sqrt{5}} = \left[\frac{2+\sqrt{5}}{2-\sqrt{5}}\right]\left[\frac{2+\sqrt{5}}{2+\sqrt{5}}\right]$$

$$= \frac{(2+\sqrt{5})^2}{(2)^2-(\sqrt{5})^2}$$

$$= \frac{4+4\sqrt{5}+5}{4-5}$$

$$= \frac{9+4\sqrt{5}}{-1} = -(9+4\sqrt{5})$$

Examples are given topic-wise to apply the concepts learned in a particular chapter

Illustrative examples solved in a logical and step-wise manner

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\sqrt{5} \times \sqrt{125} =$ _______.
2. $\dfrac{\sqrt{20}}{\sqrt{320}} =$ _______.
3. $\sqrt[3]{6} \times \sqrt[3]{6} \times \sqrt[3]{6} =$ _______.
4. $\sqrt{3}\ \surd\ 4\sqrt{12} =$ _______.
5. The sum/difference of a rational and an irrational number is __________.
6. $\dfrac{\sqrt{7}}{3\sqrt{3}}$, in rational denominator is _______.
7. Two mixed quadratic surds, $a + \sqrt{b}$ and $a - \sqrt{b}$, whose sum and product are rational, are called

16. Multiply: $\sqrt[3]{5}$ by $\sqrt[4]{2}$.
17. Which is greater, $\sqrt{2}$ or $\sqrt[3]{3}$?
18. Express the surd $2\sqrt{3}$ as a pure surd.
19. Multiply: $\sqrt{14}$ by $\sqrt{8}$.
20. $(\sqrt[3]{3})^4 =$ _______.
21. Rationalizing factor of $5^{1/3} + 5^{-1/3}$ is _______.
22. Express the following in the simplest form: $\dfrac{\sqrt[3]{81}}{\sqrt[3]{3}}$
23. Express the following in the simplest form:

Different levels of questions have been included in the *Test Your Concepts* as well as on *Concept Application* which will help students to develop the problem-solving skill

'Test Your Concepts' at the end of the chapter for classroom preparation

CONCEPT APPLICATION

Level 1

1. Which of the following fractions lie between $\frac{1}{5}$ and $\frac{1}{4}$?

   A. $\frac{7}{33}$ B. $\frac{4}{11}$ C. $\frac{13}{57}$ D. $\frac{7}{17}$

   (a) A and B (b) A and C
   (c) B, C and D (d) A, B and D

2. Express $0.\overline{34} + 0.3\overline{4}$ as a single decimal.

   (a) $0.67\overline{88}$ (b) $0.6\overline{89}$
   (c) $0.68\overline{78}$ (d) $0.6\overline{87}$

   (a) 10 (b) 25
   (c) 13 (d) 43

5. What is the value of 4^{2x-2}, if $(16)^{2x+3} = (64)^{x+3}$?

   (a) 64 (b) 256
   (c) 32 (d) 512

6. Which of the following pairs is having two equal values? (where $x \in R$) _______.

   (a) $9^{x/2}, 24^{x/3}$ (b) $(256)^{4/x}, (4^3)^{4/x}$
   (c) $(343)^{x/3}, (7^4)^{x/12}$ (d) $(36^2)^{2/7}, (6^3)^{2/7}$

'Concept Application' section with problems divided as per complexity: Level 1; Level 2; and Level 3

56. Let the usual speed of Hari be u kmph.

Let his usual time be t hours.

Distance (in km) $= (u + 5)(t - 1)$

$= (u - 2.5)\left(t + \dfrac{48}{60}\right) = ut$

$(u + 5)(t - 1) = ut$

$\therefore\ ut + 5t - u - 5 = ut$

$5t - 5 = u \qquad (1)$

$(u - 2.5)\left(t + \dfrac{48}{60}\right) = ut$

$ut - 2.5t + \dfrac{4}{5}u - 2 = ut$

$\dfrac{4}{5}u = 2.5t + 2 \qquad (2)$

$4t - 4 = 2.5t + 2$ (From Eqs. (1) and (2))

$1.5t = 6$

$t = 4$

$\therefore\ u = 15$

$\therefore\ ut = 60$

Hints and Explanation for key questions along with highlights on the common mistakes that students usually make in the examination

Series Chapter Flow

Class 7

1. Number Systems
2. Expressions and Special Products
3. Ratio and Its applications
4. Indices
5. Geometry
6. Mensuration
7. Equations and their Applications
8. Formulae
9. Statistics
10. Set Theory

Class 9

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4. Linear Equations and Inequations
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9. Statistics
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20. Ratio, Proportion and Variation
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22. Time and Work
23. Time and Distance

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22. Banking
23. Taxation
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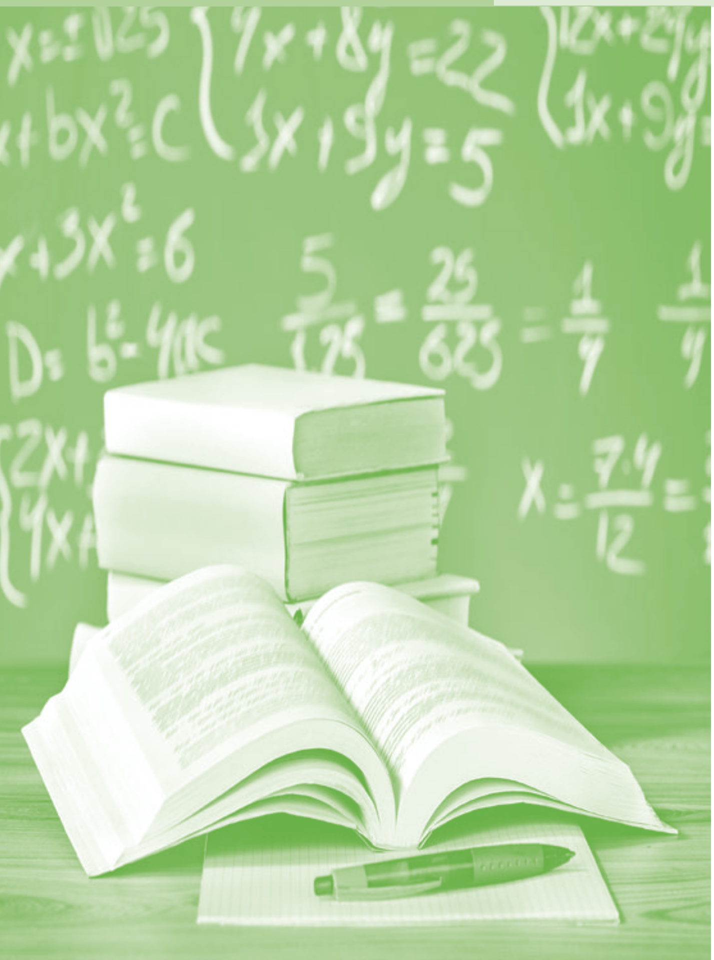
25. Shares and Dividends
26. Partial Fractions
27. Logarithms

Chapter 1

Number Systems



REMEMBER

Before beginning this chapter, you should be able to:

- Represent numbers on number lines
- Obtain LCM and HCF of numbers
- Apply operations on integers, fractions, decimals, rational and irrational numbers

KEY IDEAS

After completing this chapter, you should be able to:

- Define the numbers and represent them on a number line
- State the properties of numbers
- Understand the basic concepts of radicals and the laws of radicals
- Describe the surd, types and order of the surd, its operations, combination and conjugations
- Study rationalizing factor (RF) in surds
- Obtain square root of quadratic surds

INTRODUCTION

In this chapter, we shall discuss types of numbers, laws of indices and then about the surds, types of surds, laws of surds, the four basic operations on surds, their comparison etc. Basically, a surd is an irrational number. Hence, let us first review the types of numbers and recall the definitions.

Natural Numbers

All counting numbers are natural numbers. If N is the set of natural numbers, then $N = \{1, 2, 3, \ldots\}$

Whole Numbers

Natural numbers including zero represent the set of whole numbers. It is denoted by the symbol W. For example: $W = \{0, 1, 2, 3, \ldots\}$

Integers

The set of integers, $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers

Any number that can be expressed in the form $\frac{p}{q}$ (where $q \neq 0$ and p, q are integers) is called a rational number. All integers, all recurring decimals and all terminating decimals are rational numbers. Any integer can be expressed in the form $\frac{p}{q}(q \neq 0)$.

Example: $\frac{2}{1}, \frac{3}{1}, \frac{4}{1}$, etc.

Any recurring decimal can be expressed in the form $\frac{p}{q}(q \neq 0)$.

Example: $0.333\ldots = \frac{1}{3}$

$0.1666\ldots = \frac{1}{6}$

Any terminating decimal can be expressed in the form $\frac{p}{q}(q \neq 0)$.

Example: $0.5 = \frac{5}{10}$ or $\frac{1}{2}, 0.25 = \frac{25}{100}$ or $\frac{1}{4}$

Irrational Numbers

A number that cannot be expressed in the form $\frac{p}{q}$ (where $q \neq 0$), p and q are integers, is an irrational number or any non-terminating and non-recurring decimal is an irrational number.

Example: $\sqrt{2}, \sqrt{3}, \sqrt[3]{5}, \sqrt[4]{25}$ cannot be expressed in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers.

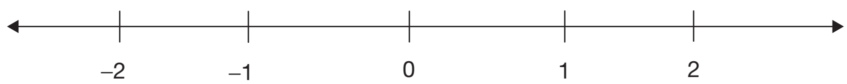
$\therefore$ These numbers are irrational numbers.

Real Numbers

A number that is either rational or irrational is a real number.

NUMBER LINE

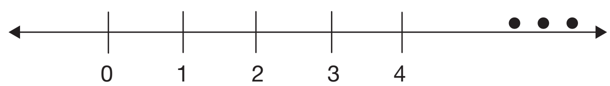
A straight line on which points are identified with real numbers is called a number line. Successive integers are placed on the number line at regular intervals. The following is an illustration of the number line:



Representation of Numbers on the Number Line

We now learn the procedure of representing real numbers on the number line.

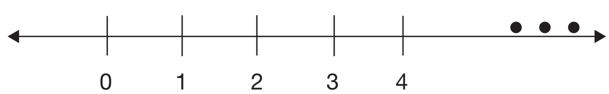
Representation of natural numbers: Draw a line. Mark a point on it which represents 0 (zero). Now on the right hand side of zero (0), mark points at equal intervals of length, as shown below:



These points represent natural numbers 1, 2, 3, … respectively. The three dots on the number line indicate the continuation of these numbers indefinitely.

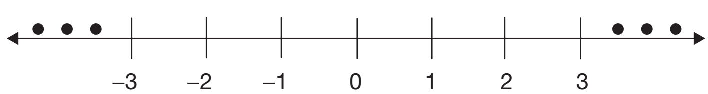
Representation of Whole Numbers

This is similar as above, but with the inclusion of 0 in the number line, it is as follows:



Representation of Integers

Draw a line. Mark a point on it which represents 0 (zero).

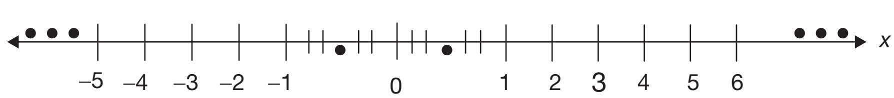


Three dots on either side show the continuation of integers indefinitely on each side.

Representation of Rational Numbers

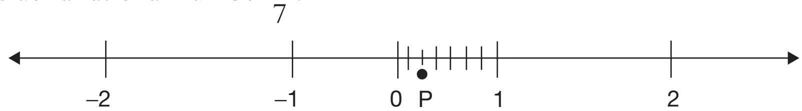
Rational numbers can be represented by some points on the number line. Draw a line. Mark a point on it which represents 0 (zero).

Set equal distances on both sides of 0. Each point on the division represents an integer as shown below.



The length between two successive integers is called unit length.

Let us consider a rational number $\frac{2}{7}$.



Divide unit length between 0 and 1 into 7 equal parts; call them sub-divisions. The point at the line indicating the second sub-division from 0 which represents $\frac{2}{7}$.

In this way any rational number can be represented on the number line.

Representation of Irrational Numbers on the Number Line

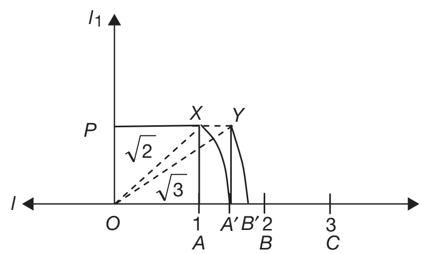
We use the Pythagoras property of a right-angled triangle, according to which, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.

Consider the number line l and a perpendicular line l_1 to it. Let $OA = 1$ unit and $OP = 1$ unit. Let $OAXP$ be a square.

$$OX = \sqrt{OA^2 + AX^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Taking O as the centre and OX as the radius, cut the number line at the point A'.

That is, $OX = OA' = \sqrt{2}$.



Representing $\sqrt{3}$ on the Number Line

Let $OA'YP$ be a rectangle.

$OA' = \sqrt{2}$ units and $A'Y = 1$ unit.

$$OY = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{3} \text{ units.}$$

Taking O as centre and OY as radius, cut the number line with an arc at the point B'. That is, $OB' = \sqrt{3}$ units. In the same way, $\sqrt{5}, \sqrt{6}, \ldots$ can be represented on the number line.

Properties of Real Numbers

Addition

1. **Closure**

   The sum of two real numbers is also a real number.

   Example: $\frac{1}{2} + 3 = 3\frac{1}{2}$.

2. **Commutative**

   If a and b are the real numbers, then $a + b = b + a$.

   Example: $1 + 2 = 2 + 1$.

3. **Associative**

   If a, b and c are three real numbers, then $(a + b) + c = a + (b + c)$.

   Example: $(1 + 2) + 3 = 1 + (2 + 3)$.

4. **Existence of identity element**

   Zero is the additive identity element for all real numbers. For every real number, there exists a number $0 \in R$ such that $a + 0 = 0 + a = a$.

5. **Existence of inverse element**

   If a is a real number, then there exists a real number $(-a)$ such that $a + (-a) = (-a) + a = 0$.

Multiplication

1. **Closure**
If a and b are real numbers, then $a \times b$ is also a real number.
Example: $2 \times \sqrt{3} = 2\sqrt{3}$.

2. **Commutative**
If a and b are real numbers, then $a \times b = b \times a$.
Example: $3 \times \sqrt{5} = \sqrt{5} \times 3$.

3. **Associative**
If a, b and c are real numbers, then $(a \times b) \times c = a \times (b \times c)$.
Example: $(3 \times 5) \times \sqrt{3} = 3\left(5 \times \sqrt{3}\right)$.

4. **Existence of identity element**
One is the identity element for all real numbers.
That is, $\forall\ a \in R$, $a \times 1 = 1 \times a = a$. ($\forall$ means *for all*)
Example: $2 \times 1 = 2 = 1 \times 2$.

5. **Existence of inverse element**
For every non-zero real number, there exists a real number, $\frac{1}{a}$ such that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$.

Distribution of Multiplication over Addition

If a, b and c are real numbers, $a \times (b + c) = a \times b + a \times c$.
Example: $13 \times (10 + 7) = 13 \times 10 + 13 \times 7$.

Laws of Indices for Real Numbers

1. $a^m \times a^n = a^{m+n}$ (Product of powers)

Examples:

(i) $2^3 \times 2^6 = 2^{3+6} = 2^9$

(ii) $\left(\frac{5}{6}\right)^4 \times \left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^{4+5} = \left(\frac{5}{6}\right)^9$

(iii) $2^3 \times 2^4 \times 2^5 \times 2^8 = 2^{(3+4+5+8)} = 2^{20}$

(iv) $\left(\sqrt{7}\right)^3 \times \left(\sqrt{7}\right)^5 = \left(\sqrt{7}\right)^{3+5} = \left(\sqrt{7}\right)^8$

Note $a^{m1} \times a^{m2} \times a^{m3} \times \cdots \times a^{mn} = a^{ml + m2 + m3 + \cdots + m^n}$

2. $a^m \div a^n = a^{m-n}$, $a \neq 0$ (Quotient of powers)

Examples:

(i) $7^8 \div 7^3 = 7^{8-3} = 7^5$

(ii) $\left(\frac{7}{3}\right)^9 \times \left(\frac{7}{3}\right)^5 = \left(\frac{7}{3}\right)^{9-5} = \left(\frac{7}{3}\right)^4$

Note We now consider, what meaning we can assign to a°. If we want these laws to be true for all values of m and n, i.e., even for $n = m$, from (2) above we get $\frac{a^m}{a^m} = a^{m-m}$ or $1 = a^\circ$. We see that if we define a° as 1, this law will be true even for $n = m$. Therefore, we define a° as 1, provided $a \neq 0$.

When $a = 0$, $a^{n-n} = \dfrac{a^n}{a^n} = \dfrac{0}{0}$, which is not defined.

$\therefore$ 0° is not defined.

Note We shall now consider, what meaning we can assign to a^n, where n is a negative integer.

We have $a^m \times a^n = a^{m+n}$.

Consider $a^n \times a^{-n} = a^{n+(-n)}$ (if we want the law to be true) $= a^\circ = 1$

$\therefore$ $a^n \times a^{-n} = 1$

$\Rightarrow a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$. If we define a^{-n} as $\dfrac{1}{a^n}$, this law is true even for negative value of n.

Example: $2^{-4} = \dfrac{1}{2^4}, 5^{-1} = \dfrac{1}{5}, a^{-n} = \dfrac{1}{a^n}$ (provided $a \neq 0$).

Note $\left(\dfrac{a}{b}\right)^{-1} = \dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

3. $(a^m)^n = a^{m \times n}$ (Power of a power)

Examples:

(i) $(5^2)^3 = 5^{2\times 3} = 5^6$

(ii) $\left[\left(\dfrac{2}{3}\right)^4\right]^5 = \left(\dfrac{2}{3}\right)^{4\times 5} = \left(\dfrac{2}{3}\right)^{20}$

Note $[(a^m)^n]^p = a^{mnp}$.

4. $(ab)^n = a^n \times b^n$ (power in the product)

Examples:

(i) $(20)^5 = (4 \times 5)^5 = 4^5 \times 5^5$.

(ii) $(42)^7 = (2 \times 3 \times 7)^7 = 2^7 \times 3^7 \times 7^7$.

In problems, we may often want to write $a^n \times b^n$ as $(ab)^n$.

Examples:

(i) $8 \times 27 = 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

(ii) $\dfrac{125}{343} \times \dfrac{729}{8} = \left[\left(\dfrac{5}{7}\right)\left(\dfrac{9}{2}\right)\right]^3 = \left(\dfrac{45}{14}\right)^3$

Note $(a\ b\ c\ d \ldots z)^n = a^n\ b^n\ c^n\ d^n \ldots z^n$.

5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ (Power in a quotient)

Examples:

(i) $\left(\dfrac{4}{5}\right)^7 = \dfrac{4^7}{5^7}$

(ii) $\left(\dfrac{6}{8}\right)^8 = \dfrac{(2^8)(3^8)}{8^8}$

In problems, we may want to write down $\dfrac{a^n}{b^n}$ as $\left(\dfrac{a}{b}\right)^n$.

Examples:

(i) $\dfrac{4^8}{5^8} = \left(\dfrac{4}{5}\right)^8$

(ii) $\dfrac{\left(\dfrac{2}{3}\right)^5}{\left(\dfrac{9}{8}\right)^5} = \left(\dfrac{\dfrac{2}{3}}{\dfrac{9}{8}}\right)^5 = \left(\dfrac{16}{27}\right)^5$

6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Examples:

(i) $\left(\dfrac{5}{9}\right)^{-3} = \left(\dfrac{9}{5}\right)^3$

(ii) $\left(\dfrac{1}{5}\right)^{-1} = \left(\dfrac{5}{1}\right)^1 = 5$

Note $\left(\dfrac{1}{a}\right)^{-1} = \left(\dfrac{a}{1}\right)^1 = a.$

Exponential Equation

1. If $a^m = a^n$, then $m = n$, if $a \neq 0$ and $a \neq -1$.

Examples:

(i) If $5^p = 5^3 \Rightarrow p = 3$

(ii) If $4^p = 256$

$\because\ 4^p = 4^4 \Rightarrow p = 4$

2. If $a^n = b^n$, then $a = b$ (when n is odd).

Examples:

(i) If $5^7 = p^7$, then $p = 5$.

(ii) If $(5)^{2n-1} = (3 \times p)^{2n-1}$, then $5 = 3p$ or $p = \dfrac{5}{3}$.

3. If $a^n = b^n$, $n \neq 0$, then $a = \pm b$ (when n is even).

Example: $2^4 = x^4 \Rightarrow x = \pm 2$.

RADICALS

A radical expression (or simply a radical) is an expression of the type $\sqrt[n]{x}$. The sign '$\sqrt[n]{\ }$' is called the radical sign. The number under this sign, i.e., 'x' is called the radicand and the number in the angular part of the sign, i.e., 'n' is the order of the radical. At present, we shall deal only with cases where x is a real number. Depending on the values of x, n can have certain corresponding values. Initially we shall consider only positive integral values of n. But remember that, $\sqrt[n]{x} = x^{1/n}$. Last year, we studied such exponential expressions and we know the values that the index can have for different values of the base. From those results, we get the following results for radicals.

If $x > 0$, n can have any real value except zero.

If $x = 0$, n can have any positive real value.

If $x < 0$, n can have any real value except zero. But, we shall consider only certain rational values. Later, we shall examine what these values are for other values of n, (some rational and all irrational) the radical does not have a real value. We shall study such expressions in higher classes.

Examples:

1. $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{7}, \sqrt[2]{8},$ are some radicals.
2. $\sqrt{0}, \sqrt[3]{0}$ are also radicals. But $\sqrt[0]{0}$ and $\sqrt[-1]{0}$ are undefined.
3. $\sqrt[3]{-2}, \sqrt[5]{-64}, \sqrt[7]{-128}$ are radicals and $\sqrt[2]{-4}, \sqrt[4]{-16}$ are also radicals, but they are not real numbers, we shall study them only in higher classes.

   In all these examples, the value of n, i.e., the order of the radical is a positive integer. But as stated above, it can have other values.

SURDS

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational numbers $\pm\sqrt[n]{a}$ are called simple surds or monomial surds.

Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same as that of irrational numbers.

Examples:

1. $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.
2. $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number.
3. π is an irrational number, but it is not a surd.
4. $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

EXAMPLE 1.1

Which of the following are surds?

(a) $\sqrt{9}$ **(b)** $\sqrt[3]{13}$ **(c)** $\sqrt[4]{25}$ **(d)** $\sqrt[6]{32}$

SOLUTION

(a) $\sqrt{9} = 3$ is not a surd. **(b)** $\sqrt[3]{13}$ is a surd.

(c) $\sqrt[4]{25}$ is a surd. **(d)** $\sqrt[6]{32}$ is a surd.

Types of Surds

1. **Unit surds and multiples of surds:** If $\sqrt[n]{a}$ is a surd, it is also referred to as a unit surd. If k is a rational number, $k\sqrt[n]{a}$ is a multiple of a surd.

   Note All multiples of surds can be expressed as unit surds as $k\sqrt[n]{a} = \sqrt[n]{k^n \cdot a}$.

2. **Mixed surds:** If a is a rational number (not equal to 0) and $\sqrt[n]{b}$ is a surd, then $a+\sqrt[n]{b}, a-\sqrt[n]{b}$, are called mixed surds. If $a = 0$, then they are called pure surds.

   Example: $2+\sqrt{3}, 5-\sqrt[3]{6}$ are mixed surds, while $\sqrt{3}, \sqrt[3]{6}$ are pure surds.

3. **Compound surd:** A surd which is the sum or difference of two or more surds is called a compound surd.

   Example: $\sqrt{2}+\sqrt[3]{3}, \sqrt{3}+\sqrt[5]{7}-\sqrt[3]{2}$ and $\sqrt{7}+\sqrt{2}-\sqrt{3}$ are compound surds.

4. **Binomial surd:** A compound surd consisting of two surds is called a binomial surd.

   Example: $\sqrt{3}, \sqrt{6}+4\sqrt{5}, \sqrt{8}-\sqrt[3]{7}$.

5. **Similar surds:** If two surds are different multiples of the same surd, they are called similar surds. Otherwise they are called dissimilar surds.

   Example: $2\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}$ are similar surds. $(\sqrt{2}+3\sqrt{3}), (2\sqrt{2}+6\sqrt{3})$ are similar surds and $1+\sqrt{2}, 2+2\sqrt{2}$ are similar surds. $3\sqrt{3}$ and $6\sqrt{5}$ are dissimilar surds.

Equality of Two Mixed Surds of the Form $a + c\sqrt{b}$ and $d + e\sqrt{b}$

Two mixed surds $a+c\sqrt{b}$ and $d+e\sqrt{b}$ are equal, if and only if their respective rational parts and the irrational parts are equal, i.e., $a = d$ and $c = e$.

EXAMPLE 1.2

Identify the following types of surds:

(a) $\sqrt{6}+5\sqrt{3}$ **(b)** $\sqrt{15}+\sqrt{8}-\sqrt{11}$ **(c)** $\sqrt{5}$ **(d)** $5+\sqrt{7}$

SOLUTION

(a) $\sqrt{6}+5\sqrt{3}$. It is the sum of two surds.

$\therefore$ It is a compound surd of two surds, i.e., a binomial surd.

(b) $\sqrt{15}+\sqrt{8}-\sqrt{11}$. It is the combination of three surds.

$\therefore$ It is a compound surd.

(c) $\sqrt{5}$. It is a monomial surd or a simple surd.

(d) $5+\sqrt{7}$. It is the sum of a rational number and a surd.

$\therefore$ It is a mixed surd.

EXAMPLE 1.3

Which of the following surds are similar?

(a) $2\sqrt{5}$ **(b)** $3\sqrt[3]{5}$ **(c)** $4\sqrt{5}$ **(d)** $5\sqrt[4]{5}$

SOLUTION

$2\sqrt{5}$ and $4\sqrt{5}$ are multiples of the same surd, $\sqrt{5}$.

$\therefore$ They are similar.

Laws of Radicals

If $a > 0$, $b > 0$ and n is a positive rational number, then

1. $(\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$
2. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$
4. $\sqrt[n]{a^p} = a^{p/n}$ and $\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}}$

The application of laws of radicals are as follows:

1. Convert the multiple of a surd into a unit surd.
2. Convert certain unit surds into multiples of other unit surds.
3. Express surds of different orders as surds of the same order.

EXAMPLE 1.4

Express the following surds in their simplest form as multiples of smaller surds:

(a) $\sqrt[3]{1458}$ **(b)** $\sqrt[3]{144}$ **(c)** $\sqrt[4]{1024}$

SOLUTION

(a) $\sqrt[3]{1458} = \sqrt[3]{2(9^3)} = 9\sqrt[3]{2}$.

(b) $\sqrt[3]{144} = \sqrt[3]{2^4(3^2)} = \sqrt[3]{2^3(2)(3^2)} = 2\sqrt[3]{18}$.

(c) $\sqrt[4]{1024} = \sqrt[4]{2^{10}} = \sqrt[4]{(2^8)(2^2)} = \sqrt[4]{2^8}\,\sqrt[4]{2^2} = 4\sqrt{2}$.

Order of a Surd

In the surd $\sqrt[n]{a}$, n is called the order of the surd. Thus, the orders of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$ are 2, 3 and 4 respectively.

Note The orders of the radicals $\sqrt[4]{9}, \sqrt[6]{16}, \sqrt[6]{27}$ are 4, 6 and 6 respectively. We note that $\sqrt[4]{9} = \sqrt[4]{3^2} = 3^{2/4} = \sqrt{3}$ and the order of $\sqrt{3}$ is 2. Thus the order of a surd is not a property of the surd itself, but of the way in which it is expressed.

Comparison of Monomial Surds

If two simple surds are of the same order, then they can easily be compared. If $a < b$, $\sqrt[n]{a} < \sqrt[n]{b}$ for all positive integral values of n.

Example: $\sqrt[4]{2} < \sqrt[4]{7}$, $\sqrt[3]{3} < \sqrt[3]{5}$, $\sqrt[5]{10} < \sqrt[5]{13}$, etc.

If two simple surds of different orders have to be compared, they have to be expressed as radicals of the same order.

Thus to compare $\sqrt[4]{6}$ and $\sqrt[3]{5}$, we express both as the radicals of 12th (LCM of 3, 4) order.

$$\sqrt[4]{6} = \sqrt[12]{6^3} \text{ and } \sqrt[3]{5} = \sqrt[12]{5^4}$$

$$\text{As } 6^3 < 5^4,\ \sqrt[4]{6} < \sqrt[3]{5}.$$

EXAMPLE 1.5

Arrange the following in ascending or descending order of magnitude:

$\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

SOLUTION

$\sqrt[4]{6} = 6^{1/4}, \sqrt[3]{7} = 7^{1/3}, \sqrt{5} = 5^{1/2}$

LCM of the denominators of the exponents of these three terms, 4, 3 and 2 is 12.

Now express the exponent of each term, as a fraction in which the denominator is 12.

$$6^{\frac{1}{4}} = 6^{\frac{3}{12}} = (6^3)^{\frac{1}{12}} = \sqrt[12]{216}$$

$$7^{\frac{1}{3}} = 7^{\frac{4}{12}} = (7^4)^{\frac{1}{12}} = \sqrt[12]{2401}$$

$$5^{\frac{1}{2}} = 5^{\frac{6}{12}} = (5^6)^{\frac{1}{12}} = \sqrt[12]{15625}$$

Now $\sqrt[4]{6} = \sqrt[12]{216}$, $\sqrt[3]{7} = \sqrt[12]{2401}$, $\sqrt{5} = \sqrt[12]{15625}$

Hence, their ascending order is

$\sqrt[12]{216}, \sqrt[12]{2401}, \sqrt[12]{15625}$, i.e., $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

$\therefore$ The descending order of magnitude of the given radicals is $\sqrt{5}, \sqrt[3]{7}, \sqrt[4]{6}$.

Addition and Subtraction of Surds

Addition and subtraction of similar surds can be done using the distributive law,

$$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}.$$

EXAMPLE 1.6

Simplify the following by combining similar surds:

(a) $2\sqrt{5}+5\sqrt{5}$ **(b)** $3\sqrt{6}+\sqrt{216}$ **(c)** $2\sqrt{3}-5\sqrt{12}+3\sqrt{48}$

SOLUTION

(a) $2\sqrt{5}+5\sqrt{5} = (2+5)\sqrt{5} = 7\sqrt{5}$.

(b) $3\sqrt{6}+\sqrt{216} = 3\sqrt{6}+\sqrt{6^2(6)} = 3\sqrt{6}+6\sqrt{6} = 9\sqrt{6}$.

(c) $2\sqrt{3}-5\sqrt{12}+3\sqrt{48}$

$= 2\sqrt{3}-5\sqrt{2^2(3)}+3\sqrt{4^2(3)}$

$= 2\sqrt{3}-5(2)\sqrt{3}+3(4)\sqrt{3}$

$= (2-10+12)\sqrt{3} = 4\sqrt{3}$.

Multiplication and Division of Surds

Surds of the same order can be multiplied according to the law, $(\sqrt[n]{x})(\sqrt[n]{y}) = \sqrt[n]{xy}$.

Note When the surds to be multiplied or divided are not of the same order, they have to be necessarily brought into the same order before the operation is performed.

EXAMPLE 1.7

(a) $\sqrt{15}\times\sqrt{35}$.

(b) $2\sqrt{3}\div 3\sqrt{27}$.

(c) Multiply $\sqrt[3]{3}$ by $\sqrt[4]{2}$.

(d) Divide $\sqrt[6]{5}$ by $\sqrt[3]{10}$.

SOLUTION

(a) $(\sqrt{15})(\sqrt{35}) = \sqrt{(15)(35)}$

$= \sqrt{(5)(3)(5)(7)} = 5\sqrt{21}.$

(b) $2\sqrt{3} \div 3\sqrt{27} = \dfrac{2\sqrt{3}}{3\sqrt{27}}$

$= \dfrac{2\sqrt{3}}{(3)\sqrt{3^2(3)}} = \dfrac{2\sqrt{3}}{(3)(3)\sqrt{3}} = \dfrac{2}{9}.$

(c) $\sqrt[3]{3} = 3^{1/3}$ and $\sqrt[4]{2} = 2^{1/4}$

The LCM of 3 and 4 is 12.

$$\therefore 3^{1/3} = 3^{4/12} = \sqrt[12]{3^4}$$

$$2^{1/4} = 2^{3/12} = \sqrt[12]{2^3}$$

$$(\sqrt[3]{3})(\sqrt[4]{2}) = (\sqrt[12]{3^4})(\sqrt[12]{2^3})$$

$$= \sqrt[12]{(3^4)(2^3)}$$

$$= \sqrt[12]{(81)(8)} = \sqrt[12]{648}.$$

(d) $\sqrt[6]{5} = 5^{1/6}$

$\sqrt[3]{10} = 10^{1/3}$

LCM of 3 and 6 is 6.

$$\sqrt[3]{10} = 10^{1/3} = 10^{2/6} = \sqrt[6]{10^2} = \sqrt[6]{100}$$

$$\therefore \frac{\sqrt[6]{5}}{\sqrt[3]{10}} = \frac{\sqrt[6]{5}}{\sqrt[6]{100}}$$

$$= \sqrt[6]{\frac{5}{100}} = \sqrt[6]{\frac{1}{20}}.$$

Rationalizing Factor (RF)

If the product of two surds is a rational number, then each of the two surds is a RF of the other.

1. RF is not unique.
2. If one RF of a surd is known, then the product of this factor and any non-zero rational number is also the RF of the given surd.
3. It is convenient to use the simplest of all RFs of the given surd to convert it to a rational number.

Examples:

1. $(3\sqrt{3})(\sqrt{3}) = (3)(3) = 9$, a rational number.

   $\therefore$ $\sqrt{3}$ is a RF of $3\sqrt{3}$.

2. $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$, a rational number

   $\therefore$ $\sqrt{3} - \sqrt{2}$ is a RF of $\sqrt{3} + \sqrt{2}$

   and $\sqrt{3} + \sqrt{2}$ is a RF of $\sqrt{3} - \sqrt{2}$

EXAMPLE 1.8

Find the simplest RF of:

(a) $\sqrt[4]{216}$ and **(b)** $\sqrt[5]{16}$

SOLUTION

(a) $\sqrt[4]{216} = \sqrt[4]{(2^3)(3^3)} = 2^{3/4} \times 3^{3/4}$

So RF is $2^{1/4} \times 3^{1/4} = (2 \times 3)^{1/4} = \sqrt[4]{6}$.

$\therefore$ $\sqrt[4]{6}$ is the simplest RF of $\sqrt[4]{216}$.

(b) $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{4/5}$

$\therefore$ RF is $2^{1/5} \Rightarrow (2^{4/5})(2^{1/5}) = 2^{5/5} = 1$.

$\therefore$ $\sqrt[5]{2}$ is the simplest RF of $\sqrt[5]{16}$

Notes

1. $\sqrt[n]{a}$ is a RF of $\sqrt[n]{a^{n-1}}$ and vice-versa.
2. $\sqrt[n]{a^m}$ is a RF of $\sqrt[n]{a^{n-m}}$ and vice-versa.
3. $\sqrt{a} + \sqrt{b}$ is a RF of $\sqrt{a} - \sqrt{b}$ and vice-versa.
4. $\sqrt[3]{a} + \sqrt[3]{b}$ is a RF of $a^{2/3} - a^{1/3} \cdot b^{1/3} - b^{2/3}$ and vice-versa.
5. $\sqrt[3]{a} - \sqrt[3]{b}$ is a RF of $a^{2/3} + a^{1/3} \cdot b^{1/3} + b^{2/3}$ and vice-versa.

EXAMPLE 1.9

Express the following surds with rational denominators:

(a) $\dfrac{2}{\sqrt{14}}$ **(b)** $\dfrac{2\sqrt[3]{3}}{\sqrt[3]{25}}$

SOLUTION

(a) $\dfrac{2}{\sqrt{14}} \times \dfrac{\sqrt{14}}{\sqrt{14}} = \dfrac{2\sqrt{14}}{14} = \dfrac{\sqrt{14}}{7}$.

(b) $\dfrac{2\sqrt[3]{3}}{\sqrt[3]{25}} = \dfrac{2\sqrt[3]{3}}{\sqrt[3]{25}} \times \dfrac{\sqrt[3]{5}}{\sqrt[3]{5}} = \dfrac{2\sqrt[3]{3 \times 5}}{\sqrt[3]{5^3}} = \dfrac{2\sqrt[3]{15}}{5}$.

EXAMPLE 1.10

Given that $\sqrt{2}$ = 1.414, find the value of $\dfrac{3}{\sqrt{2}}$ up to three decimal places.

SOLUTION

$$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{2} = 1.5\sqrt{2}$$

$$= 1.5\,(1.414) = 2.121.$$

Rationalization of Mixed Surds

$a+\sqrt{b}$ is the rationalizing factor of $a-\sqrt{b}$, where a and b are rational.

EXAMPLE 1.11

Rationalize the denominator of $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}$.

SOLUTION

$$\frac{2+\sqrt{5}}{2-\sqrt{5}}=\left[\frac{2+\sqrt{5}}{2-\sqrt{5}}\right]\left[\frac{2+\sqrt{5}}{2+\sqrt{5}}\right]$$

$$=\frac{(2+\sqrt{5})^2}{(2)^2-(\sqrt{5})^2}$$

$$=\frac{4+4\sqrt{5}+5}{4-5}$$

$$=\frac{9+4\sqrt{5}}{-1}=-(9+4\sqrt{5})$$

EXAMPLE 1.12

Given $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{5}=2.236$, $\sqrt{6}=2.449$ and $\sqrt{10}=3.162$.

Find the value of $\dfrac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}$ upto three decimal places.

SOLUTION

We have to rationalize the denominator.

The RF of $\sqrt{3}-\sqrt{5}$ is $\sqrt{3}+\sqrt{5}$.

$$\therefore \frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}=\left[\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}\right]\left[\frac{(\sqrt{3}+\sqrt{5})}{(\sqrt{3}+\sqrt{5})}\right]$$

$$=\frac{\sqrt{6}-\sqrt{3}+\sqrt{10}-\sqrt{5}}{(\sqrt{3})^2-(\sqrt{5})^2}=\frac{\sqrt{10}+\sqrt{6}-\sqrt{3}-\sqrt{5}}{3-5}$$

$$=\frac{3.162+2.449-1.732-2.236}{-2}=-0.822.$$

Conjugate of the Surd of the Form $\sqrt{a}+\sqrt{b}$

Two binomial quadratic surds $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$, are called conjugate surds. The product of conjugate surds is rational.

EXAMPLE 1.13

Write the conjugate of:

(a) $\sqrt{3}+\sqrt{5}$

(b) $5+\sqrt[3]{7}$

SOLUTION

(a) $\sqrt{3}-\sqrt{5}$ is the conjugate of $\sqrt{3}+\sqrt{5}$ and $\sqrt{5}-\sqrt{3}$ is also the conjugate of $\sqrt{3}+\sqrt{5}$.

(b) $5-\sqrt[3]{7}$ is conjugate of $5+\sqrt[3]{49}$.

The following formulae are helpful in finding the rationalizing factors of mixed quadratic and cubic surds:

1. $(a+b)(a-b) = a^2 - b^2$
2. $(a+b)(a^2 - ab + b^2) = a^3 + b^3$
3. $(a-b)(a^2 + ab + b^2) = a^3 - b^3$

EXAMPLE 1.14

(a) Find the RF of $2^{1/3}+2^{-1/3}$

(b) Find the RF of $5^{1/3}-5^{-1/3}$

SOLUTION

(a) $2^{1/3}+2^{-1/3}$

Let $a = 2^{1/3}$ and $b = 2^{-1/3}$

$a^3 = (2^{1/3})^3 = 2$

$b^3 = (2^{-1/3})^3 = 2^{-1} = \dfrac{1}{2}$

But $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\therefore a^2 - ab + b^2 = (2^{1/3})^2 - (2^{1/3}\cdot 2^{-1/3}) + (2^{-1/3})^2 = 2^{2/3} - 1 + 2^{-2/3}$

$\therefore$ RF of $2^{1/3}+2^{-1/3}$ is $2^{2/3}-1+2^{-2/3}$.

(b) $5^{1/3}-5^{-1/3}$

We have $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$\therefore$ RF of $5^{1/3}-5^{-1/3}$ is $[5^{1/3}]^2 + [5^{1/3}\cdot 5^{-1/3}] + [5^{-1/3}]^2$

$= 5^{2/3} + 1 + 5^{-2/3}$.

Comparison of Compound Surds

EXAMPLE 1.15

Among $\sqrt{7}-\sqrt{3}$ and $\sqrt{11}-\sqrt{7}$, which is greater?

SOLUTION

By rationalizing,

$$\sqrt{7}-\sqrt{3}=\frac{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)}{\sqrt{7}+\sqrt{3}}=\frac{4}{\sqrt{7}+\sqrt{3}}$$

$$\sqrt{11}-\sqrt{7}=\frac{\left(\sqrt{11}-\sqrt{7}\right)\left(\sqrt{11}+\sqrt{7}\right)}{\sqrt{11}+\sqrt{7}}=\frac{4}{\sqrt{11}+\sqrt{7}}$$

The numerator of each of the irrational number is 4.

But $\sqrt{11}+\sqrt{7}>\sqrt{7}+\sqrt{3}$

$$\therefore \frac{4}{\sqrt{7}+\sqrt{3}}>\frac{4}{\sqrt{11}-\sqrt{7}}$$

$$\sqrt{7}-\sqrt{3}>\sqrt{11}-\sqrt{7}.$$

EXAMPLE 1.16

Compare the surds $A=\sqrt{8}+\sqrt{7}$ and $B=\sqrt{10}+\sqrt{5}$.

SOLUTION

Since there is a positive sign, by squaring both the surds, we get,

$$A^2=\left(\sqrt{8}+\sqrt{7}\right)^2=8+7+2\sqrt{56}=15+2\sqrt{56}$$

$$B^2=\left(\sqrt{10}+\sqrt{5}\right)^2=10+5+2\sqrt{50}=15+2\sqrt{50}$$

As $56>50, 15+2\sqrt{56}>15+2\sqrt{50}\Rightarrow A>B.$

i.e., $\sqrt{8}+\sqrt{7}>\sqrt{10}+\sqrt{5}.$

Rationalizing the Numerator

EXAMPLE 1.17

Rationalize the numerator of $\dfrac{2-\sqrt{3+x}}{x-1}$.

SOLUTION

Rationalizing factor of $2-\sqrt{3+x}$ is $2+\sqrt{3+x}$.

$$\therefore \frac{2-\sqrt{3+x}}{x-1}=\left[\frac{2-\sqrt{3+x}}{x-1}\right]\left[\frac{2+\sqrt{3+x}}{2+\sqrt{3+x}}\right]$$

$$=\frac{(2)^2-\left(\sqrt{3+x}\right)^2}{(x-1)\left(2+\sqrt{3+x}\right)}=\frac{4-(3+x)}{(x-1)\left(2+\sqrt{3+x}\right)}$$

$$=\frac{1-x}{(x-1)\left(2+\sqrt{3+x}\right)}=\frac{-1}{2+\sqrt{3+x}}.$$

EXAMPLE 1.18

Express $E=\dfrac{1}{\sqrt{5}+\sqrt{3}-\sqrt{8}}$ with a rational denominator.

SOLUTION

The denominator is a trinomial surd, an expression having all the three terms as surds. We group any two of the three terms, say $\sqrt{5}$ and $\sqrt{3}$.

Thus, $\sqrt{5}+\sqrt{3}-\sqrt{8}=(\sqrt{5}+\sqrt{3})-\sqrt{8}$

Consider the product,

$$\left[\left(\sqrt{5}+\sqrt{3}\right)-\sqrt{8}\right]\left[\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{8}\right]=\left(\sqrt{5}+\sqrt{3}\right)^2-\left(\sqrt{8}\right)^2$$

$$=5+3+2\sqrt{5}\sqrt{3}-8=2\sqrt{15}$$

$$\therefore \frac{1}{\sqrt{5}+\sqrt{3}-\sqrt{8}}=\frac{\sqrt{5}+\sqrt{3}+\sqrt{8}}{\left(\sqrt{5}+\sqrt{3}-\sqrt{8}\right)\left(\sqrt{5}+\sqrt{3}+\sqrt{8}\right)}$$

$$=\frac{(\sqrt{5}+\sqrt{3}+\sqrt{8})}{2\sqrt{15}}.$$

Rationalizing the denominator,

$$E=\frac{\sqrt{5}+\sqrt{3}+\sqrt{8}}{2\sqrt{15}}\left(\frac{\sqrt{15}}{\sqrt{15}}\right)$$

$$=\frac{5\sqrt{3}+3\sqrt{5}+2\sqrt{30}}{30}.$$

EXAMPLE 1.19

If both a and b are rational numbers, find the value of a and b in each of the following:

(a) $\dfrac{3+\sqrt{5}}{3-\sqrt{5}}=a+b\sqrt{5}$ **(b)** $\dfrac{3+2\sqrt{3}}{5-2\sqrt{3}}=a+b\sqrt{3}$

SOLUTION

(a) $\dfrac{3+\sqrt{5}}{3-\sqrt{5}}$

$3+\sqrt{5}$ is the rationalizing factor of $3-\sqrt{5}$.

$$\therefore \frac{3+\sqrt{5}}{3-\sqrt{5}}=\frac{3+\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{\left(3+\sqrt{5}\right)^2}{(3)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{9+5+6\sqrt{5}}{9-5}=\frac{14+6\sqrt{5}}{4}=\frac{14}{4}+\frac{6}{4}\sqrt{5}=\frac{7}{2}+\frac{3}{2}\sqrt{5}=a+b\sqrt{5}$$

$\therefore a=\dfrac{7}{2}$ and $b=\dfrac{3}{2}$.

(b) $\dfrac{3+2\sqrt{3}}{5-2\sqrt{3}}$

$5+2\sqrt{3}$ is the RF of $5-2\sqrt{3}$.

$$\therefore \frac{3+2\sqrt{3}}{5-2\sqrt{3}}=\frac{3+2\sqrt{3}}{5-2\sqrt{3}}\times\frac{5+2\sqrt{3}}{5+2\sqrt{3}}=\frac{15+10\sqrt{3}+12+6\sqrt{3}}{(5)^2-\left(2\sqrt{3}\right)^2}$$

$$=\frac{27+16\sqrt{3}}{25-12}=\frac{27+16\sqrt{3}}{13}=a+b\sqrt{3}$$

$$\Rightarrow\ a=\frac{27}{13}\text{ and }b=\frac{16}{13}.$$

Square root of a Quadratic Surd

Consider the real number $a+\sqrt{b}$, where a and b are rational numbers and $\sqrt{b}$ is a surd. Equate the square root of $a+\sqrt{b}$ to $\sqrt{x}+\sqrt{y}$, where x and y are rational numbers, i.e., $\sqrt{a+\sqrt{b}}=\sqrt{x}+\sqrt{y}$.

Squaring both sides, $a+\sqrt{b}=x+y+2\sqrt{xy}$

Equating the rational numbers on the two sides of the above equation, we get

$$a=x+y \tag{1}$$

and equating the irrational numbers, we get

$$\sqrt{b}=2\sqrt{xy} \tag{2}$$

By solving (1) and (2) we get the values of x and y.

Similarly, $\sqrt{a-\sqrt{b}}=\sqrt{x}-\sqrt{y}$.

Square Root of a Trinomial Quadratic Surd

Consider the real number $a+\sqrt{b}+\sqrt{c}+\sqrt{d}$, where a is a rational number and $\sqrt{b},\sqrt{c}$ and $\sqrt{d}$ are surds. Let, $\sqrt{a+\sqrt{b}+\sqrt{c}+\sqrt{d}}=\sqrt{x}+\sqrt{y}+\sqrt{z}$.

By squaring both sides, and comparing rational and irrational parts on either sides, we get,

$$x+y+z=a.$$

$$x=\frac{1}{2}\sqrt{\frac{bd}{c}},\ y=\frac{1}{2}\sqrt{\frac{bc}{d}}\text{ and }z=\frac{1}{2}\sqrt{\frac{cd}{b}}.$$

EXAMPLE 1.20

(a) Find the square root of $7+4\sqrt{3}$.

SOLUTION

Let $\sqrt{7+4\sqrt{3}}=\sqrt{x}+\sqrt{y}$

Squaring both the sides,

$$7+4\sqrt{3}=x+y+2\sqrt{xy}$$
$$\Rightarrow x+y=7 \text{ and } \sqrt{xy}=2\sqrt{3} \Rightarrow xy=12.$$

By solving, we get $x = 4$ and $y = 3$

$$\sqrt{x}+\sqrt{y}=\sqrt{4}+\sqrt{3}=2+\sqrt{3}.$$

(b) Find the square root of $10+\sqrt{24}+\sqrt{60}+\sqrt{40}$.

SOLUTION

Let the given expression be equal to $a+\sqrt{x}+\sqrt{y}+\sqrt{z}$.

As per the method discussed, $a = 10, b = 24, c = 60$ and $d = 40$.

$$x = \frac{1}{2}\sqrt{\frac{bd}{c}} = \frac{1}{2}\sqrt{\frac{24 \times 40}{60}} = 2$$
$$y = \frac{1}{2}\sqrt{\frac{bc}{d}} = \frac{1}{2}\sqrt{\frac{24 \times 60}{40}} = 3$$
$$z = \frac{1}{2}\sqrt{\frac{cd}{b}} = \frac{1}{2}\sqrt{\frac{60 \times 40}{24}} = 5$$

$\therefore \sqrt{x}+\sqrt{y}+\sqrt{z} = \sqrt{2}+\sqrt{3}+\sqrt{5}.$

Alternative method:

$$= \sqrt{10+\sqrt{24}+\sqrt{60}+\sqrt{40}}$$
$$= \sqrt{10+2\sqrt{6}+2\sqrt{15}+2\sqrt{10}}$$
$$= \sqrt{(2+3+5)+2\sqrt{2(3)}+2\sqrt{3(5)}+2\sqrt{2(5)}}$$
$$= \sqrt{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2}$$
$$= \sqrt{2}+\sqrt{3}+\sqrt{5}.$$

EXAMPLE 1.21

If $x = 2+2^{1/3}+2^{2/3}$, then find the value of $x^3 - 6x^2 + 6x - 2$.

HINTS

Cubing on both sides for $(x - 2) = (2^{1/3}+2^{2/3})$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\sqrt{5}\times\sqrt{125} =$ ________.
2. $\dfrac{\sqrt{20}}{\sqrt{320}} =$ ________.
3. $\sqrt[3]{6}\times\sqrt[3]{6}\times\sqrt[3]{6} =$ ________.
4. $\sqrt{3}\ \surd\ 4\sqrt{12} =$ ________.
5. The sum/difference of a rational and an irrational number is ___________.
6. $\dfrac{\sqrt{7}}{3\sqrt{3}}$, in rational denominator is ________.
7. Two mixed quadratic surds, $a+\sqrt{b}$ and $a-\sqrt{b}$, whose sum and product are rational, are called _______ surds.
8. $10\sqrt{3}$ and $11\sqrt{3}$ are __________ surds. (similar/dissimilar)
9. $x+\sqrt{y}$ is a pure surd, if $x =$ _________. (zero/one)
10. Conjugate surd of $\sqrt{5}-3$ is ________.
11. If $x+\sqrt{5} = 4+\sqrt{y}$, then $x+y =$ ________. (where x and y are rational)
12. If the product of two surds is a rational number, then each of the two is a ________ of the other.
13. $\sqrt{6}-\sqrt{7}$ is the conjugate surd of $\sqrt{6}+\sqrt{7}$. (True/False)
14. Express the surd $\dfrac{3}{\sqrt{11}}$ with rational denominator.
15. Find the smallest rationalizing factor of $\sqrt{28}$.
16. Multiply: $\sqrt[3]{5}$ by $\sqrt[4]{2}$.
17. Which is greater, $\sqrt{2}$ or $\sqrt[3]{3}$?
18. Express the surd $2\sqrt{3}$ as a pure surd.
19. Multiply: $\sqrt{14}$ by $\sqrt{8}$.
20. $(\sqrt[3]{3})^4 =$ ________.
21. Rationalizing factor of $5^{1/3}+5^{-1/3}$ is ________.
22. Express the following in the simplest form: $\dfrac{\sqrt[3]{81}}{\sqrt[3]{3}}$
23. Express the following in the simplest form: $\sqrt[3]{625}$
24. Express the following as a pure surd: $\dfrac{2}{3}\sqrt[3]{16}$
25. $\sqrt[3]{3+\sqrt{2}}$ is a surd. (True/False)
26. Express the surd $\dfrac{\sqrt[3]{5}}{\sqrt[3]{6}}$ with rational denominator.
27. If $p = 2+\sqrt{3}$ and pq is a rational number, then q is a unique surd. (True/False)
28. Divide: $\sqrt[6]{144}$ by $\sqrt[6]{4}$.
29. Express the following in the simplest form: $\sqrt[3]{\sqrt{15625}}$
30. Which is smaller, $\sqrt{2}-1$ or $\sqrt{3}-\sqrt{2}$?

Short Answer Type Questions

31. $\sqrt{7+\sqrt{48}} =$ ________.
32. Find the positive square root of $6-\sqrt{20}$.
33. Express the following in the simplest form: $\sqrt[4]{\sqrt[5]{1048576}}$
34. Simplify: $2\sqrt{12}-3\sqrt{32}+2\sqrt{48}$.
35. If $x = c\sqrt{b}+4$, find $x+\dfrac{1}{x}$.
36. Arrange the following surds in an ascending order of magnitude:
    $\sqrt[3]{9}, \sqrt[9]{5}, \sqrt[3]{7}$
37. If $x = \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$, find the value of $x+\dfrac{1}{x}$.
38. Which of the two expressions, $\sqrt{11}-\sqrt{10}$ and $\sqrt{12}-\sqrt{11}$ is greater?
39. Simplify the following:
$$\frac{5\sqrt{5}}{\sqrt{11}+\sqrt{6}}-\frac{3\sqrt{3}}{\sqrt{6}+\sqrt{3}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$
40. Arrange the following surds in an ascending order of magnitude:
$$\sqrt[3]{4}, \sqrt[4]{5}, \sqrt{8}$$

41. Find the positive square root of the following:

$$10+2\sqrt{6}+\sqrt{60}+2\sqrt{10}$$

42. If $x=\dfrac{11}{4-\sqrt{5}}$, find the value of $x^2-8x+11$.

43. If both a and b are rational numbers, find the values of a and b in the following equation:

$$\frac{2+3\sqrt{5}}{4+5\sqrt{5}}=a+b\sqrt{5}$$

44. Given $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$, find the value, correct to three decimals, of the following:

$$\frac{1-\sqrt{3}}{\sqrt{5}-\sqrt{2}}$$

45. If $x=3\sqrt{3}+\sqrt{26}$, find the value of $\dfrac{1}{2}\left(x+\dfrac{1}{x}\right)$.

Essay Type Questions

46. If $x=\dfrac{1}{7+4\sqrt{3}}$, $y=\dfrac{1}{7-4\sqrt{3}}$, find the value of $5x^2-7xy-5y^2$.

47. Rationalize the denominator of the following:

$$\frac{1}{3+\sqrt{2}-3\sqrt{3}}$$

48. Rationalize the denominator of the following:

$$\frac{\sqrt{2}+3\sqrt{5}}{3\sqrt{7}+5\sqrt{3}}$$

49. Given $\sqrt{3}=1.7321$, find the value of the following surd, correct to three decimal places.

$$\frac{\sqrt{3}+1}{\sqrt{3}+1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{4+\sqrt{3}}{4-\sqrt{3}}$$

CONCEPT APPLICATION

Level 1

1. Which of the following fractions lie between $\dfrac{1}{5}$ and $\dfrac{1}{4}$?

A. $\dfrac{7}{33}$ B. $\dfrac{4}{11}$ C. $\dfrac{13}{57}$ D. $\dfrac{7}{17}$

(a) A and B (b) A and C

(c) B, C and D (d) A, B and D

2. Express $0.\overline{34}+0.3\overline{4}$ as a single decimal.

(a) $0.6\overline{788}$ (b) $0.\overline{689}$

(c) $0.6\overline{878}$ (d) $0.\overline{687}$

3. If $\sqrt{5^n}=125$, then $5^{\sqrt[n]{64}}$ = ______.

(a) 25 (b) $\dfrac{1}{125}$

(c) 625 (d) $\dfrac{1}{25}$

4. If $x^4+1=1297$ and $y^4-1=2400$, then y^2-x^2 = ______.

(a) 10 (b) 25

(c) 13 (d) 43

5. What is the value of 4^{2x-2}, if $(16)^{2x+3}=(64)^{x+3}$?

(a) 64 (b) 256

(c) 32 (d) 512

6. Which of the following pairs is having two equal values? (where $x \in R$) ______.

(a) $9^{x/2}, 24^{x/3}$ (b) $(256)^{4/x}, (4^3)^{4/x}$

(c) $(343)^{x/3}, (7^4)^{x/12}$ (d) $(36^2)^{2/7}, (6^3)^{2/7}$

7. The expression $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{7}-\sqrt{2}\right)$ when simplified becomes a

(a) simple surd. (b) mixed surd.

(c) compound surd. (d) binomial surd.

8. If m and n are positive integers, then for a positive number a, $\left\{\sqrt[m]{\left(\sqrt[n]{a}\right)}\right\}^{mn}$ = ______.

(a) a^{mn} (b) a
(c) $a^{m/n}$ (d) 1

9. If $2^{-m} \times \frac{1}{2^m} = \frac{1}{4}$, then $\frac{1}{14}\left[(4^m)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right] =$ ______

(a) $\frac{1}{2}$ (b) 2
(c) 4 (d) $\frac{-1}{4}$

10. The surds $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[5]{5}$, in their descending order are

(a) $\sqrt[3]{3}, \sqrt[5]{5}, \sqrt{2}$ (b) $\sqrt{2}, \sqrt[3]{3}, \sqrt[5]{5}$
(c) $\sqrt{2}, \sqrt[5]{5}, \sqrt[3]{3}$ (d) $\sqrt[3]{3}, \sqrt{2}, \sqrt[5]{5}$

11. $2[(16-15)^{-1} + 25\ (13-8)^{-2}]^{-1} + (1024)^0 =$ ______.

(a) 2 (b) 3
(c) 1 (d) 5

12. If $x = 2$ and $y = 4$, then $\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x} =$ ______.

(a) 4 (b) 8
(c) 12 (d) 2

13. In which of the following pairs of surds are the given two surds similar?

(a) $\sqrt{5}, 7\sqrt{5}$ (b) $\sqrt[3]{7}, \sqrt[2]{7}$
(c) $\sqrt{7}, \sqrt{28}$ (d) Both (a) and (c)

14. Which of the following is the greatest?

(a) 7^2 (b) $(49)^{3/2}$
(c) $\left(\frac{1}{343}\right)^{-1/3}$ (d) $(2401)^{-1/4}$

15. $(\sqrt[6]{5})(\sqrt[3]{2})(\sqrt{3})(\sqrt[12]{6}) =$

(a) $\sqrt[12]{1749600}$ (b) $\sqrt[3]{2} \times \sqrt[12]{109350}$
(c) $\sqrt[12]{177960}$ (d) Both (a) and (b)

16. If $p = 3$ and $q = 2$, then $(3p-4q)^{q-p} \div (4p-3q)^{2q-p}$ = ______.

(a) 1 (b) 6
(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

17. $\left[\frac{(32)^{0.2} + (81)^{0.25}}{(256)^{0.5} - (121)^{0.5}}\right] =$ ______.

(a) 2 (b) 5
(c) 1 (d) 11

18. Which of the following surd is the smallest?
$\sqrt{10}-\sqrt{5}, \sqrt{19}-\sqrt{14}, \sqrt{22}-\sqrt{17}$ and $\sqrt{8}-\sqrt{3}$

(a) $\sqrt{10}-\sqrt{5}$ (b) $\sqrt{19}-\sqrt{14}$
(c) $\sqrt{22}-\sqrt{17}$ (d) $\sqrt{8}-\sqrt{3}$

19. If $\sqrt{m} = \sqrt{a} + \sqrt{c}$ and $\sqrt{m}$, $\sqrt{a}$ and $\sqrt{c}$ are three surds, then

(a) $\sqrt{m}$ is dissimilar to $\sqrt{a}$ and $\sqrt{c}$.
(b) $\sqrt{a}$ and $\sqrt{c}$ are similar to $\sqrt{m}$.
(c) only $\sqrt{a}$ is similar to $\sqrt{m}$.
(d) None of these

20. The surd obtained after rationalizing the numerator of $\frac{4-\sqrt{25-a}}{a-9}$ is equal to

(a) $\frac{a-9}{4-\sqrt{25-a}}$

(b) $\frac{1}{4-\sqrt{25-a}}$

(c) $\frac{1}{(a-9)(4+\sqrt{25-a})}$

(d) $\frac{1}{4+\sqrt{25-a}}$

21. If $\sqrt{13 - x\sqrt{10}} = \sqrt{8} + \sqrt{5}$, then what is the value of x?

(a) −5 (b) −6
(c) −4 (d) −2

22. If the surds $\sqrt[4]{4}, \sqrt[6]{5}, \sqrt[8]{6}$ and $\sqrt[12]{8}$ are arranged in ascending order from left to right, then the third surd from the left is

(a) $\sqrt[12]{8}$ (b) $\sqrt[4]{4}$
(c) $\sqrt[8]{6}$ (d) $\sqrt[6]{5}$

23. $\sqrt{11\sqrt{11\sqrt{11 \ldots 4 \text{ terms}}}} =$

(a) $\sqrt[16]{11^5}$ (b) $\sqrt[16]{11}$
(c) $\sqrt[16]{11^{14}}$ (d) $\sqrt[16]{11^{15}}$

24. If $\frac{5-\sqrt{3}}{2+\sqrt{3}} = x + y\sqrt{3}$, then (x, y) is

(a) (13, −7) (b) (−13, 7)
(c) (−13, −7) (d) (13, 7)

PRACTICE QUESTIONS

25. The simplified form of $\sqrt{125}+\sqrt{125}-\sqrt{845}$ is

(a) $\sqrt{15}$ (b) $2\sqrt{5}$

(c) $-\sqrt{5}$ (d) $-2\sqrt{5}$

26. Which of the following statements is true?

I. If x is a conjugate surd of y, then x can be a RF of y.

II. If x is a RF of y, then x need not be the conjugate of y.

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

27. If $\frac{3-2\sqrt{5}}{6-\sqrt{5}} = a+b\sqrt{5}$ where a and b are rational numbers, then what are the values of a and b?

(a) $\frac{8}{35}, \frac{-9}{35}$ (b) $\frac{8}{31}, \frac{-9}{31}$

(c) $\frac{-8}{31}, \frac{9}{31}$ (d) $\frac{-8}{35}, \frac{9}{35}$

28. If $\frac{3^{5x}\times(81)^2\times 6561}{3^{2x}} = 3^7$, then $x =$________

(a) 3 (b) −3

(c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

29. If $\sqrt{2^n} = 1024$, then $3^{\left(\frac{n}{4}-4\right)} =$ ________.

(a) 3 (b) −3

(c) 27 (d) 81

30. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^m$, then $m =$ ________.

(a) $\frac{-1}{3}$ (b) $\frac{1}{4}$

(c) −3 (d) 2

Level 2

31. $\left[\left\{\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}}\right\}^{a+b}\right]^{\frac{1}{(a+b)^2}} =$ ________.

(a) x^2 (b) $\frac{1}{x}$

(c) 7^3 (d) $\frac{1}{x^2}$

32. If $\frac{2^{m+n}}{2^{m-n}} = 16$ and $a = 2^{\frac{1}{10}}$ then $\frac{(a^{2m+n-p})^2}{(a^{m-2n+2p})^{-1}} =$ ________.

(a) 2 (b) $\frac{1}{4}$

(c) 9 (d) $\frac{1}{8}$

33. Simplify

$$\left[(p^{-1}+q^{-1})(p^{-1}-q^{-1}) \div \left(\frac{1}{p^{-1}}-\frac{1}{q^{-1}}\right)\left(\frac{1}{p^{-1}}+\frac{1}{q^{-1}}\right)\right](pq)^2.$$

(a) $(pq)^2$ (b) −1

(c) $-(pq)^{-2}$ (d) 1

34. If $x = \frac{2}{\sqrt{10}-\sqrt{8}}$, $y = \frac{2}{\sqrt{10}+2\sqrt{2}}$, then $(x-y)^2 =$

(a) $4\sqrt{2}$ (b) 32

(c) $8\sqrt{2}$ (d) 64

35 If $a=\sqrt{6}-\sqrt{3}$, $b=\sqrt{3}-\sqrt{2}$ and $c=\sqrt{2}-\sqrt{6}$, then find the value of $a^3+b^3+c^3-2abc$.

(a) $3\sqrt{2}-5\sqrt{3}-\sqrt{6}$

(b) $3\sqrt{2}-5\sqrt{3}-\sqrt{6}$

(c) $3\sqrt{2}-4\sqrt{3}+\sqrt{6}$

(d) $3\sqrt{2}+4\sqrt{3}+\sqrt{6}$

36. $\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}}}}}\cdots\infty =$

(a) $\frac{81}{64}$ (b) $\frac{9}{8}$

(c) $\frac{3}{2}$ (d) $\frac{3}{2\sqrt{2}}$

37. If $a^p = b^q = c^r = abc$, then $pqr =$ ________.

(a) p^2q+q^2r

(b) $pq+qr+pr$

(c) $(pq+qr+rp)^2$

(d) $pq(qr+rp)$

PRACTICE QUESTIONS

38. The value of $\left[(23+2^2)^{2/3}+(140-29)^{1/2}\right]^2$ is ______.

(a) 196 (b) 289
(c) 324 (d) 400

39. If $x=\sqrt{6}+\sqrt{5}$, then $x^2+\dfrac{1}{x^2}-2=$

(a) $2\sqrt{6}$ (b) $2\sqrt{5}$
(c) 24 (d) 20

40. $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$ is equal to ______.

(a) -3 (b) 3
(c) 6 (d) 2

41. Simplify

$$\frac{1}{\sqrt{19-\sqrt{360}}}-\frac{1}{\sqrt{21-\sqrt{440}}}+\frac{2}{\sqrt{20+\sqrt{396}}}=$$

______.

(a) 1 (b) 2
(c) 0 (d) None of these

42. If $a=\sqrt{17}-\sqrt{16}$ and $b=\sqrt{16}-\sqrt{15}$ then

(a) $a<b$ (b) $a>b$
(c) $a=b$ (d) None of these

43. $\left(\sqrt[6]{15-2\sqrt{56}}\right)\cdot\left(\sqrt[3]{\sqrt{7}+2\sqrt{2}}\right)=$ ______.

(a) 0 (b) 1
(c) -1 (d) 2

44. $\sqrt{\sqrt{63}+\sqrt{56}}=$

(a) $\sqrt[4]{7}(\sqrt{3}+\sqrt{5})$ (b) $\sqrt[4]{7}(\sqrt{3}+1)$
(c) $\sqrt[4]{7}(\sqrt{3}+\sqrt{5})$ (d) $\sqrt[4]{7}(\sqrt{2}+1)$

45. If $\dfrac{\sqrt{7}+2\sqrt{3}}{2\sqrt{7}-\sqrt{5}}=\dfrac{c+\sqrt{p}+\sqrt{q}+\sqrt{r}}{23}(p<q<r)$, where p, q, r are rational numbers, then $q+r-p=$

(a) 361 (b) 302
(c) 418 (d) 426

46. The following are the steps involved in finding the value of $x-y$ from $\dfrac{8-\sqrt{5}}{8+\sqrt{5}}=x-y\sqrt{40}$. Arrange them in sequential order.

(A) $\dfrac{13-2\sqrt{40}}{8-5}=x-y\sqrt{40}$

(B) $\dfrac{(\sqrt{8})^2+(\sqrt{5})^2-2(\sqrt{8})(\sqrt{5})}{(\sqrt{8})^2-(\sqrt{5})^2}=x-y\sqrt{40}$

(C) $x-y=\dfrac{11}{3}$

(D) $x=\dfrac{13}{3}$ and $y=\dfrac{2}{3}$

(E) $\dfrac{(\sqrt{8}-\sqrt{5})(\sqrt{8}-\sqrt{5})}{(\sqrt{8}+\sqrt{5})(\sqrt{8}-\sqrt{5})}=x-y\sqrt{40}$

(a) EABDC
(b) EBADC
(c) ABDEC
(d) DEBAC

47. The following are the steps involved in finding the least among $\sqrt{3}$, $\sqrt[3]{4}$ and $\sqrt[6]{15}$. Arrange them in sequential order.

(A) $\therefore \sqrt[6]{15}$ is the smallest.

(B) $\therefore 3^{1/2}=3^{3/6},\ 4^{1/3}=4^{2/6},\ 15^{1/6}=15^{1/6}$

(C) The LCM of the denominators of the exponents is 6.

(D) $\sqrt{3}=3^{1/2},\ \sqrt[3]{4}=4^{1/3},\ \sqrt[6]{15}=15^{1/6}$

(E) $\therefore \sqrt{3}=\sqrt[6]{27},\ \sqrt[3]{4}=\sqrt[6]{16},\ \sqrt[6]{15}=\sqrt[6]{15}$

(a) DCABE (b) DABEB
(c) DCBEA (d) DBCAE

48. If $y=3-\sqrt{8}$, then $\left(y-\dfrac{1}{y}\right)^2=$ ______.

(a) 9 (b) 81
(c) 4 (d) 32

49. The following are the steps involved in finding the value of $a+b$ from $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}=a+b\sqrt{3}$. Arrange them in sequential order.

(A) $\dfrac{2^2+(\sqrt{3})^2+2\times2\times\sqrt{3}}{2^2-(\sqrt{3})^2}=a+b\sqrt{3}$

(B) $a+b=7+4=11$

(C) $\dfrac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = a + b\sqrt{3}$

(D) $\dfrac{7+4\sqrt{3}}{4-3} = a + b\sqrt{3}$

(E) $a = 7$ and $b = 4$

(a) CDAEB (b) CAEBD
(c) CADEB (d) CEDAB

50. The following are the steps involved in finding the greatest among $\sqrt[3]{2}$, $\sqrt[6]{3}$ and $\sqrt{6}$. Arrange them in sequential order.

(A) The LCM of the denominators of the exponents is 6.

(B) $\therefore \sqrt[6]{216}$ i.e., $\sqrt{6}$ is the greatest.

(C) $\sqrt[3]{2} = 2^{1/3}$, $\sqrt[6]{3} = 3^{1/6}$, $\sqrt{6} = 6^{1/2}$

(D) $2^{1/3} = 2^{2/6}$, $3^{1/6} = 3^{1/6}$, $6^{1/2} = 6^{3/6}$

(E) $\sqrt[3]{2} = \sqrt[6]{4}$ $\sqrt[6]{3} = \sqrt[6]{3}, \sqrt{6} = \sqrt[6]{216}$

(a) CADEB (b) CDABE
(c) DCAEB (d) DACBE

51. If $x = \dfrac{1}{\sqrt{3}+2}$, then $\left(x + \dfrac{1}{x}\right)^2 =$ ______.

(a) 16 (b) 3
(c) 12 (d) 6

52. If $\sqrt[x]{3} \times \sqrt[y]{5} = 10125$, then $12xy =$ ______.

(a) 1 (b) $\dfrac{1}{3}$
(c) 2 (d) $\dfrac{1}{2}$

53. If $x = \dfrac{1}{5+2\sqrt{6}}$, then $x^2 - 10x + 1 =$ ______.

(a) 1 (b) −1
(c) 0 (d) 10

54. If $x = \dfrac{2}{\sqrt{3}-\sqrt{5}}$ and $y = \dfrac{2}{\sqrt{3}+\sqrt{5}}$, then $x + y =$ ______.

(a) 3 (b) $4\sqrt{3}$ (c) $-2\sqrt{3}$ (d) 6

55. $\dfrac{3}{7}$ lies between the fractions ______.

(a) $\dfrac{4}{9}, \dfrac{5}{9}$ (b) $\dfrac{43}{99}, \dfrac{4}{9}$
(c) $\dfrac{42}{99}, \dfrac{4}{9}$ (d) $\dfrac{41}{99}, \dfrac{42}{99}$

Level 3

56. If $\displaystyle\sum_{k=4}^{143} \frac{1}{\sqrt{k}+\sqrt{k+1}} = a - \sqrt{b}$, then a and b respectively are

(a) 10 and 0 (b) −10 and 4
(c) 10 and 4 (d) −10 and 0

57. The surd $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$, after rationalizing the denominator becomes

(a) $\sqrt{5}-\sqrt{2}+\sqrt{10}+1$

(b) $\sqrt{5}+\sqrt{10}+\sqrt{2}+1$

(c) $\sqrt{10}+\sqrt{2}+\sqrt{5}+1$

(d) $\sqrt{5}-\sqrt{10}-\sqrt{2}-1$

58. If $A^{1/A} = B^{1/B} = C^{1/C}$, $A^{BC} + B^{AC} + C^{AB} = 729$.

Which of the following equals $A^{1/A}$?

(a) $\sqrt[ABC]{81}$ (b) $\sqrt{2}$
(c) $\sqrt[ABC]{27}$ (d) $\sqrt[ABC]{9}$

59. If $x = \dfrac{1}{2-\sqrt{3}}$, the value of $x^3 - 2x^2 - 7x + 10$ is equal to

(a) $2+\sqrt{3}$ (b) 10
(c) $7+2\sqrt{3}$ (d) 8

60. If $x = 1 + 5^{1/3} + 5^{2/3}$, then find the value of $x^3 - 3x^2 - 12x + 6$.

(a) 22 (b) 20
(c) 16 (d) 14

61. $\dfrac{4}{\sqrt{10-2\sqrt{21}}} =$ ______.

(a) $\dfrac{1}{4}(\sqrt{7}+\sqrt{3})$ (b) $\dfrac{1}{4}(\sqrt{7}-\sqrt{3})$
(c) $\sqrt{7}+\sqrt{3}$ (d) $\sqrt{7}-\sqrt{3}$

62. If $y = 3^{1/3} + 3$, then $y^3 - 9y^2 + 27y =$ _____.

(a) 27 (b) -27

(c) -30 (d) 30

63. $\dfrac{1}{\sqrt{8+2+\sqrt{15}}} =$ _______.

(a) $\dfrac{1}{2}(\sqrt{5}+\sqrt{3})$ (b) $\dfrac{1}{2}(\sqrt{5}-\sqrt{3})$

(c) $\dfrac{1}{2}(\sqrt{5}+1)$ (d) $\dfrac{1}{2}(\sqrt{5}-1)$

64. If $x = 2^{1/3} - 2$, then $x^3 + 6x^2 + 12x =$ _______.

(a) 6 (b) -6

(c) 8 (d) -8

65. $\dfrac{3}{\sqrt{19-2\sqrt{88}}} - \dfrac{8}{\sqrt{14+2\sqrt{33}}} =$ _______.

(a) $\sqrt{19+2\sqrt{33}}$

(b) $\sqrt{14-2\sqrt{88}}$

(c) $\sqrt{11+2\sqrt{24}}$

(d) $\sqrt{11-2\sqrt{55}}$

66. $\sqrt{\sqrt[x]{2^{x}\sqrt[x^2]{3^{x^3}\sqrt[x^3]{6^{x^6}\sqrt[x^4]{9^{x^{10}}}}}}} =$

(a) 18 (b) 54

(c) 24 (d) 36

67. $\sqrt{7+2\sqrt{6}} + \sqrt{7-2\sqrt{6}} =$ _______.

(a) 14 (b) $\sqrt{6}$

(c) $2\sqrt{6}$ (d) 7

68. $\sqrt{3^2\sqrt{9^2\sqrt{(81)^2\sqrt{16^{16}}}}} =$ _______.

(a) 6×2^4 (b) $3^3 \times 2$

(c) $6^3 \times 2^3$ (d) $6^3 \times 2$

69. $\sqrt[6]{15-2\sqrt{56}}\,\sqrt[3]{\sqrt{7}+2\sqrt{2}} =$ _______.

(a) 0 (b) $\sqrt{2}$

(c) 1 (d) $\sqrt[6]{2}$

70. If $p = 7 - 4\sqrt{3}$, then $\dfrac{p^2+1}{7p} =$ _______.

(a) 2 (b) 1

(c) 7 (d) $\sqrt{3}$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 25
2. $\frac{1}{4}$
3. 6
4. $\frac{1}{8}$
5. irrational
6. $\frac{\sqrt{21}}{9}$
7. conjugate
8. similar
9. zero
10. $-3-\sqrt{5}$
11. 9
12. rationalizing factor
13. True
14. $\frac{3\sqrt{11}}{11}$
15. $\sqrt{7}$
16. $\sqrt[12]{5000}$
17. $\sqrt[3]{3}$
18. $\sqrt{12}$
19. $4\sqrt{7}$
20. $3\times\sqrt[3]{3}$
21. $5^{2/3}-1+5^{-2/3}$
22. 3
23. $5\sqrt[3]{5}$
24. $\sqrt[3]{\frac{128}{27}}$
25. False
26. $\frac{\sqrt[3]{180}}{6}$
27. False
28. $\sqrt[6]{36}$
29. 5
30. $\sqrt{3}-\sqrt{2}$

Shot Answer Type Questions

31. $2+\sqrt{3}$
32. $\sqrt{5}-1$
33. 2
34. $12\sqrt{3}-12\sqrt{2}$
35. $5+\frac{3\sqrt{3}}{2}$
36. $\sqrt[9]{5}, \sqrt[6]{9}, \sqrt[3]{7}$
37. 14
38. $\sqrt{11}-\sqrt{10}$
39. $\sqrt{55}-3\sqrt{2}-3$
40. $\sqrt[4]{5}, \sqrt[3]{4}, \sqrt{8}$
41. $\sqrt{2}+\sqrt{3}+\sqrt{5}$
42. 0
43. $a=\frac{67}{109}, b=\frac{-2}{109}$
44. -0.891
45. $3\sqrt{3}$

Essay Type Questions

46. $-7(1+80\sqrt{3})$
47. $\frac{-7\sqrt{2}-24\sqrt{3}-9\sqrt{6}-30}{92}$
48. $\frac{3\sqrt{14}+9\sqrt{35}-5\sqrt{6}-15\sqrt{15}}{-12}$
49. 6.527

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (d)
11. (a)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (d)
21. (c)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (a)

Level 2

31. (b)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (d)	**40.** (b)
41. (c)	**42.** (a)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (c)	**50.** (a)
51. (a)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (c)					

Level 3

56. (a)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (a)	**61.** (c)	**62.** (d)	**63.** (b)	**64.** (b)	**65.** (c)
66. (a)	**67.** (c)	**68.** (d)	**69.** (c)	**70.** (a)					

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Convert them into decimal form.
2. Express them in $\frac{p}{q}$ form.
3. Find n.
4. Find x and y.
5. Find the value of x.
6. Simplify the numbers given in options.
7. Find the product and recall the types of surds.
8. Recall the laws of radicals.
9. Find the value of m.
10. Convert the given surds into similar surds
11. Simplify the expression.
12. Substitute the values of x and y.
13. Recall the definition of similar surds.
14. Convert the base into same number.
15. Convert all factors into similar surds.
16. Substitute the values of p and q.
17. Apply laws of indices.
18. Rationalize each binomial.
19. Recall the definitions of similar and dissimilar surds.
20. RF of $a-\sqrt{b}$ is $a+\sqrt{b}$.
21. Taking squares on both the sides.
22. Convert the surds into similar surds.
23. $\sqrt{a\sqrt{a\sqrt{a\ldots}}}$ n terms $= a^{\frac{2^n-1}{2^n}}$.
24. Rationalize the denominator of $\frac{(5-\sqrt{3})}{(2+\sqrt{3})}$.
25. Factorize each rational part in the compound surd.
26. Recall the definitions of RF and conjugate.
27. Rationalize the denominator of $\frac{(3-2\sqrt{5})}{(6-\sqrt{5})}$.
28. Apply laws of indices.
29. Find n.
30. Apply laws of indices.

Level 2

31. (i) $(a^m)^n = a^{mn}$.
    (ii) Use $\left((a^m)^n\right)^s = a^{mns}$.
32. (i) Apply laws of indices
    (ii) $\frac{2^{m+n}}{2^{m-n}} = 16 \Rightarrow 2^{2m} = 2^4$. Find the value of m and apply in the given expression.
33. (i) $\frac{1}{a^{-m}} = a^m$ and $a^{-m} = \frac{1}{a^m}$.
    (ii) $p^{-1}+q^{-1} = \frac{1}{p}+\frac{1}{q}$ and $\frac{1}{p^{-1}}+\frac{1}{q^{-1}} = p+q$.
34. (i) Rationalize the denominators of x and y
    (ii) Rationalize the denominators of both x and y.
35. (i) If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$.
    (ii) If $a+b+c=0$, then $a^3+b^3+c^3-3abc=0$.
    (iii) $a^3+b^3+c^3-2abc = abc$.
36. (i) Let the given expression be y and square on both sides.
    (ii) Let $y = \sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\ldots\infty}}}$
    (iii) $y^2 = \frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\ldots\infty}}$
    $\Rightarrow y^2 = \frac{81}{64}y$.
37. (i) Assume each of the power as k then find pqr.
    (ii) Using, $a^p = b^q = c^r = abc$, find the values of a, b and c in terms of (abc).
    (iii) Multiply the obtained a, b and c values and compare the exponents.
38. (i) Simplify the numbers in the brackets first and then write into exponential forms.
    (ii) $23+2^2 = 27 = 3^3$; $140-19 = 121 = 11^2$.

HINTS AND EXPLANATION

39. (i) Rationalize the denominator of $\frac{1}{x}$.

(ii) $x=\sqrt{6}+\sqrt{5}\Rightarrow\frac{1}{x}=\sqrt{6}-\sqrt{5}$.

(iii) $\left(x^2+\frac{1}{x^2}-2\right)=\left(x-\frac{1}{x}\right)^2$.

40. (i) Say, the given expression is y and square on both sides.

(ii) Let $y=\sqrt{6\sqrt{6\sqrt{6+\ldots\infty}}}$.

(iii) $y^2=6+\sqrt{6+\sqrt{6+\ldots\infty}}\Rightarrow y^2=6+y$.

41. (i) Express every denominator in the form $\sqrt{a}-\sqrt{b}$ or $\sqrt{a}+\sqrt{b}$ by finding the square root and rationalize them.

(ii) Simplify the surds in each denominator and then rationalize.

42. (i) If $a>b$, then $\frac{1}{a}<\frac{1}{b}$.

(ii) Rationalize the numerators of given surds and then compare.

43. (i) Use '$\sqrt[6]{x}=\sqrt[3]{\sqrt{x}}$' and simplify by finding the square root wherever necessary.

(ii) $\sqrt[6]{x}=\sqrt[3]{\sqrt[2]{x}}$

(iii) $\sqrt[3]{x+y}\,.\sqrt[3]{x-y}=\sqrt[3]{(x^2-y^2)}$.

44. (i) Bring a possible term out of the root.

(ii) $\sqrt{\sqrt{63}+\sqrt{56}}=\sqrt[4]{7}\sqrt{\sqrt{9}+\sqrt{8}}$

(iii) Let $\sqrt{3+2\sqrt{2}}=\sqrt{x}+\sqrt{y}$.

45. (i) Rationalize the denominator of the LHS of the equation.

(ii) Rationalize the denominator of LHS and compare it with RHS.

46. EBADC is the required sequential order.

47. DCBEA is the required sequential order.

48. Given that, $y=3-\sqrt{8}$

$$\frac{1}{y}=\frac{1}{3-\sqrt{8}}=\frac{3+\sqrt{8}}{(3-\sqrt{8})(3+\sqrt{8})}$$

$$=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}.$$

Now, $\left(y-\frac{1}{y}\right)^2=(3-\sqrt{8}-3-\sqrt{8})^2$

$$=(-2\sqrt{8})^2=32.$$

49. CADEB is the required sequential order.

50. CADEB is the required sequential order.

51. Given that $x=\frac{1}{\sqrt{3}+2}$

$$\Rightarrow x=\frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\Rightarrow x=\frac{2-\sqrt{3}}{4-3}$$

$$\Rightarrow x=2-\sqrt{3}.$$

And $\frac{1}{x}=\sqrt{3}+2$

Now $\left(x+\frac{1}{x}\right)^2=(2-\sqrt{3}+\sqrt{3}+2)^2$

$$=(4)^2=16.$$

52. $\sqrt[x]{3}\times\sqrt[y]{5}=10125$

$3^{1/x}\cdot 51^{1/y}=3^4\times 5^3$

$$\Rightarrow\frac{1}{x}=4,\ \frac{1}{y}=3$$

$$\Rightarrow 4x=1,\ 3y=1$$

$$\Rightarrow 12xy=1.$$

53. $x=\frac{1}{5+2\sqrt{6}}$

$$x=\frac{5-2\sqrt{6}}{25-24}\Rightarrow x=5-2\sqrt{6}$$

$$\frac{1}{x}=5+2\sqrt{6}\Rightarrow x+\frac{1}{x}=10$$

$$x^2+1=10x\Rightarrow x^2-10x+1=0.$$

54. $x=\frac{2}{\sqrt{3}-\sqrt{5}},\ y=\frac{2}{\sqrt{3}+\sqrt{5}}$

$$x+y=\frac{2}{\sqrt{3}-\sqrt{5}}+\frac{2}{\sqrt{3}+\sqrt{5}}$$

$$=\frac{2\sqrt{3}+2\sqrt{5}+2\sqrt{3}-2\sqrt{5}}{3-5}=\frac{4\sqrt{3}}{-2}$$

$$(x+y)=-2\sqrt{3}.$$

55. $\frac{3}{7}=0.\overline{428571}$

(a) $\frac{4}{9}=0.444\ldots\qquad\frac{5}{9}=0.555\ldots$

(b) $\frac{43}{99}=0.434343\ldots$ $\frac{4}{9}=0.4444\ldots$

(c) $\frac{4}{9}=0.4444\ldots$ $\frac{42}{99}=0.424242\ldots$

(d) $\frac{41}{99}=0.414141\ldots$ $\frac{42}{99}=0.424242\ldots$

Level 3

56. (i) Substitute the values of k and rationalize every term of LHS.

(ii) $\sum_{k=4}^{143}\frac{1}{\sqrt{k}+\sqrt{k+1}}$

$$=\frac{1}{\sqrt{5}+\sqrt{4}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\cdots+\frac{1}{\sqrt{144}+\sqrt{143}}.$$

(iii) Rationalize the denominator of each term and simplify.

57. (i) Rationalize the denominator two times.

(ii) RF of denominator is $(3+\sqrt{5}-2\sqrt{2})$.

(iii) Rationalize the denominator twice and simplify.

61. $\frac{4}{\sqrt{10-2\sqrt{21}}}=\frac{4}{\sqrt{(\sqrt{7})^2+(\sqrt{3})^2-2\sqrt{7\times 3}}}$

$$=\frac{4}{\sqrt{(\sqrt{7}-\sqrt{3})^2}}=\frac{4}{\sqrt{7}-\sqrt{3}}$$

$$=\frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}$$

$$=\frac{4(\sqrt{7}+\sqrt{3})}{4}=\sqrt{7}+\sqrt{3}.$$

62. Given, $y=3^{1/3}+3$

$$\Rightarrow y-3=3^{1/3}$$

Taking the cubes on both sides

$$\Rightarrow (y-3)^3=(3^{1/3})^3$$

$$\Rightarrow y^3-9y^2+27y-27=3$$

$$\Rightarrow y^3-9y^2+27y=30.$$

63. $\frac{1}{\sqrt{8+2\sqrt{15}}}=\frac{1}{\sqrt{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5\times 3}}}$

$$=\frac{1}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{1}{2}(\sqrt{5}-\sqrt{3}).$$

64. Given $x=2^{1/3}-2$

$$\Rightarrow x+2=2^{1/3}$$

Taking the cubes of the terms on both the sides,

$$\Rightarrow (x+2)^3=(2^{1/3})^3$$

$$\Rightarrow x^3+6x^2+12x+8=2$$

$$\Rightarrow x^3+6x^2+12x=-6.$$

65. $\frac{3}{\sqrt{19-2\sqrt{88}}}-\frac{8}{\sqrt{14+2\sqrt{33}}}$

$$=\frac{3}{\sqrt{11}-\sqrt{8}}-\frac{8}{\sqrt{11}+\sqrt{3}}$$

$$=\frac{3(\sqrt{11}+\sqrt{8})}{11-8}-\frac{8(\sqrt{11}-\sqrt{3})}{11-3}$$

$$=\sqrt{11}+\sqrt{8}-\sqrt{11}+\sqrt{3}$$

$$=\sqrt{8}+\sqrt{3}=\sqrt{11+2\sqrt{24}}.$$

66. $\sqrt{\sqrt[x]{2^{x}\sqrt[x^2]{3^{x^3}\sqrt[x^3]{6^{x^6}\sqrt[x^4]{9^{x^{10}}}}}}}$

$$=\sqrt{2^{x/x}\cdot 3^{x^3/x^3}\cdot 6^{x^6/x^6}\cdot 9^{x^{10}/x^{10}}}=18.$$

67. $\sqrt{7+2\sqrt{6}}+\sqrt{7-2\sqrt{6}}$

$$=\sqrt{6+1+2\sqrt{6}}+\sqrt{6+1-2\sqrt{6}}$$

$$=\sqrt{6}+1+\sqrt{6}-1=2\sqrt{6}.$$

68. $\sqrt{3^2\sqrt{9^2\sqrt{(81)^2\sqrt{16^{16}}}}}$

$$=(3^2)^{1/2}\times(9^2)^{1/4}\times\left[(81)^2\right]^{1/8}\times\left[(16)^{16}\right]^{1/16}$$

$$=3\times 3\times 3\times 16=6^3\times 2.$$

HINTS AND EXPLANATION

69. $(\sqrt[6]{15-2\sqrt{56}})(\sqrt[3]{\sqrt{7}+2\sqrt{2}})$

$\sqrt[3]{\sqrt{15-2\sqrt{56}}}\ \sqrt[3]{\sqrt{7}+2\sqrt{2}}$

$=\sqrt[3]{\sqrt{8}-\sqrt{7}}\ \sqrt[3]{\sqrt{7}+\sqrt{8}}=\sqrt[3]{8-7}=1.$

70. $P=7-4\sqrt{3}$

$\frac{1}{P}=\frac{1}{7-4\sqrt{3}}=\frac{7+4\sqrt{3}}{49-48}$

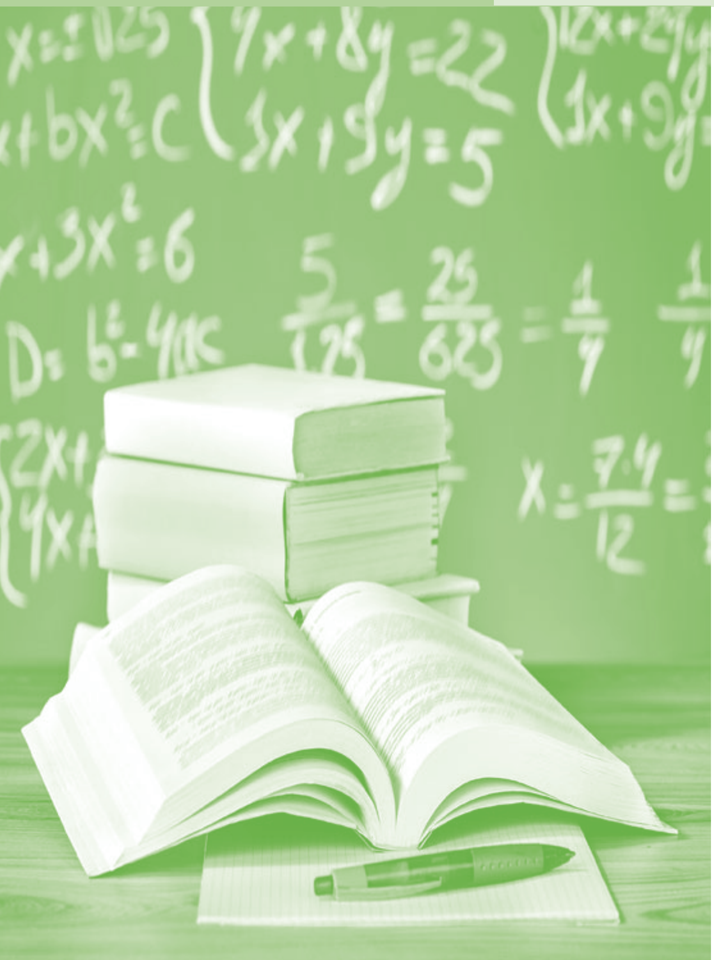
$\therefore \frac{1}{P}=7+4\sqrt{3}$

$P+\frac{1}{P}=\frac{P^2+1}{P}=7-4\sqrt{3}+7+4\sqrt{3}=14$

$\Rightarrow \frac{P^2+1}{7P}=\frac{14}{7}=2.$

Chapter

2 Logarithms



REMEMBER

Before beginning this chapter, you should be able to:

- Explain laws of indices
- Solve exponential equations

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the system of logarithms
- Learn properties and laws of logarithms
- Understand variation of $\log_a x$ with x and learn the signs of $\log_a x$ for different values of x and a
- Find the log of a number using log tables obtain the antilog from a given problem

INTRODUCTION

Please recall that you have earlier learnt about indices. One of the results we learnt is that if $2^x = 2^3$, $x = 3$ and if $4^x = 4^y$, then $x = y$, i.e., if two powers of the same base are equal and the base is not equal to −1, 0 or 1, then the indices are equal.

But when $3^x = 5^2$, just by using the knowledge of indices, we cannot find the numerical value of x. The necessity of the concept of logarithms arises here. Logarithms are useful in long calculations involving multiplication and division.

Definition

The logarithm of any positive number to a given base (a positive number not equal to 1) is the index of the power of the base which is equal to that number. If N and a ($\neq 1$) are any two positive real numbers and for some real number x, $a^x = N$, then x is said to be logarithm of N to the base a. It is written as $\log_a N = x$, i.e., if $a^x = N$, then $x = \log_a N$.

If in a particular relation, all the log expressions are to the same base, we normally do not specify the base.

Examples:

1. $2^3 = 8 \Rightarrow 3 = \log_2 8$
2. $5^4 = 625 \Rightarrow \log_5 625 = 4$

From the definition of logs, we get the following results:

When $a > 0$, $b > 0$ and $b \neq 1$,

1. $\log_a a^n = n$, e.g., $\log_4 4^3 = 3$
2. $a^{\log_a b} = b$, e.g., $2^{\log_2 16} = 16$.

SYSTEM OF LOGARITHMS

Though we can talk of the log of a number to any positive base not equal to 1, there are two systems of logarithms, natural logarithms and common logarithms, which are used most often.

1. **Natural logarithms:** These were discovered by Napier. They are calculated to the base e which is approximately equal to 2.71. These are used in higher mathematics.
2. **Common logarithms:** Logarithms to the base 10 are known as common logarithms. This system was introduced by Briggs, a contemporary of Napier.

For the rest of this chapter, we shall use the short form log rather than logarithm.

Properties

1. Logs are defined only for positive real numbers.
2. Logs are defined only for positive bases different from 1.
3. In $\log_b a$, neither a nor b is negative but the value of $\log_b a$ can be negative.

   Example: As $10^{-2} = 0.01$, $\log_{10} 0.01 = -2$

4. Logs of different numbers to the same base are different, i.e., if $a \neq b$, then $\log_m a \neq \log_m b$. In other words, if $\log_m a = \log_m b$, then $a = b$.

   Example: $\log_{10} 2 \neq \log_{10} 3$

   $\log_{10} 2 = \log_{10} y \Rightarrow y = 2$

5. Logs of the same number to different bases have different values, i.e., if $m \neq n$, then $\log_m a \neq \log_n a$. In other words, if $\log_m a = \log_n a$, then $m = n$.

   Example: $\log_2 16 \neq \log_4 16$

   $\log_2 16 = \log_n 16 \Rightarrow n = 2$

6. Log of 1 to any base is 0.

   Example: $\log_2 1 = 0$ ($\therefore$ $2^0 = 1$)

7. Log of a number to the same base is 1

   Example: $\log_4 4 = 1$.

8. Log of 0 is not defined.

Laws

1. $\log_m(ab) = \log_m a + \log_m b$

   Example: $\log 15 = \log(5 \times 3) = \log 5 + \log 3$

2. $\log_m\left(\dfrac{a}{b}\right) = \log_m a - \log_m b$

   Example: $\log\left(\dfrac{15}{20}\right) = \log 15 - \log 20$

3. $\log a^m = m \log a$

   Example: $\log 36 = \log 6^2 = 2\log 6$

4. $\log_b a \log_c b = \log_c a$ (chain rule)

   Example: $\log_2 4 \log_4 16 = \log_2 16$

5. $\log_b a = \dfrac{\log_c a}{\log_c b}$

   Example: $\log_4 16 = \dfrac{\log_2 16}{\log_2 4}$ (change of base rule)

   In this relation, if we take $a = c$, we get the following result:

$$\log_b a = \frac{1}{\log_a b}.$$

Variation of $\log_a x$ with x

For $1 < a$ and $0 < p < q$, $\log_a p < \log_a q$
For $0 < a < 1$ and $0 < p < q$, $\log_a p > \log_a q$

Example: $\log_{10} 2 < \log_{10} 3$ and $\log_{0.1} 2 > \log_{0.1} 3$

Bases which are greater than 1 are called strong bases and bases which are less than 1 are called weak bases. Therefore, for strong bases log increases with number and for weak bases log decreases with number.

Sign of $\log_a x$ for Different Values of x and a

Strong bases ($a > 1$)

1. If $x > 1$, $\log_a x$ is positive.

   Example: $\log_2 10$, $\log_5 25$ are positive.

2. If $0 < x < 1$, then $\log_a x$ is negative.

   Example: $\log_3 0.2$, $\log_{10} 0.25$ are negative.

   Consider $\log_3 0.2 = \dfrac{\log 0.2}{\log 3} = \dfrac{\log 2 - \log 10}{\log 3}$

   $\log 2 < \log 10$ and $0 < \log 3$ for strong bases.

   As $\dfrac{\log 2 - \log 10}{\log 3} < 0$, $\log_3 0.2 < 0$

Weak bases ($0 < a < 1$)

3. If $x > 1$ and, then $\log_a x$ is negative.
   Example: Consider $\log_{0.4} 2$

   $$= \frac{\log 2}{\log 0.4}$$

   $$= \frac{\log 2}{\log 4 - \log 10}$$

   $\log 4 < \log 10$ (for any base)
   $\log 4 - \log 10 < 0$
   $\dfrac{\log 2}{\log 4 - \log 10} < 0$. (for strong bases)

4. If $0 < x < 1$, then $\log_a x$ is positive. For example, $\log_{0.1} 0.2$, $\log_{0.4} 0.3$ are positive. To summarize, logs of big numbers (>1) to strong bases and small numbers (<1) to weak bases are positive.

EXAMPLE 2.1

If $x^2 + y^2 = 3xy$, then choose the correct answer of $2\log(x - y)$ from the following options:

(a) $\log x - \log y$ **(b)** $\log x + \log y$ **(c)** $\log(xy)$ **(d)** Both (b) and (c)

HINT

Find $(x - y)^2$.

EXAMPLE 2.2

Choose the correct answer from the following option for: $\log_2[\log_4\{\log_3(\log_3 27)\}] =$

(a) 0 **(b)** 1 **(c)** $\log_2 3$ **(d)** undefined

SOLUTION

$\log_2[\log_4\{\log_3(\log_3 27)\}]$
$= \log_2[\log_4\{\log_3(\log_3 3^3)\}]$
$= \log_2[\log_4\{\log_3 3(\log_3 3)\}]$
$= \log_2[\log_4\{\log_3 3(1)\}]$
$= \log_2[\log_4\{\log_3 3\}]$
$= \log_2[\log_4 1] = \log_2 0$, which is not defined.

EXAMPLE 2.3

If $\log 2 = 0.301$, then find the number of digits in 2^{1024} from the following options:

(a) 307 **(b)** 308 **(c)** 309 **(d)** 310

SOLUTION

Let $x = 2^{1024}$
$\Rightarrow \log x = \log 2^{1024}$
$= 1024 \log 2 = 1024(0.301)$
$\Rightarrow \log x = 308.22$
$\Rightarrow$ The characteristic is 308
$\therefore$ The number of digits in 2^{1024} is 309.

To Find the log of a Number to Base 10

Consider the following numbers:
2, 20, 200, 0.2 and 0.02.

We see that $20 = 10(2)$ and $200 = 100(2)$

$\therefore \log 20 = 1 + \log 2$ and $\log 200 = 2 + \log 2$

Similarly, $\log 0.2 = -1 + \log 2$ and $\log 0.02 = -2 + \log 2$

From the tables, we see that $\log 2 = 0.3010$. (Using the tables, this is explained in more detail in later examples.)

$\therefore \log 20 = 1.3010$, $\log 200 = 2.3010$, $\log 0.2 = -1 + 0.3010$ and $\log 0.02 = -2 + 0.3010$.

We note two points:

1. Multiplying or dividing by a power of 10 changes only the integral part of the log, not the fractional part.
2. For numbers less than 1, (for example 0.2) it is more convenient to leave the log value as $-1 + 0.3010$ instead of changing it to -0.6090. We refer to the first form (in which the fraction is positive) as the standard form and the second form as the normal form. Both the forms represent the same number.

   For numbers less than 1, it is more convenient to express the log in the standard form. As the negative sign refers only to the integral part, it is written above the integral part, rather than in front, i.e., $\log 0.2 = \bar{1}.3010$ and not -1.3010.

   The convenience of the standard form will be clear when we learn how to take the antilog, which is explained in more detail later.

   antilog (-0.6090) = antilog $(-1 + 0.3010)$ = antilog $\bar{1}.3010 = 0.2$.

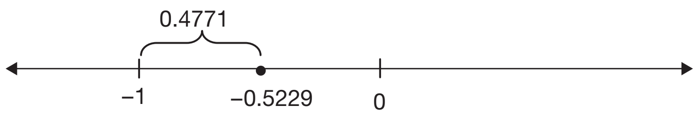
When the logs of numbers are expressed in the standard form, (for numbers greater than 1, the standard form of the log is the same as the normal form), the integral part is called the characteristic and the fractional part (which is always positive) is called the *mantissa*.

EXAMPLE 2.4

Express -0.5229 in the standard form and locate it on the number line.

SOLUTION

$$-0.5229 = -1 + 1 - 0.5229 = \bar{1}.4771$$



The Rule to Obtain the Characteristic of log x

1. If $x > 1$ and there are n digits in x, the characteristic is $n - 1$.
2. If $x < 1$ and there are m zeroes between the decimal point and the first non-zero digit of x, the characteristic is $-m$, more commonly written as $\bar{m}$.

Note $-4 = \bar{4}$ but $-4.01 \neq \bar{4}.01$

To Find the log of a Number from the log Tables

EXAMPLE 2.5

Find the value of $\log 25$, $\log 250$ and $\log 0.025$.

SOLUTION

In the log table, we find the number 25 in the left-hand column. In this row, in the next column (under 0), we find 0.3979. (The decimal point is dropped in other columns)

For the log of all numbers whose significant digits are 25 and this number 0.3979, is the *mantissa*. Prefixing the characteristics we have,

$$\log 25 = 1.3979$$
$$\log 250 = 2.3979$$
$$\log 0.025 = \bar{2}.3979.$$

EXAMPLE 2.6

Find the value of $\log 2.54$, $\log 0.254$ and $\log 25400$.

SOLUTION

In the log table, we locate 25 in the first column. In this row, in the column under 4 we find 0.4048. As done in the earlier example, the same line as before gives the *mantissa* of logarithms of all numbers which begin with 25. From this line we pick out the *mantissa* which is located in the column numbered 4. This gives 4048 as the *mantissa* for all numbers whose significant digits are 254.

$$\log 2.54 = 0.4048$$
$$\log 0.254 = \bar{1}.4048$$
$$\log 25400 = 4.4048.$$

EXAMPLE 2.7

Find the value of $\log 2.546$ and $\log 25460$.

SOLUTION

As found in the above example, we can find the mantissa for the sequence of digits 4048.

Since there are four significant digits in 2546, in the same row where we have found 4048, under column 6 in the mean difference column, we find the number 10, the mantissa of the logarithm of 2546 will be $4048 + 10 = 4058$.

Thus,
$$\log 2.546 = 0.4058$$
$$\log 0.2546 = \bar{1}.4058$$
$$\log 25460 = 4.4058.$$

ANTILOG

As $\log_2 8 = 3$, 8 is the antilogarithm of 3 to the base 2. Antilog b to base m is m^b.

In example 3 above, we saw that, $\log 2.546 = 0.4058$. Therefore, Antilog $0.4058 = 2.546$

To Find the Antilog

EXAMPLE 2.8

To find the antilog of 1.301.

SOLUTION

Step 1: In the antilog table, we find the number 30 in the left hand column. In that row in the column under 1, we find 2000.

Step 2: As the characteristic is 1, we place the decimal after two digits from the left. That is, antilog $1.301 = 20.00$.

If the characteristic was 2, we would place the decimal after three digits from the left. That is, antilog $2.301 = 200.0$.

If the characteristic was 3, we would place the decimal after four digits from the left. That is, antilog $3.301 = 2000$.

EXAMPLE 2.9

To find the antilog of 2.3246.

SOLUTION

We have to locate 0.32 in the left hand column and slide along the horizontal line and pick out the number in the vertical column headed by 4. We see that the number is 2109. The mean difference for 6 in the same line is 3.

The significant digits in the required number are $= 2109 + 3 = 2112$. As the characteristic is 2, the required antilog is 211.2.

EXAMPLE 2.10

Find the value of $\dfrac{5.431 \times 0.061}{12.38 \times 0.041}$ to four significant digits.

SOLUTION

log of a fraction = (log of numerator) − (log of denominator)

log of numerator = $\log 5.431 + \log 0.061 = 0.7349 + \bar{2}.7853 = \bar{1}.5202$

log of denominator = $\log 12.38 + \log 0.041 = 1.0927 + \bar{2}.6128 = \bar{1}.7055$

log of given expression = $\bar{1}.5202 - \bar{1}.7055 = \bar{1}.8147$

antilog of 8147 is 6516 + 11 = 6527

Since the characteristic is $\bar{1}$, the decimal should be kept before the four digits.

$\therefore$ The answer is 0.6527.

EXAMPLE 2.11

If $\log_{10}3 = 0.4771$, and $\log_{10}2 = 0.3010$, Find the value of $\log_{10}48$.

SOLUTION

$\log_{10}48 = \log_{10}16 + \log_{10}3$

$= \log_{10}2^4 + \log_{10}3 = 4\log_{10}2 + \log_{10}3$

$= 4(0.3010) + 0.4771 = 1.6811.$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\log_x A^n =$ ______.
2. Expand $\log_3\left(\dfrac{xy^2}{z^3}\right)$.
3. Can we write $\log_x \dfrac{a}{b}$ as $\dfrac{\log_x a}{\log_x b}$?
4. Express $0.001 = (0.1)^3$ in the logarithmic form.
5. $(5)^{2\log_5 2} =$ ________.
6. $\log_5 2 + \log_5 20 - \log_5 8 =$ ________.
7. The value of $\dfrac{3+\log_{10}(10)^2}{\log_5 5}$ is ________.
8. $\log_x ab = (\log_x a) \times (\log_x b)$. State True or False.
9. If $\log_{10}2 = 0.3010$, then $\log_{10} 2000 =$ ________.
10. Evaluate $3 - \log_{10}100$.
11. Given $3 = \log_2 x + 4\log_2 8$. Then the value of $x =$ ________.
12. If $\log_{10}2 = 0.3010$, then $\log_{10}5 =$ ________.
13. If $x = \log_5 3$ and $y = \log_5 8$, then $\log_5 24$ in terms of x, y is equal to ________.
14. If $\log_{16}25 = k \log_2 5$, then $k =$ ________.
15. If $5\log 3 + \log x = 5\log 6$, then $x =$ ________.
16. If $\log x = \left(\dfrac{\log_a y}{\log_a x}\right)^k$, then $k =$ ________.
17. If $a > 1$ and $m > n$, then which is greater, $\log_a m$ (or) $\log_a n$?
18. If $2\log x + 2\log y = k$ and $xy = 1$, then $k =$ ________.
19. If $\log 198.9 = 2.2987$, then the characteristic of $\log 198.9 =$ and mantissa of $\log 198.9 =$ ________.
20. When $0 < a < 1$ and $m < n$, then which is greater, $\log_a m$ (or) $\log_a n$?
21. Given $\log_{10}x = y$. If the characteristic of y is 10, then the number of digits to the left the decimal point in x is ______.
22. Find the value of $\log_{\sqrt{8}}16$.
23. $\log_y x \times \log_z y \times \log_x z =$ ______.
24. If the characteristics of the logarithm of two numbers $abcd \cdot abef$ and $a \cdot bcdabef$ are x and y respectively, then $x - y =$ ______.
25. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, then find the values of $\log 210$.
26. Given, antilog$(2.375) = x$. Characteristic of $\log x$ is ______.
27. If $\log(21.73) = 1.3371$, then find the values of $\log(2.173)$.
28. If antilog $(0.2156) = 1.643$, then find the values of antilog (1.2156).
29. Without using the logarithm tables find the value of $3\log_3 27$.
30. Find the value of $\log_{0.6}\left(\dfrac{9}{25}\right)$.

Short Answer Type Questions

31. Prove that $\log 5040 = 4\log 2 + 2\log 3 + \log 5 + \log 7$.
32. Find the value of $\log_{2^{-1}}(0.0625)$.
33. Express the following as a single logarithm. $\dfrac{1}{3}\log x - \dfrac{8}{5}\log y + \dfrac{7}{2}\log z$.
34. If $x^2 + y^2 = 25xy$, then prove that $2\log(x + y) = 3\log 3 + \log x + \log y$.
35. If $x^2 + y^2 = z^2$, then prove that $\log_y(z + x) + \log_y(z - x) = 2$.
36. Prove that $\log_2[\log_4\{\log_5(625)^4\}] = 1$.
37. If $\log_{10}2 = 0.3010$, then find the number of digits in $(16)^{10}$.
38. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, then find the value of $\log 75$.
39. If $x^4 + y^4 = 83x^2y^2$, then prove that $\log\left(\dfrac{x^2 - y^2}{9}\right) = \log x + \log y$.
40. Prove that

$$2\log\frac{35}{192} + 2\log\frac{114}{91} + \log 48 + 2\log\left(\frac{13}{19}\right) = \log\left(\frac{75}{64}\right).$$

PRACTICE QUESTIONS

41. Solve for x:

$\log x + \log 5 = 2 + \log 64$

42. If $x^6 - y^6 = z^6$, then prove that

$\log_z(x^2 - y^2) + \log_z(x^2 + y^2 - xy) + \log_z(x^2 + y^2 + xy) = 6$.

43. Using the tables find the value of $\sqrt[4]{(32)^3}$.

44. Using the tables find the value of $\sqrt{(0.12)^3}$.

45. Given $\log 3 = 0.4771$, then the number of digits in 3^{1000} is ______.

Essay Type Questions

46. If $\log_{x+1} 2x - 1 + \log_{2x-1} x + 1 = 2$, find x.

47. If $a = b^{1/3} = c^{1/5} = d^{1/7} = e^{1/9}$, find $\log_a abcde$.

48. Arrange the following in ascending order.

$A = \log_9 6561$ $\qquad B = \log_{\sqrt{5}} 625$

$C = \log_{\sqrt{3}} 243$ $\qquad D = \log_{\sqrt{2}} 256$

49. If $\log_y x - \log_{y^3} x^2 = 9(\log_x y)^2$ and $x = 9y$, find y.

50. $\log\left(\frac{a^2}{b}\right) + \log\left(\frac{a^4}{b^3}\right) + \log\left(\frac{a^6}{b^3}\right) + \cdots + \log\left(\frac{a^{2n}}{b^n}\right) = ?$

CONCEPT APPLICATION

Level 1

1. $\log_y a \times \log_x y =$ ______.

(a) $\log_a y$ (b) $\log_x a$
(c) $\log_y a$ (d) $\log_a x$

2. If $\log x = 123.242$, then the characteristic of $\log x$ is

(a) 0.242 (b) 122
(c) 123 (d) 124

3. Pick up the false statement.

(A) Logarithms are defined only for positive real numbers.
(B) $\log_a N$ is always unique.
(C) The log form of $2^3 = 8$ is $3 = \log_8 2$.
(D) $\log 1 = 0$

(a) B (b) C
(c) D (d) A

4. $\log\left(\frac{169}{9}\right) - 2\log 13 + 2\log 3 = ?$

(a) 1 (b) 0
(c) $\log\left(\frac{13}{3}\right)$ (d) $\log\left(\frac{x}{yz}\right)$

5. $\log_{z^2} x^2 y^2 = ?$

(a) $2(\log x + \log y - \log z)$
(b) $\log x + \log y - \log z$
(c) $\frac{\log x^2 + \log y}{\log z}$
(d) $\frac{\log x + \log y}{\log z}$

6. If $x^3 - y^3 = 3xy(x - y)$, then $\log(x - y)^3 =$ ______.

(a) 0 (b) 1
(c) Undefined (d) None of these

7. $\log(a^3 + b^3) - \log(a + b) - \log(a^2 - ab + b^2) =$ ______.

(a) $a^3 - b^3$
(b) 0
(c) $\log 1$
(d) Both (b) and (c)

8. Which is greatest among the following:

(a) $\log_2 20$ (b) $\log_7 35$
(c) $\log_5 70$ (d) $\log_3 68$

9. $\log(a + b) + \log(a - b) - \log(a^2 - b^2) =$ ______.

(a) 0 (b) 1
(c) $(a^2 - b^2)$ (d) $a^2 + b^2$

PRACTICE QUESTIONS

10. If $x^3 + y^3 = 4xy(x + y)$. Then $\log(x + y)^3 =$ _______.

(a) $\log x + \log y + \log(x + y) - \log 7$

(b) $\log(x) - \log y + \log(x + y) + \log 7$

(c) $\log x + \log y + \log(x - y) + \log 7$

(d) $\log x + \log y + \log(x + y) + \log 7$

11. If $\frac{\log x}{\log y} = \frac{\log 49}{\log 7}$, then the relation between x and y.

(a) $x = \sqrt{y}$

(b) $x = y^3$

(c) $y = x^2$

(d) $x = y^2$

12. $\log(x) - \log(2x - 3) = 1$, then $x =$?

(a) $\frac{30}{19}$ (b) $\frac{20}{19}$

(c) $\frac{19}{30}$ (d) $\frac{19}{20}$

13. If $2\log(x + 4) = \log 16$, then $x =$?

(a) 0, –8 (b) –8

(c) –2 (d) 0

14. The value of x when $\log_x 343 = 3$, is

(a) 7 (b) 8

(c) 3 (d) 27

15. $\log_{16}3 \cdot \log_{17}4 \cdot \log_9 17 =$ _______.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{2}{3}$

16. $\log_2 [\log_2 \{\log_2 (\log_3 81)\}] =$

(a) 1

(b) 0

(c) $\log 3$

(d) Undefined

17. $\log_{11}3 \cdot \log_3 1331 =$ _______.

(a) 3 (b) 11

(c) 121 (d) 9

18. $\log_{121}\left(\frac{\sqrt{14641}}{121}\right) =$ _______.

(a) 11 (b) 121

(c) 0 (d) 1

19. $\frac{\log_3 729 + \log_6 216}{4 + \log_2 16 - 2\log_4 64} =$ _______.

(a) 9 (b) 4

(c) $\frac{9}{2}$ (d) $\frac{1}{2}$

20. If $\log_{x^n} y^m = k\log_x y$, then the value of k is

(a) $\frac{m}{n}$ (b) mn

(c) m^n (d) n^m

21. If $\log_y x = 2$, then ${}_a\log_a{}^{(\log_x y)} =$ _______.

(a) -2 (b) 4

(c) $\frac{1}{2}$ (d) $\frac{-1}{4}$

22. If $p = \log_6 216$ and $q = \log_3 25$ then $p^q =$ _______.

(a) 3 (b) 25

(c) 15 (d) Cannot be determined

23. $2^{(16 - \log_2 1024)} =$ _______.

(a) 16 (b) 32

(d) 64 (d) 8

24. $2^{3\log_2 2} + 3^{2\log_3 2} =$ _______.

(a) 8 (b) 4

(c) 9 (d) 2

25. $\log_{a+b}(a^3 + b^3) - \log_{a+b}(a^2 - ab + b^2) =$ _______.

(a) $\log_{a+b}(a - b)$ (b) 2

(c) $a + b$ (d) 1

26. $4^{\log_{16}25} =$ _______.

(a) 25 (b) 5

(c) 16 (d) 4

27. $\frac{1}{\log_{xy} x} + \frac{1}{\log_{xy} y} =$ _______?

(a) 1 (b) 2

(c) 0 (d) $\frac{1}{\log_{xy}x \times \log_{xy}y}$

28. If $y = \log_{x-3}(x^2 - 6x + 9)$, then find y.

(a) 4 (b) 8

(c) 2 (d) 32

PRACTICE QUESTIONS

Level 2

29. If $\log_{10}2 = 0.3010$, then the number of digits in 16^{12} is

(a) 14 (b) 15

(c) 13 (d) 16

30. If $\log_{64} p^2 = 1\frac{2}{3}$, then $\log_2 \frac{p}{16} =$ _______.

(a) 16 (b) 2

(c) 32 (d) 1

31. $\log_2 \log_2 \log_5 125 =$ _______.

(a) 4 (b) 8

(c) −1 (d) 1

32. If $\log_x y = \frac{\log_a y}{p}$, then the value of p is

(a) $\log_y x$ (b) $\log_x a$

(c) $\log_a x$ (d) $\log_a y$

33. $\log\left[\frac{\sqrt[3]{x^2} \times y}{\sqrt[5]{z^2}}\right] =$ _______

(a) $\log x^{2/3} - \log z^{2/5} + \log y$

(b) $\log x^{3/2} + \log y - \log z^{5/2}$

(c) $\log x^{2/3} - \log y + \log z^{2/5}$

(d) None of these

34. If $\log[4 - 5\log_{32}(x + 3)] = 0$, find x.

(a) 32 (b) 8

(c) 3 (d) 5

35. If $x = \log_3 \log_2 \log_2 256$, then $2^{\log_4 2^{2x}} =$ _______.

(a) 4 (b) 8

(c) 2 (d) 1

36. If $\log_a\left(\frac{13^2}{\sqrt{2^3} \times 5}\right) = 2\log_a 13 - \log_a 5 - x$ then

(a) $a^x = 2^{3/2}$ (b) $x^a = 2^{3/2}$

(c) $a^x = 2^{2/3}$ (d) $x^a = 2^{2/3}$

37. If $\log 81 - \log 3 = \log a$, then $4_9^{\log a} =$ _______.

(1) 4 (2) 16

(3) 2 (4) 8

38. If $a^{\log_a n} = 3$ then $a^{2\log b} - b^{\log a} =$ _______.

(a) 6 (b) 9

(c) 3 (d) Cannot be determined

39. If $\log_3(x - 5) + \log_3(x + 2) = \log_3 8$, then $x =$ _______.

(a) −3 (b) 6

(c) 6, −3 (d) 3, −6

40. If $\log(x + y) = \log x + \log y$, then $x =$ _______.

(a) $\frac{-y}{1+y}$ (b) $\frac{-y}{1-y}$

(c) 1 (d) $\frac{y}{1+y}$

41. If $2^{\log 5} \cdot 5^{\log 2} = 2^{\log x}$, then $\log_5 \sqrt[3]{x^2} =$ _______.

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) 3

42. If $3^{\log x} + x^{\log 3} = 54$, find $\log x$.

(a) 3 (b) 2

(c) 4 (d) Cannot be determined

43. If $\log_{10}x - \log_{10}y = 1$ and $x + y = 11$, then $x =$ _______.

(a) 10 (b) 1

(c) 11 (d) 2

44. If $\log_{49}3 \times \log_9 7 \times \log_2 8 = x$, then find the value of $\frac{4x}{3}$.

(a) 3 (b) 7

(c) 8 (d) 1

45. The value of $\log_{a-b}(a^3 - b^3) - \log_{a-b}(a^2 + ab + b^2)$ is _______. $(a > b)$

(a) 0 (b) 1

(3) 3 (c) Undefined

46. If $\log_3 2 = x$, then the value of $\frac{\log_{10} 72}{\log_{10} 24}$ is

(a) $\frac{1+x}{1-x}$ (b) $\frac{2+3x}{1-3x}$

(c) $\frac{2-3x}{2+3x}$ (d) $\frac{3x+1}{3x+2}$

PRACTICE QUESTIONS

47. $\log\left(\frac{1}{2}\right)+\log\left(\frac{2}{3}\right)+\log\left(\frac{3}{4}\right)+\cdots+\log\left(\frac{99}{100}\right)$ = ______.

(a) −2 (b) −1

(c) 0 (d) 2

48. If $x^2 - y^2 = 1$, $(x > y)$, then find the value of $\log_{(x-y)}(x+y)$.

(a) −2 (b) 2

(c) −1 (d) 1

49. If $3^{\log_3 5} + 5^{\log_x 3} = 8$, then find the value of x.

(a) 3 (b) 5

(c) 4 (d) 8

50. $\log_2 1 \cdot \log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \ldots \log_{200} 199$ = ______.

(a) ∞ (b) 0

(c) 1 (d) Cannot be determined

51. If $x^2 + y^2 - 3xy = 0$ and $x > y$, then find the value of $\log_{xy}(x - y)$.

(a) $\frac{1}{4}$ (b) 4

(c) $\frac{1}{2}$ (d) 2

52. If $\log 3 = 0.4771$, then find the number of digits in 3^{100}.

(a) 47 (b) 48

(c) 49 (d) 50

Level 3

53. If $\log_5 x - \log_5 y = \log_5 4 + \log_5 2$ and $x - y = 7$, then x = ______.

(a) 1 (b) 8

(c) 7 (d) 6

54. If $\log_2\left[-1+\sqrt{x^2-14x+49}\right]=4$, then x = ______.

(a) 24 (b) −10

(c) 24, −10 (d) 10

55. If $\frac{\log p}{2}=\frac{\log q}{4}=\frac{\log r}{8}=k$ and $pqr=100$, then k = ______.

(a) 14 (b) $\frac{1}{6}$

(c) $\frac{1}{7}$ (d) 2

56. If $\log 2 = 0.3010$, and $\log 3 = 0.4771$ then $\log 150$ = ______.

(a) 2.1761 (b) 2.8751

(c) 2.5762 (d) 2.6126

57. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10}\left(\frac{2^3 \times 3^2}{5^2}\right)$ is

(a) 0.4592. (b) 0.5492.

(c) 0.4529. (d) 0.5429.

58. The value of $\frac{1}{1+\log_{ab} c}+\frac{1}{1+\log_{ac} b}+\frac{1}{1+\log_{bc} a}$ equals

(a) 2 (b) 0

(c) 1 (d) $\log abc$

59. If $x^2 + y^2 = z^2$, then $\frac{1}{\log_{(z+x)} y}+\frac{1}{\log_{(z-x)} y}$ = ______.

(a) 4 (b) 3

(c) 2 (d) 1

60. If $\log 2 = 0.301$, then find the number of digits in 2^{1024}.

(a) 307 (b) 308

(c) 309 (d) 310

61. If $x^2 - y^2 = 1$, $(x > y)$, then find the value of $\log_{(x-y)}(x + y)$.

(a) −2 (b) 2

(c) −1 (d) 1

62. If $x^2 + y^2 - 3xy = 0$ and $x > y$, then find the value of $\log_{xy}(x - y)$.

(a) $\frac{1}{4}$ (b) 4

(c) $\frac{1}{2}$ (d) 2

63. If $2^{\log_3 9} + 25^{\log_9 3} = 8^{\log_x 9}$, then $x =$ ______.

(a) 9 (b) 8

(c) 3 (d) 2

64. If $\log_a x = m$ and $\log_b x = n$, then $\log_{\left(\frac{a}{b}\right)} x =$ ______.

(a) $\dfrac{m}{m-n}$ (b) $\dfrac{mn}{m-n}$

(c) $\dfrac{n}{m-n}$ (d) $\dfrac{mn}{n-m}$

65. $\dfrac{\log_5 6}{\log_5 2 + 1} =$

(a) $\log_2 6$ (b) $\log_2 5$

(c) $\log_{10} 6$ (d) $\log_{10} 30$

66. If $x = \log_3 27$ and $y = \log_9 27$, then $\dfrac{1}{x} + \dfrac{1}{y} =$ ______.

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{9}$

(c) 3 (d) 1

67. If $\log_6 x + 2\log_{36} x + 3\log_{216} x = 9$, then $x =$ ______.

(a) 6 (b) 36

(c) 216 (d) None of these

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $n\log_x A$
2. $\log_3 x + 2\log_3 y - 3\log_3 z$
3. No
4. $\log_{0.1}(0.001) = 3$
5. 4
6. 1
7. 5
8. False
9. 3.3010
10. 1
11. $\dfrac{1}{512}$
12. 0.6990
13. $x + y$
14. 1/2
15. 32
16. −1
17. $\log_a m$
18. 0
19. 2; 0.2987
20. $\log_a m$
21. 11
22. $\dfrac{8}{3}$
23. 1
24. 3
25. 2.3222
26. 2
27. 0.3371
28. 16.43
29. 27
30. 2

Short Answer Type Questions

32. 4
33. $\log\left(\dfrac{x^{1/3}z^{7/2}}{y^{8/5}}\right)$
36. 1
37. 13
38. 1.8751
41. $x = 1280$
43. 13.45
44. 0.04158
45. 478

Essay Type Questions

46. 2
47. 25
48. ABCD
49. 3
50. $\log\dfrac{a^{n(n+1)}}{b^{\frac{n(n+1)}{2}}}$

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (d)
11. (d)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (c)		

Level 2

29. (b)	**30.** (d)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (d)	**38.** (a)
39. (b)	**40.** (b)	**41.** (a)	**42.** (a)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (c)
49. (b)	**50.** (b)	**51.** (c)	**52.** (b)						

Level 3

53. (b)	**54.** (c)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (c)	**61.** (c)	**62.** (c)
63. (b)	**64.** (d)	**65.** (c)	**66.** (d)	**67.** (c)					

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Apply laws of logarithm.
2. Recall the definition of characteristic.
3. Verify all the options.
4. Apply laws of logarithm.
5. Apply laws of logarithm.
6. (i) Apply log on both the sides
   (ii) $x^3 - y^3 = 3xy(x - y) \Rightarrow (x - y)^3 = 0$.
7. (i) $\log x - \log y - \log z = \log\left(\dfrac{x}{yx}\right)$
   (ii) $\log M - \log N = \log\left(\dfrac{M}{N}\right)$ and $\log M + \log N = \log MN$.
8. (i) $\log_2 16 < \log_2 20 < \log_2 32$
   (ii) Given four options are to be compared.
   (iii) For example $\log_2 16 < \log_2 20 < \log_2 32 \Rightarrow 4 < \log_2 20 < 5$.
   (iv) $7 < 35 < 7^2$; $5^2 < 70 < 5^3$; $3^3 < 68 < 3^4$ apply logarithms to suitable bases and evaluate $\log_7 15$, $\log_5 70$ and $\log_3 68$.
9. (i) Use the identity $\log m + \log n - \log p = \log mn/p$
   (ii) Use $\log x + \log y = \log xy$ and $\log p - \log q = \log(p/q)$.
10. Find $(x + y)^3$.
11. (i) Use the identity $\dfrac{\log a}{\log b} = \log_b a$
    (ii) $\dfrac{\log x}{\log y} = 2 \Rightarrow \log_y x = 2$
    (iii) $\log_b a = n \Rightarrow a = b^n$.
12. (i) Use $\log m - \log n = \log \dfrac{m}{n}$
    (ii) Use $\log m - \log n = \log \dfrac{m}{n}$ and simplify LHS.
    (iii) Write 1 as $\log_{10} 10$ and proceed.
13. (i) Use $\log m = \log n \Rightarrow m = n$.
    (ii) Remove logarithms to set $(x + 4)^2 = 16$, then find x.
14. (i) Use $\log m = \log n \Rightarrow m = n$.
    (ii) Use, $\log_b a = N \Rightarrow a = b^N$.
15. Apply laws of logarithms.
16. (i) $\log_b a^m = m\log_b a$ and $\log_a a = 1$
    (ii) Express 81 as 3^4 and then use $\log a^m = m \log a$ and proceed until all the brackets are removed.
17. Apply laws of logarithms.
18. Find $\sqrt{14641}$.
19. Simplify the expression by applying laws of logarithms.
20. Apply laws of logarithm.
21. Apply laws of logarithm.
22. Find p and q.
23. Write 1024 as a power of 2.
24. Apply laws of logarithm.
25. Apply laws of logarithm.
26. (i) Recall the laws of logarithm.
    (ii) Use $a^{\log_a N} = N$ after simplifying the given term.
27. (i) Simplify and then use laws of logarithm.
    (ii) Take LCM and simplify.
    (iii) Use $\log_a x + \log_a y = \log_a xy$.
29. (i) Use $\log_a a = 1$ to find y and then substitute y value.
    (ii) Write $x^2 - 6x + 9$ as $(x - 3)^2$ and use $\log_a a = 1$.

Level 2

29. Find the value of 16^{12} by applying logarithm.

30. Find p.

31. Apply laws of logarithm.

32. (i) Recall the laws of logarithm.

(ii) $p = \dfrac{\log a^y}{\log x^y}$. Use $\log p^q = q \log p$.

33. (i) Use the identities $\log mn = \log m + \log n$, $\log \dfrac{m}{n} = \log m - \log n$ and $\log x^n = n \log x$

(ii) $\log\left(\dfrac{\sqrt[3]{x^2}\cdot y}{\sqrt[5]{z^2}}\right)\log\left(\left(\dfrac{x^{2/3}\cdot y}{z^{2/5}}\right)\right)$

(iii) Use $\log\left(\dfrac{a}{b}\right) = \log a - \log b$ and $\log ab = \log a + \log b$ simplify.

34. (i) $\log x = 0 \Rightarrow x = 1$

(ii) $\log x = 0 \Rightarrow x = 1$

(iii) Use $\log_b a = \dfrac{1}{n}\log_b a$

(iv) $x = a \log n \Rightarrow a^x = n$.

35. Find x.

36. Apply laws of logarithm.

37. Find a.

38. Substitute the value of $a^{\log b}$.

39. Apply laws of logarithm.

40. (i) $\log(a) = \log(b) \Rightarrow a = b$

(ii) $\log(x + y) = \log xy$

(iii) $\Rightarrow x + y = xy$.

41. (i) $a^{\log b} = b^{\log a}$

(ii) $5^{\log 2} = 2^{\log 5}$

(iii) $a^m = a^n \Rightarrow m = n$.

42. (i) $a^{\log b} = b^{\log a}$

(ii) $x^m = x^n \Rightarrow m = n$.

43. Apply laws of logarithm and solve for x.

44. $\log_{49}3 \times \log_9 7 \times \log_2 8 = x$

$$\frac{\log 3}{2\log 7} \times \frac{\log 7}{2\log 3} \times \frac{3\log 2}{\log 2} = x$$

$$\frac{3}{4} = x \Rightarrow \frac{4x}{3} = \frac{4}{3} \times \frac{3}{4} = 1.$$

45. $\log_{a-b}(a^3 - b^3) - \log_{a-b} a^2 + ab + b^2$

$$= \log_{a-b}\left(\frac{a^3 - b^3}{a^2 + ab + b^2}\right)$$

$$= \log_{a-b}\frac{(a-b)(a^2 + ab + b^2)}{(a^2 + ab + b^2)}$$

$$= \log_{a-b}(a - b) = 1.$$

46. Given, $\log_3 2 = x$

$$\frac{\log_{10} 72}{\log_{10} 24} = \log_{24} 72 = \log_{24}(24 \times 3)$$

$$= \log_{24} 24 + \log_{24} 3 = 1 + \frac{\log 3}{\log 24}$$

$$= 1 + \frac{\log_3 3}{\log_3 24}$$

$$= 1 + \frac{1}{\log_3(3 \times 8)}$$

$$= 1 + \frac{1}{\log_3 3 + \log_3 8}$$

$$= 1 + \frac{1}{1 + \log_3 2^3}$$

$$= 1 + \frac{1}{1 + 3\log_3 2}$$

$$= \frac{1 + 3\log_3 2 + 1}{1 + 3\log_3 2} = \frac{2 + 3x}{1 + 3x}.$$

47. $\log\left(\dfrac{1}{2}\right) + \log\left(\dfrac{2}{3}\right) + \cdots + \log\left(\dfrac{99}{100}\right)$

$$= \log 1 - \log 2 + \log 2 - \log 3 + \cdots + \log 99 - \log 100$$

$$= \log 1 - \log 100 = 0 - \log 10^2$$

$$= -2\log 10 = -2.$$

48. Given $x^2 - y^2 = 1$

Applying log on both the sides, we get

$(x^2 - y^2) = \log 1$

$\Rightarrow \log[(x + y)(x - y)] = 0$

$\Rightarrow \log(x+y)+\log(x-y)=0$

$\Rightarrow \log(x+y)=-\log(x-y)$

$\frac{\log(x+y)}{\log(x-y)}=-1$

$\Rightarrow \log_{(x-y)}(x+y)=-1.$

49. $3^{\log_3 5}+5^{\log_x 3}=8$

$\Rightarrow 5+5^{\log_x 3}=8$

$\Rightarrow 5^{\log_x 3}=3$

$3^{\log_x 5}=3$

$\Rightarrow \log_x 5=1$

$\Rightarrow x=5.$

50. As $\log_2 1=0$, $\log_2 1\cdot\log_3 2\cdot\log_4 3\ldots\log_{199}200=0$

51. Given $x^2+y^2-3xy=0$

$x^2+y^2-2xy=xy$

Applying log on both sides

$\log(x-y)^2=\log xy$

$2\log(x-y)=\log xy$

$\Rightarrow \frac{\log(x-y)}{\log xy}=\frac{1}{2}$

$\Rightarrow \log_{xy}(x-y)=\frac{1}{2}.$

52. Let $x=3^{100}$

$\Rightarrow \log x=\log 3^{100}$

$\Rightarrow \log x=100\log 3$

$\Rightarrow \log x=100\ (0.4771)$

$\Rightarrow \log x=47.71$

The characteristic is 47

The number of digits in 3^{100} is 48.

Level 3

53. Apply laws of logarithm.

54. (i) $\log_a x=b\Rightarrow a^b=x.$

(ii) Write $\sqrt{x^2-14x+49}=16+1$

$=17$ and solve for x.

(iii) Then verify for what values of x, $\log f(x)$ is defined.

55. (i) Find the value of p, q, r then find pqr.

(ii) Express p, q and r in terms of k.

(iii) Substitute the above values in $pqr=100$ and find k.

56. (i) Express as product of powers of 2 and 3 then use the values of $\log 2$ and $\log 3$

(ii) $\log 150=2\log 5+\log 2+\log 3.$

(iii) Also, $\log 5=\log 10-\log 2.$

57. (i) Apply laws of logarithm.

(ii) Use $\log\left(\frac{a}{b}\right)=\log a-\log b$ and $\log a^m=m\log a.$

58. $\frac{1}{1+\log_{ab}c}=\frac{1}{\log_{ab}ab+\log_{ab}c}=\frac{1}{\log_{ab}abc}$

$=\log_{abc}ab\ \frac{1}{1+\log_{ac}b}=\frac{1}{\log_{ac}ac+\log_{ac}b}$

$=\frac{1}{\log_{ac}abc}=\log_{abc}ac\ \frac{1}{1+\log_{bc}a}+$

$\frac{1}{\log_{bc}bc+\log_{bc}a}=\frac{1}{\log_{bc}abc}\log_{abc}bc$

Hence the value of the required expression

$=\log_{abc}ab+\log_{abc}ac+\log_{abc}bc$

$=\log_{abc}[(ab)(ac)(bc)]=\log_{abc}(abc)^2=2.$

59. Given that, $x^2+y^2=z^2$

$\Rightarrow z^2-x^2=y^2$ (1)

$\therefore \frac{1}{\log_{z+x}y}+\frac{1}{\log_{(z-x)}y}$

$=\log_y(z+x)+\log_y(z-x)$

$=\log_y(z^2-x^2)=\log_y y^2$ {from (1)}

$=2.$

60. Let $x=2^{1024}$

$\Rightarrow \log x=\log 2^{1024}$

$=1024\log 2=1024(0.301)$

$\Rightarrow \log x=308.22$

$\Rightarrow$ The characteristic is 308

$\therefore$ The number of digits in 2^{1024} is 309.

61. Given $x^2 - y^2 = 1$

Applying log on both the sides, we get $\log(x^2 - y^2) = \log 1$

$\Rightarrow \log[(x+y)(x-y)] = 0$

$\Rightarrow \log(x+y) + \log(x-y) = 0$

$\Rightarrow \log(x+y) = -\log(x-y)$

$$\frac{\log(x+y)}{\log(x-y)} = -1$$

$\Rightarrow \log_{(x-y)}(x+y) = 1.$

62. Given, $x^2 + y^2 - 3xy = 0$

$x^2 + y^2 - 2xy = xy$

Applying log on both sides,

$\log(x-y)^2 = \log xy$

$2\log(x-y) = \log xy$

$$\Rightarrow \frac{\log(x-y)}{\log xy} = \frac{1}{2}$$

$$\Rightarrow \log_{xy}(x-y) = \frac{1}{2}.$$

63. $2^{\log_3 9} + 25^{\log_9 3} = 8^{\log_x 9}$

$\Rightarrow 2^{\log_3 3^2} + 25^{\log_{3^2} 3} = 8^{\log_x 9}$

$\Rightarrow 2^2 + 25^{1/2} = 8^{\log_x 9}$

$\Rightarrow 9 = 8^{\log_x 9}$

$\Rightarrow \log_x 9 = \log_8 9$

$\Rightarrow x = 8.$

64. $\log_a x = m$, $\log_b x = n$

$$\log_{\left(\frac{a}{b}\right)} x = \frac{\log x}{\log\left(\frac{a}{b}\right)} = \frac{\log_x x}{\log_x\left(\frac{a}{b}\right)}$$

$$= \frac{1}{\log_x a - \log_x b}$$

$$= \frac{1}{\frac{1}{\log_x a} - \frac{1}{\log_b x}}$$

$$= \frac{1}{\frac{1}{m} - \frac{1}{n}} = \frac{1}{\left(\frac{n-m}{mn}\right)} = \frac{mn}{n-m}.$$

65. $$\frac{\log_5 6}{\log_5 2 + 1} = \frac{\log_5 6}{\log_5 2 + \log_5 5} = \frac{\log_5 6}{\log_5(2\times 5)}$$

$$= \frac{\log_5 6}{\log_5 10}\log_{10} 6.$$

66. $x = \log_3 27$

$\Rightarrow x = \log_3 3^3$

$\Rightarrow x = 3$

$y = \log_9 27$

$y = \log_{3^2} 3^3$

$y = \frac{3}{2}$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{3} + \frac{1}{\left(\frac{3}{2}\right)} = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 1.$$

67. $\log_6 x + 2\log_{36} x + 3\log_{216} x = 9$

$\Rightarrow \log_6 x + \log_6 x + \log_6 x = 9$

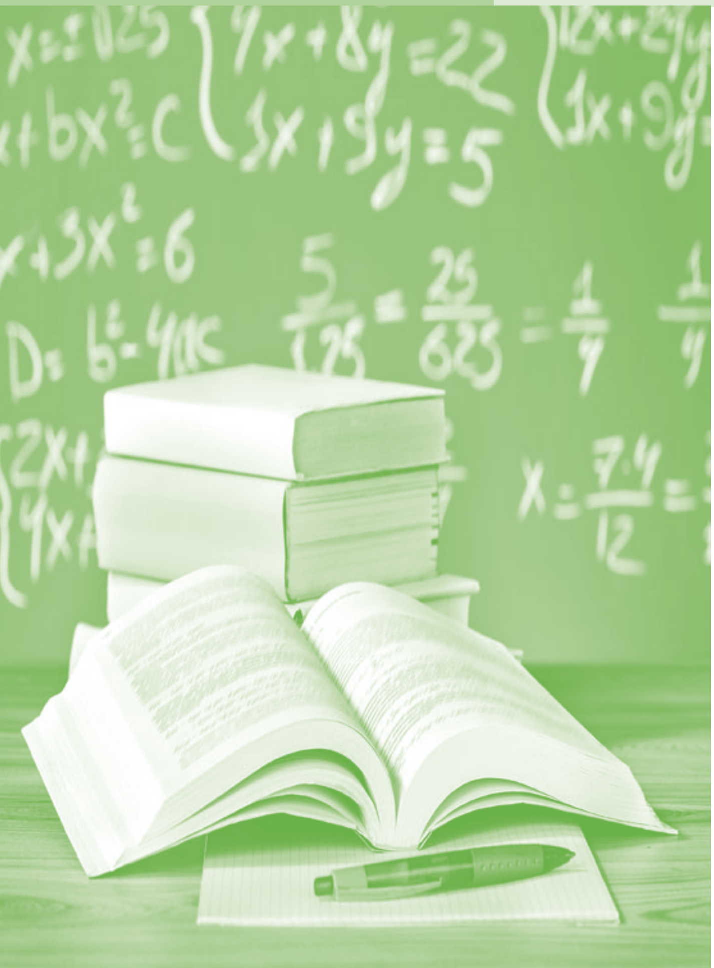
$\Rightarrow 3\log_6 x = 9 \Rightarrow \log_6 x = 3$

$\Rightarrow x = 6^3$

$\therefore x = 216.$

Chapter 3

Polynomials and Square Roots of Algebraic Expressions



REMEMBER

Before beginning this chapter, you should be able to:

- Apply basic operations on polynomials
- Solve basic factorization of polynomials

KEY IDEAS

After completing this chapter, you should be able to:

- State the types of polynomials and operations on polynomials
- Find the factorization of polynomials and obtain HCF and LCM of polynomials
- Understand different methods for finding square roots of algebraic expressions
- Learn about homogeneous, symmetric and cyclic expressions

INTRODUCTION

Before learning the meaning and scope of polynomials, the terms like, constants, variables, algebraic expressions etc., have to be understood.

Constant

A number having a fixed numerical value is called a constant.

Example: 7, $\frac{1}{2}$, 4.7, 16.5, etc.

Variable

A number which can take various numerical values is known as variable.

Example: x, y, z, a, b, c, etc.

A variable raised to any non-zero real number is also a variable.

Example: x^5, $y^{10/3}$, $z^{0.9}$, etc.

A number which is the product of a constant and a variable is also a variable.

Example: $8x^3$, $-7x^5$, $4x^{10}$, etc.

A combination of two or more variables separated by a (+) sign or a (–) sign is also a variable.
Example: $x^2 - y^3 + z$, $x^3 - y^3$, etc.

Algebraic Expression

A combination of constants and variables connected by +, –, × and ÷ signs is known as an algebraic expression.
Example: $8x + 7$, $11x^2 - 13x$, $5x^5 + 8x^2y$, etc.

Terms

The parts of an algebraic expression separated by + or – signs are called the terms of the expression.
Example: In the expression $3x + 4y - 7$, we call $3x$, $4y$ and -7 as terms.

Coefficient of a Term

Consider the term $8x^2$. In this case, 8 is called the numerical coefficient and x^2 is said to be the literal coefficient.

In case of $9xy$, we have the numerical coefficient as 9 and the literal coefficient as xy.

Like Terms

Terms having the same literal coefficients are called like terms.

Examples:

1. $15x^2$, $-19x^2$ and $35x^2$ are all like terms.
2. $8x^2y$, $5x^2y$ and $-7x^2y$ are all like terms.

Unlike Terms

Terms having different literal coefficients are called unlike terms.

Example: $5x^2$, $-10x$ and $15x^3$ are unlike terms.

POLYNOMIAL

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

Example: $5x^2 - 8x + 7$, $3x^3 + 5x^2 - 9$, $3y^2 - 5y + z$, etc.

The expression $3x^5 - 8x + \frac{4}{x} + 11\,x^{5/2}$ is not a polynomial. Since the exponents of x are negative integers and fractions.

A polynomial with one variable is known as a polynomial in that variable.

Example: $5x^4 + 7x^3 + 3x - 9$ is a polynomial in the variable x.

$3y^3 + y^2 + y$ is a polynomial in the variable y.

$4x^2y^2 + 3xy^2 - 7xy$ is a polynomial in variables x and y.

Degree of a Polynomial in One Variable

The highest index of the variable in a polynomial of one variable is called the degree of the polynomial.

Examples:

1. $11x^3 - 7x^2 + 5x + 2$ is a polynomial of degree 3.
2. $15x^6 - 8x + 7$ is a polynomial of degree 6.

Types of Polynomials with Respect to Degree

1. **Linear polynomial:** A polynomial of degree one is called a linear polynomial.

   Example: $11x - 5$, $10y + 7$ and $13z + 4$ are polynomials of degree one and hence they can be called as linear polynomials.

2. **Quadratic polynomial:** A polynomial of degree two is called a quadratic polynomial.

   Example: $5x^2 - 8x + 3$ and $13y^2 - 8y + 3$ are polynomials of degree two and hence can be called as quadratic polynomials.

3. **Cubic polynomial:** A polynomial of degree three is called a cubic polynomial.

   Example: $5x^3 + 6x^2 + 7x + 8$ and $4y^3 - 9y^2 + 3$ are polynomials of degree three and hence can be called as cubic polynomials.

4. **Biquadratic polynomial:** A polynomial of degree four is called a biquadratic polynomial.

   Example: $3x^4 - x^3 + 7x^2 - 2x + 1$ and $5x^4 - 2x + 7$ are polynomials of degree four and hence can be called as biquadratic polynomials.

5. **Constant polynomial:** A polynomial having only one term which is a constant is called a constant polynomial. Degree of a constant polynomial is 0.

   Example: 10, −11 are constant polynomials.

Types of Polynomials with Respect to Number of Terms

1. **Monomial:** An expression containing only one term is called a monomial.

   Example: $8x$, $-11x^2y$, $-15x^2y^3z^2$, etc.

2. **Binomial:** An expression containing two terms is called a binomial.
   Example: $3x - 8y$, $4xy - 5x$, $9x + 5x^2$, etc.

3. **Trinomial:** An expression containing three terms is called a trinomial.
   Example: $5x - 2y + 3z$, $x^2 + 2xy - 5z$, etc.

Addition of Polynomials

The sum of two or more polynomials can be obtained by arranging the terms and then adding the like terms.

EXAMPLE 3.1

Add $7x^2 - 8x + 5$, $3x^2 - 8x + 5$ and $-6x^2 + 15x - 5$.

SOLUTION

$$\begin{array}{r}
7x^2 - 8x + 5 \\
3x^2 - 8x + 5 \\
-6x^2 + 15x - 5 \\
\hline
4x^2 - x + 5 \\
\hline
\end{array}$$

$\therefore$ The required sum is $4x^2 - x + 5$.

Subtraction of Polynomials

The difference of two polynomials can be obtained by arranging the terms and subtracting the like terms.

EXAMPLE 3.2

Subtract $11x^3 - 7x^2 + 10x$ from $16x^3 + 4x^2 - 11x$.

SOLUTION

$$\begin{array}{r}
16x^3 + 4x^2 - 11x \\
11x^3 - 7x^2 + 10x \\
- \quad + \quad - \\
\hline
5x^3 + 11x^2 - 21x \\
\hline
\end{array}$$

$\therefore$ The required difference is $5x^3 + 11x^2 - 21x$.

Multiplication of Two Polynomials

The result of multiplication of two polynomials is obtained by multiplying each term of the polynomial by each term of the other polynomial and then taking the algebraic sum of these products.

EXAMPLE 3.3

Multiply $(5x^2 - 8x + 7)$ with $(2x - 5)$.

SOLUTION

$$\begin{array}{l} 5x^2 - 8x + 7 \\ 2x - 5 \\ \hline 10x^3 - 16x^2 + 14x \\ \quad\;\; - 25x^2 + 40x - 35 \\ \hline 10x^3 - 41x^2 + 54x - 35 \\ \hline \end{array}$$

$\therefore$ The required product is $10x^3 - 41x^2 + 54x - 35$.

This is true for all real values of x, such equations are called algebraic identities.

Division of a Polynomial by a Monomial

To divide a polynomial by a monomial, we need to divide each term of the polynomial by the monomial.

EXAMPLE 3.4

Divide $18x^4 - 15x^3 + 24x^2 + 9x$ by $3x$.

SOLUTION

$$\frac{18x^4 - 15x^3 + 24x^2 + 9x}{3x}$$

$$= \frac{18x^4}{3x} - \frac{15x^3}{3x} + \frac{24x^2}{3x} + \frac{9x}{3x}$$

$$= 6x^3 - 5x^2 + 8x + 3$$

$\therefore$ The required result is $6x^3 - 5x^2 + 8x + 3$.

Division of a Polynomial by a Polynomial

Factor Method

In this method, we factorize the polynomial to be divided so that one or more of the factors is equal to the polynomial by which we wish to divide.

EXAMPLE 3.5

Divide $4x^2 + 7x - 15$ by $x + 3$.

SOLUTION

$$4x^2 + 7x - 15 = 4x^2 + 12x - 5x - 15$$

$$4x(x + 3) - 5(x + 3) = (4x - 5)(x + 3)$$

$$\therefore \frac{4x^2 + 7x - 15}{x + 3} = \frac{(4x - 5)(x + 3)}{x + 3} = 4x - 5.$$

Note The factor method for division of polynomials is used only when the remainder is zero.

Long Division Method

Step 1: First arrange the terms of the dividend and the divisor in the descending order of their degrees.

Step 2: Now the first term of the quotient is obtained by dividing the first term of the dividend by the first term of the divisor.

Step 3: Then multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

Step 4: Consider the remainder as new dividend and proceed as before.

Step 5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

EXAMPLE 3.6

Divide $2x^3 + 9x^2 + 4x - 15$ by $2x + 5$.

SOLUTION

$$
\begin{array}{l}
2x + 5\ )\ 2x^3 + 9x^2 + 4x - 15\ (x^2 + 2x - 3 \\
\qquad\qquad 2x^3 + 5x^2 \\
\qquad\qquad \underline{(-)\quad (-)\qquad\qquad} \\
\qquad\qquad\qquad 4x^2 + 4x \\
\qquad\qquad\qquad 4x^2 + 10x \\
\qquad\qquad\qquad \underline{(-)\quad (-)\qquad\qquad} \\
\qquad\qquad\qquad\qquad -6x - 15 \\
\qquad\qquad\qquad\qquad -6x - 15 \\
\qquad\qquad\qquad\qquad \underline{(+)\quad (+)\qquad} \\
\qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

$\therefore$ $(2x^3 + 9x^2 + 4x - 15) \div (2x + 5) = x^2 + 2x - 3$.

Horner's Method of Synthetic Division

EXAMPLE 3.7

Divide $27x^3 - 81x^2 + 45x + 23$ by $(x - 2)$.

SOLUTION

$$
\begin{array}{c|r|rrrl}
 & 27 & -81 & 45 & 23 & \\
x = 2 & 0 & 54 & -54 & -18 & \\
\hline
 & 27 & -27 & -9 & 5 & \text{remainder}
\end{array}
$$

coefficients of quotient

Step 1: We first write the coefficients of the dividend arranging them in descending powers of x with zero as the coefficient for missing power of x.

Step 2: To divide by $x - 2$, we write $x = 2$.

Step 3: Bring down the leading coefficient of the dividend, multiply by 2 and add the second coefficient which results -27.

Step 4: Now multiply -27 by 2 and add this to the third coefficient to get -9.

Step 5: This process is continued until the final sum.

Step 6: Thus, we get the quotient as $27x^2 - 27x - 9$ and the remainder as 5.

Factorization

Factorization is expressing a given polynomial as a product of two or more polynomials.

Example: $x^3 - 15x^2 = x^2(x - 15)$

$\Rightarrow x^2$ and $x - 15$ are the factors of $x^3 - 15x^2$.

1. Factorization of polynomials of the form $x^2 - y^2$.
   $x^2 - y^2 = (x + y)(x - y)$
   $\Rightarrow x + y$ and $x - y$ are the factors of $x^2 - y^2$.

EXAMPLE 3.8

Factorize $81x^2 - 225y^2$

SOLUTION

Let $a = 9x$ and $b = 15y$

$a^2 - b^2 = (a + b)(a - b)$

$\therefore\ 81x^2 - 225y^2$

$= (9x)^2 - (15y)^2$

$= (9x + 15y)(9x - 15y)$

$\therefore\ 9x + 15y$ and $9x - 15y$ are the factors of $81x^2 - 225y^2$.

2. Factorization of polynomials by grouping of terms: In this method we group the terms of the polynomials in such a way that we get a common factor out of them.

EXAMPLE 3.9

(a) Factorize $a^2 - (b - 8)a - 8b$

SOLUTION

$\Rightarrow a^2 - (b - 8)a - 8b$

$= a^2 - ab + 8a - 8b$

$= a(a - b) + 8(a - b)$

$= (a + 8)(a - b)$

$\therefore\ a^2 - (b - 8)a - 8b = (a + 8)(a - b).$

(b) Factorize $4x^3 - 10y^3 - 8x^2y + 5xy^2$

SOLUTION

$4x^3 - 8x^2y + 5xy^2 - 10y^3$

$= 4x^2(x - 2y) + 5y^2(x - 2y)$

$= (4x^2 + 5y^2)(x - 2y).$

3. Factorization of a trinomial that is a perfect square. A trinomial of the form $x^2 \pm 2xy + y^2$ is equivalent to $(x \pm y)^2$. This identity can be used to factorize perfect square trinomials.

EXAMPLE 3.10

(a) Factorize $49x^2 + 9y^2 + 42xy$

SOLUTION

$49x^2 + 9y^2 + 42xy$

$= (7x)^2 + (3y)^2 + 2(7x)(3y)$

$= (7x + 3y)^2.$

(b) Factorize $16x^2 + \dfrac{1}{16x^2} - 2$

SOLUTION

$$16x^2 + \frac{1}{16x^2} - 2$$

$$= (4x)^2 + \left(\frac{1}{4x}\right) - 2(4x)\left(\frac{1}{4x}\right)$$

$$= (4x)^2 - 2(4x)\left(\frac{1}{4x}\right) + \left(\frac{1}{4x}\right)^2$$

$$= \left(4x - \frac{1}{4x}\right)^2$$

$$\therefore 16x^2 + \frac{1}{16x^2} - 2 = \left(4x - \frac{1}{4x}\right)^2.$$

4. Factorization of a polynomial of the form

$x^2 + (a + b)x + ab.$

As we have already seen,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$\therefore\ x^2 + (a + b)x + ab$ can be fatcorized as $(x + a)(x + b).$

EXAMPLE 3.11

(a) Factorize $x^2 + 25x + 144$

SOLUTION

Here, the constant term is $144 = (16 \times 9)$ and the coefficient of x is $25 = (16 + 9)$

$\therefore\ x^2 + 25x + 144$

$= x^2 + 16x + 9x + 144$

$= x(x + 16) + 9(x + 16)$

$= (x + 16)(x + 9)$.

(b) Factorize $x^2 - 8x + 15$

SOLUTION

Here, the constant term is $15 = (-5)(-3)$ and the coefficient of x is $-8 = -5 - 3$.

$\Rightarrow x^2 - 8x + 15$

$= x^2 - 5x - 3x + 15$

$= x(x - 5) -3(x - 5)$

$= (x - 3)(x - 5)$

$\therefore\ x^2 - 8x + 15 = (x - 3)(x - 5)$.

(c) Factorize $x^2 - 5x - 14$

SOLUTION

Constant term is $-14 = (-7)(2)$

Coefficient of x is $-5 = -7 + 2$

$\Rightarrow x^2 - 5x - 14$

$= x^2 - 7x + 2x - 14$

$= x(x - 7) + 2(x - 7)$

$= (x + 2)(x - 7)$.

5. Factorization of polynomials of the form $ax^2 + bx + c$.

Step 1: Take the product of the constant term and the coefficient of x^2, i.e., ac.

Step 2: Now this product ac is to split into two factors m and n such that $m + n$ is equal to the coefficient of x, i.e., b.

Step 3: Then we pair one of them, say mx, with ax^2 and the other nx, with c and factorize.

EXAMPLE 3.12

(a) $6x^2 + 19x + 15$

SOLUTION

Here, $6 \times 15 = 90 = 10 \times 9$ and $10 + 9 = 19$

$\therefore\ 6x^2 + 19x + 15$

$= 6x^2 + 10x + 9x + 15$

$= 2x(3x + 5) + 3(3x + 5)$

$= (2x + 3)(3x + 5).$

(b) Factorize $7 - 17x - 12x^2$

SOLUTION

Here, $(7)(-12) = -84 = (-21)(4)$ and

$-17 = -21 + 4$

$7 - 17x - 12x^2$

$= 7 - 21x + 4x - 12x^2$

$= 7(1 - 3x) + 4x(1 - 3x) = (1 - 3x)(7 + 4x).$

6. Factorization of expressions of the form $x^3 + y^3$ (or) $x^3 - y^3$.

$x^3 + y^3 = (x + y)\,(x^2 - xy + y^2)$

$\Rightarrow x^3 + y^3$ has factors $(x + y)$ and $(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$

$\Rightarrow x^3 - y^3$ has factors $(x - y)$ and $(x^2 + xy + y^2)$.

EXAMPLE 3.13

(a) Factorize $27a^3 + 125x^3$

SOLUTION

$27a^3 + 125x^3$

$= (3a)^3 + (5x)^3$

$= (3a + 5x)\{(3a)^2 + (5x)^2 - (3a)(5x)\}$

$= (3a + 5x)(9a^2 + 25x^2 - 15ax).$

(b) Factorize $216x^3 - 64y^3$

SOLUTION

$216x^3 - 64y^3$

$= (6x)^3 - (4y)^3$

$= (6x - 4y)\{(6x)^2 + (4y)^2 + (6x)(4y)\}$

$= (6x - 4y)(36x^2 + 16y^2 + 24xy).$

7. Factorization of expressions of the form

$x^3 + y^3 + z^3$ when $x + y + z = 0$.

(Given $x + y + z = 0$)

As $x + y + z = 0$, $z = -(x + y)$

$$\begin{aligned}x^3 + y^3 + z^3 &= x^3 + y^3 + \{-(x + y)\}^3\\ &= x^3 + y^3 - (x + y)^3\\ &= x^3 + y^3 - \{x^3 + y^3 + 3xy(x + y)\}\\ &= -3xy(x + y)\\ &= -3xy(-z)\ \{\text{Since } x + y = -z\}\\ &= 3xyz\end{aligned}$$

$\therefore$ If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

HCF of Given Polynomials

For two given polynomials, $f(x)$ and $g(x)$, $r(x)$ can be taken as the highest common factor, if

1. $r(x)$ is a common factor of $f(x)$ and $g(x)$ and
2. every common factor of $f(x)$ and $g(x)$ is also a factor of $r(x)$.

Highest common factor is generally referred to as HCF.

Method for Finding HCF of the Given Polynomials

Step 1: Express each polynomial as a product of powers of irreducible factors which also requires the numerical factors to be expressed as the product of the powers of primes.

Step 2: If there is no common factor then HCF is 1 and if there are common irreducible factors, we find the least exponent of these irreducible factors in the factorized form of the given polynomials.

Step 3: Raise the common irreducible factors to the smallest or the least exponents found in step 2 and take their product to get the HCF.

EXAMPLE 3.14

(a) Find the HCF of $48x^5y^2$ and $112x^3y$.

SOLUTION

Let $f(x) = 48x^5y^2$ and $g(x) = 112x^3y$

Writing $f(x)$ and $g(x)$ as a product of powers of irreducible factors.

$$f(x) = 2^4 \cdot 3 \cdot x^5 \cdot y^2$$
$$g(x) = 2^4 \cdot 7 \cdot x^3 \cdot y$$

The common factors with the least exponents are 2^4, x^3 and y

$\therefore$ HCF $= 16x^3y$.

(b) Find the HCF of $51x^2(x + 3)^3(x - 2)^2$ and $34x(x - 1)^5(x - 2)^3$.

SOLUTION

Let $f(x) = 51x^2(x + 3)^3(x - 2)^2$ and $g(x) = 34x(x - 1)^5(x - 2)^3$

Writing $f(x)$ and $g(x)$ as the product of the powers of irreducible factors.

$$f(x) = 17 \cdot 3 \cdot x^2(x+3)^3 \cdot (x-2)^2$$
$$g(x) = 17 \cdot 2 \cdot x(x-1)^5 \cdot (x-2)^3$$

The common factors with the least exponents are 17, x and $(x-2)^2$

$\therefore$ The HCF of the given polynomials $= 17 \cdot x \cdot (x-2)^2 = 17x(x-2)^2$.

LCM of the Given Polynomials

Least Common Multiple or the Lowest Common Multiple is the product of all the factors (taken once) of the polynomials given with their highest exponents respectively.

Method to Calculate LCM of the Given Polynomials

Step 1: First express each polynomial as a product of powers of irreducible factors.

Step 2: Consider all the irreducible factors (only once) occurring in the given polynomials. For each of these factors, consider the greatest exponent in the factorized form of the given polynomials.

Step 3: Now raise each irreducible factor to the greatest exponent and multiply them to get the LCM.

EXAMPLE 3.15

(a) Find the LCM of $18x^3y^2$ and $45x^5y^2z^3$.

SOLUTION

Let $f(x) = 18x^3y^2$ and

$g(x) = 45x^5y^2z^3$

Writing $f(x)$ and $g(x)$ as the product of the powers of irreducible factors.

$f(x) = 2 \cdot 3^2 \cdot x^3 \cdot y^2$

$g(x) = 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3$

Now all the factors (taken only once) with the highest exponents are 2, 3^2, 5, x^5, y^2 and z^3.

$\therefore$ The LCM of the given polynomials $= 2 \cdot 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3 = 90x^5y^2z^3$.

(b) Find the LCM of $51x^2(x+3)^3\ (x-2)^2$ and $34x(x-1)^5\ (x-2)^3$

SOLUTION

Writing $f(x)$ and $g(x)$ as the product of powers of irreducible factors.

$f(x) = 17 \cdot 3x^2(x+3)^3 \cdot (x-2)^2$

$g(x) = 17 \cdot 2(x-1)^5 \cdot (x-2)^3$

Now all the factors (taken only once) with the highest exponents are 2, 3, 17, $x^2(x-1)^5$, $(x-2)^3$ and $(x+3)^3$.

$\therefore$ The LCM of the given polynomials $= 2 \cdot 3 \cdot 17 \cdot x^2(x-1)^5 \cdot (x-2)^3 \cdot (x+3)^3 = 102x^2(x-2)^3(x-1)^5(x+3)^3$.

Relation between the HCF, the LCM and the Product of Polynomials

If $f(x)$ and $g(x)$ are two polynomials then we have the relation,

(HCF of $f(x)$ and $g(x)$) $\times$ (LCM of $f(x)$ and $g(x)$) $= \pm(f(x) \times g(x))$.

Example: Let $f(x) = (x + 5)^2(x - 7)(x + 8)$ and

$g(x) = (x + 5)(x - 7)^2(x - 8)$ be two polynomials.

The common factors with the least exponents are $x + 5$ and $x - 7$.

$\therefore$ HCF $= (x + 5)\ (x - 7)$

All the factors (taken only once) with the highest exponents are $(x + 5)^2$, $(x - 7)^2$, $(x - 8)$ and $(x + 8)$.

$\Rightarrow$ LCM $= (x + 5)^2(x - 7)^2(x - 8)\ (x + 8)$

Now $f(x) \cdot g(x) = (x + 5)^2(x - 7)(x + 8)(x + 5)(x - 7)^2(x - 8)$

$= (x + 5)^3(x - 7)^3(x + 8)(x - 8)$

LCM $\times$ HCF $= (x + 5)^2(x - 7)^2(x - 8)(x + 8) \times (x + 5)(x - 7) = (x + 5)^3(x - 7)^3(x - 8)(x + 8)$

Thus, we say

(LCM of two polynomials) $\times$ (HCF of two polynomials) = Product of the two polynomials.

Concept of Square Roots

If x is any variable, then x^2 is called the square of the variable and for x^2, x is called the square root.

Square root of x^2 can be denoted as $\sqrt{x^2} \cdot x$ and $-x$ can both be considered as the square roots of x^2 because
$(x) \cdot (x) = x^2$ and $(-x)(-x) = x^2$.

In this study we restrict $\sqrt{x^2}$ to x, i.e., positive value of x.

Square Root of Monomials

The square root of a monomial can be directly calculated by finding the square roots of the numerical coefficient and that of the literal coefficients and then multiplying them.

EXAMPLE 3.16

Find the square root of $1296b^4$.

SOLUTION

Now, the given monomial is $1296b^4$.

Square root of

$$1296b^4 = \sqrt{1296b^4} = \sqrt{1296} \times \sqrt{b^4}$$

$$= \sqrt{(36)^2} \times \sqrt{(b^2)} = 36 \times b^2 \Rightarrow \sqrt{1296b^4} = 36b^2.$$

EXAMPLE 3.17

Find the square root of $\frac{81b^2a^4}{36x^2y^6}$.

SOLUTION

Square root of

$$\begin{aligned}\frac{81b^2a^4}{36x^2y^6} &= \sqrt{\frac{81b^2a^4}{36x^2y^6}}\\ &= \sqrt{\frac{81b^2a^4}{36x^2y^6}}\\ &= \frac{\sqrt{81}\times\sqrt{b^2a^4}}{\sqrt{36}\times\sqrt{x^2y^6}}\\ &= \frac{\sqrt{9^2}\times\sqrt{(ba^2)^2}}{\sqrt{6^2}\times\sqrt{(xy^3)^2}}\\ &= \frac{9ba^2}{6xy^3} = \frac{3ba^2}{2xy^3}\end{aligned}$$

$\Rightarrow$ Square root of $\frac{81b^2a^4}{36x^2y^6} = \frac{3ba^2}{2xy^3}$.

Methods of Finding the Square Roots of Algebraic Expressions Other than Monomials

We have four methods to find the square root of an algebraic expression which is not a monomial. They are

1. Method of inspection (using algebraic identities).
2. Method of factorization
3. Method of division
4. Method of undetermined coefficients

Method of Inspection In this method, the square root of the given algebraic expression is found by using relevant basic algebraic identities after proper inspection.

EXAMPLE 3.18

Find the square root of $x^2 + 12xy + 36y^2$.

SOLUTION

$x^2 + 12xy + 36y^2 = (x)^2 + 2(x)(6y) + (6y)^2$

We know that $a^2 + 2ab + b^2 = (a + b)^2$

Now,

$$\sqrt{x^2 + 12xy + 36y^2} = \sqrt{(x)^2 + 2(x)(6y) + (6y)^2}$$

$$= \sqrt{(x+6y)^2} = (x+6y)$$

$\therefore \sqrt{x^2 + 12xy + 36y^2} = x + 6y.$

EXAMPLE 3.19

Find the square root of $a^2x^2 - 2ayx^2 + x^2y^2$.

SOLUTION

$a^2x^2 - 2ayx^2 + x^2y^2 = x^2(a^2 - 2ay + y^2)$

We know that $a^2 - 2ab + b^2 = (a - b)^2$

Now,

$$\sqrt{a^2x^2 - 2ayx^2 + x^2y^2} = \sqrt{x^2(a^2 - 2ay + y^2)} = \sqrt{x^2(a-y)^2}$$

$$= \sqrt{[x(a-y)]^2} = x(a-y)$$

$\Rightarrow \sqrt{a^2x^2 - 2ayx^2 + x^2y^2} = x(a-y).$

Method of Factorization

EXAMPLE 3.20

(a) Find the square root of $(x^2 - 8x + 15)(2x^2 - 11x + 5)(2x^2 - 7x + 3)$.

SOLUTION

Step 1: Factorize each expression in the given product, i.e.,

$$\begin{aligned} x^2 - 8x + 15 &= x^2 - 5x - 3x + 15 \\ &= x(x-5) - 3(x-5) \\ &= (x-5)(x-3) \\ 2x^2 - 11x + 5 &= 2x^2 - 10x - x + 5 \\ &= 2x(x-5) - 1(x-5) \\ &= (2x-1)(x-5) \\ 2x^2 - 7x + 3 &= 2x^2 - 6x - x + 3 \\ &= 2x(x-3) - 1(x-3) \\ &= (x-3)(2x-1) \end{aligned}$$

Step 2: Write all the factors in a row.

$\therefore$ The given expression is $(x-5)(x-3)(2x-1)(x-5)(x-3)(2x-1)$, i.e.,

$(x-3)^2(x-5)^2(2x-1)^2$

Step 3: Evaluate the square root.

Hence, the square root of the given expression is $(x-3)(x-5)(2x-1)$.

(b) Find the square root of $(x - 1)(x - 2)(x - 3)(x - 4) + 1$.

SOLUTION

Step 1: First we select the terms in such a way that, they have a common expression in their product. In the given expression, we consider terms $(x - 1)$ and $(x - 4)$ and $(x - 2)(x - 3)$.

Step 2: Find their product, i.e.,

$$x^2 - 5x + 4 \text{ and } x^2 - 5x + 6 = (x^2 - 5x + 4)(x^2 - 5x + 6)$$

Step 3: Take the common expression in the products as a $(= x^2 - 5x)$ and substitute in the given expression.

$\therefore$ The given expression becomes $(a + 4)(a + 6) + 1$

Step 4: Factorize the resultant expression and find its square root, i.e.,

$$a^2 + 10a + 25 = (a + 5)^2$$

$$\therefore \sqrt{(a+5)^2} = a + 5.$$

Step 5: Resubstitute the value of a, which is the required square root.

$\therefore$ The square root of the given expression is $x^2 - 5x + 5$.

Method of Division We discuss the method of division to find the square root of an algebraic expression using the following example.

EXAMPLE 3.21

Find the square root of $x^2 - 18x + 81$.

SOLUTION

$x - 9$

$$\begin{array}{r|r l}
x & x^2 - 18x + 81 & \quad (x) \\
 & x^2 & \\
\hline
2x - 9 & -18x + 81 & \quad (-9) \\
 & -18x + 81 & \\
\hline
 & 0 &
\end{array}$$

$$\therefore \sqrt{x^2 - 18x + 81} = x - 9.$$

Step 1: First the given expression is arranged in the descending powers of x.

Step 2: Then the square root of the first term in the expression is calculated. In the above problem first term is x^2 whose square root is x. This is now the first term of the square root of the expression.

Step 3: Then the square of x, i.e., x^2 is written below the first term of the expression and subtracted. The difference is zero. Then the next two terms in the expression $-18x + 81$ are brought down as the dividend for the next step. Double the first term of the square root and put it down as the first term of the next divisor, i.e., $2(x) = 2x$ is to be written as the first term of the next divisor. Now the first term $-18x$ of the dividend $-18x + 81$ is to be divided by the

first term $2x$ (of the new divisor). Here we get -9 which is the second term of the square root of the given expression and the second term of the new divisor.

Step 4: Thus the new divisor becomes $2x - 9$. Multiply $(2x - 9)$ by (-9) and the product $-18x + 81$ is to be brought down under the second dividend $-18x + 81$ and subtracted where we get 0.

Step 5: Thus $x - 9$ is the square root of the given expression $x^2 - 18x + 81$.

EXAMPLE 3.22

Find the square root of $4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4$.

SOLUTION

Follow the steps indicated in the previous example.

$$
\begin{array}{r|l l}
 & 2x^3 - 3x^2 + 2 & \\
\hline
2x^3 & 4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4 & (2x^3) \\
 & 4x^6 & \\
 & - & \\
\hline
4x^3 - 3x^2 & \quad -12x^5 + 9x^4 & (-3x^2) \\
 & \quad -12x^5 + 9x^4 & \\
 & \quad + \qquad - & \\
\hline
4x^3 - 6x^2 + 2 & \qquad\qquad\qquad 8x^3 - 12x^2 + 4 & (+2) \\
 & \qquad\qquad\qquad 8x^3 - 12x^2 + 4 & \\
 & \qquad\qquad\qquad\quad + \qquad - & \\
\hline
 & \qquad 0 &
\end{array}
$$

$\therefore \sqrt{4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4} = 2x^3 - 3x^2 + 2.$

Method of Undetermined Coefficients The method of undetermined coefficients to find the square root of an algebraic expression is explained in the following examples.

EXAMPLE 3.23

(a) Find the square root of $x^4 + 4x^3 + 10x^2 + 12x + 9$.

SOLUTION

The degree of the given expression is 4, its square root will hence be an expression of degree 2. Let us assume the square root to be $ax^2 + bx + c$.

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = (ax^2 + bx + c)^2$

We know that $(p + q + r)^2 = p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

Here, $p = ax^2$, $q = bx$, $r = c$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = a^2x^4 + b^2x^2 + 2abx^3 + 2cax^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equality sign, we have

$x^4 = a^2x^4$

$\Rightarrow a^2 = 1 \Rightarrow a = 1$

$4x^3 = 2abx^3 \Rightarrow 2ab = 4$

$\Rightarrow ab = 2$, but $a = 1 \Rightarrow b = 2$

$b^2 + 2ca = 10 \Rightarrow 2^2 + 2c = 10$

$\Rightarrow 2c = 6 \Rightarrow c = 3$

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $x^2 + 2x + 3$.

(b) Find the square root of $4x^4 - 4x^3 + 5x^2 - 2x + 1$.

SOLUTION

The degree of the given expression is 4, its square root will hence be an expression in degree 2.

Let $\sqrt{4x^4 - 4x^3 + 5x^2 - 2x + 1} = ax^2 + bx + c$

$\Rightarrow (4x^4 - 4x^3 + 5x^2 - 2x + 1) = (ax^2 + bx + c)^2$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1 = a^2x^4 + 2abx^3 + (b^2 + 2ac)x^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equation, we have

$4x^4 = a^2x^4 \Rightarrow a^2 = 4 \Rightarrow a = 2$

$c^2 = 1 \Rightarrow c = 1$

$2bcx = -2x \Rightarrow 2bc = -2 \Rightarrow bc = -1$

$\Rightarrow b = \dfrac{-1}{c} \Rightarrow b = -1 \; (\therefore c = 1)$

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $2x^2 - x + 1$.

Rational Integral Function of x

A polynomial in x, the exponents in powers of x are non-negative integers and the coefficients of the various powers of x are integers.

Example:

$11x^2 - 8x + 3$, $4x^2 - 5x + 1$, $8x^5 - 7x^3 + 8x^2 + 4x + 5$, etc.

Remainder Theorem

$q(x)$ is a rational integral function of x.

If $q(x)$ is divided by $x - a$, then the remainder is $q(a)$.

EXAMPLE 3.24

Find the remainder when $x^3 - 8x^2 + 5x + 1$ is divided by $x - 1$.

SOLUTION

Let $q(x) = x^3 - 8x^2 + 5x + 1$

If $q(x)$ is divided by $x - 1$, then the remainder is $q(1)$.

$\therefore\ q(1) = (1)^3 - 8(1)^2 + 5(1) + 1 = 1 - 8 + 5 + 1$

$q(1) = -1.$

Note If $q(x)$ is divided by $ax - b$, then the remainder is $q\left(\frac{b}{a}\right)$.

EXAMPLE 3.25

Find the remainder when $x^2 - 8x + 6$ is divided by $2x - 1$.

SOLUTION

Let $q(x) = x^2 - 8x + 6$

$\therefore$ Remainder $= q\left(\frac{1}{2}\right)$

$$\text{i.e., } q\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 6$$

$$= \frac{1}{4} - 4 + 6 = \frac{1}{4} + 2$$

$$\therefore\ q\left(\frac{1}{2}\right) = \frac{9}{4}.$$

Factor Theorem

$q(x)$ is a rational integral function of x and if $q(\alpha) = 0$, then $x - \alpha$ is the factor of $q(x)$.

EXAMPLE 3.26

(a) Is $x - 2$ a factor of $x^3 + x^2 - 4x - 4$?

SOLUTION

Let $q(x) = x^3 + x^2 - 4x - 4$

$q(2) = 8 + 4 - 8 - 4 = 0$

$\therefore\ x - 2$ is a factor of $q(x)$.

(b) Find the value of m, if $x + 2$ is a factor of $x^3 - 4x^2 + 3x - 5m$.

SOLUTION

Let $q(x) = x^3 - 4x^2 + 3x - 5m$.

Given $x + 2$ is a factor of $q(x)$.

$\therefore\ q(-2) = 0$

$$\Rightarrow (-2)^3 - 4(-2)^2 + 3(-2) - 5m = 0$$

$$-8 - 16 - 6 - 5m = 0$$

$$-5m = 30$$

$$m = -6.$$

(c) Factorize $x^3 - 2x^2 - 5x + 6$.

SOLUTION

Let $f(x) = x^3 - 2x^2 - 5x + 6$

Step 1: First we find one of the factors of $f(x)$ by substituting the value of x as ± 1, ± 2 and so on till the remainder is zero.

Here,

$$f(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$

$$= 1 - 2 - 5 + 6$$

$$f(1) = 0$$

$\therefore\ x - 1$ is a factor of $f(x)$.

Step 2: To find the other two factors, we use synthetic division.

	1	–2	–5	6
$x = 1$	0	1	–1	–6
	1	–1	–6	0

$\therefore$ The other two factors are $x^2 - x - 6$, i.e., $(x - 3)(x + 2)$.

$\therefore$ The factorization of the given expression is $(x - 1)(x - 3)(x + 2)$.

(d) Factorize $x^4 - x^3 - 11x^2 + 9x + 18$.

SOLUTION

Let $f(x) = x^4 - x^3 - 11x^2 + 9x + 18$

$$f(-1) = (-1)^4 - (-1)^3 - 11(-1)^2 + 9(-1) + 18$$

$$= 1 + 1 - 11 - 9 + 18 = 20 - 20 = 0$$

$\therefore\ x + 1$ is a factor of $f(x)$.

Now,

$$f(2) = (2)^4 - 2^3 - 11(2)^2 + 9(2) + 18$$

$$= 16 - 8 - 44 + 18 + 18 = 52 - 52 = 0$$

$\therefore\ x - 2$ is a factor of $f(x)$.

	1	−1	−11	9	18
$x = -1$	0	−1	2	9	−18
$x = 2$	1	−2	−9	18	0
	0	2	0	−18	
	1	0	−9	0	

$\therefore$ The other two factors of $f(x)$ are $(x + 3)$ and $(x - 3)$.

Hence, the factorization of the given expression, is $(x + 1)(x - 2)(x - 3)(x + 3)$.

Homogeneous Expression

An algebraic expression in which, the degree of all the terms is equal is a homogeneous expression.

Example: $bx + ay$ is a first degree homogeneous expression.

$ax^2 + bxy + cy^2$ is a second degree homogeneous expression.

Notes

1. A homogeneous expression is complete if it contains all the possible terms in it.
2. The product of two homogeneous expressions is a homogeneous expression.
3. The degree of the product of two or more homogeneous expressions is the sum of degrees of all the expressions involved in product.

Symmetric Expressions

$f(x, y)$ is an expression in variables x and y.

If $f(x, y) = f(y, x)$, then $f(x, y)$, is called a symmetric expression.

i.e., If an expression remains same after interchanging the variables x and y is said to be a symmetric expression.

EXAMPLE 3.27

Consider the expressions given below and find if the expressions are symmetric or not:

(a) $ax + ay + b$

(b) $ax^2 + bxy + ay^2$

SOLUTION

(a) Let $f(x, y) = ax + ay + b$

$$f(y, x) = ay + ax + b$$

$$= ax + ay + b$$

$$\Rightarrow f(y, x) = f(x, y)$$

$\therefore$ $ax + ay + b$ is symmetric.

(b) $f(x, y) = ax^2 + bxy + ay^2$

$f(y, x) = ay^2 + byx + ax^2$

$= ax^2 + bxy + ay^2$

$\therefore f(y, x) = f(x, y)$

Hence, $ax^2 + bxy + ay^2$ is symmetric.

Notes

1. An expression which is homogeneous and symmetric is called a homogeneous symmetric expression.

   Example: $ax + ay$, $ax^2 + bxy + ay^2$

2. The sum, difference, product and quotient of two symmetric expressions is always symmetric.

Cyclic Expressions

$f(x, y, z)$ is an expression in variables x, y and z.

If $f(x, y, z) = f(y, z, x)$, then $f(x, y, z)$ is cyclic.

Example:

$a^2(a - b) + b^2(b - c) + c^2(c - a)$

Let $f(a, b, c) = a^2(a - b) + b^2(b - c) + c^2(c - a)$

Now, $f(b, c, a) = b^2(b - c) + c^2(c - a) + a^2(a - b)$

$= a^2(a - b) + b^2(b - c) + c^2(c - a)$

$f(b, c, a) = f(a, b, c)$

$\therefore f$ is cyclic.

Cyclic expressions are lengthy to write, so we use symbols Σ (read as sigma) and π(pi) to abbreviate them.

Σ is used for sum of terms and π is used for product of terms.

Example:

$x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$ can be represented as $\sum_{x,y,z} x^2(y^2 - z^2)$

$\therefore \Sigma x^2(y^2 - z^2) = x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$

Example:

$(x^2 + y^3)(y^2 + z^3)(z^2 + x^3)$ can be represented as $\sum_{x,y,z} (x^2 + y^3) = (x^2 + y^3)(y^2 + z^3)(z^2 + x^3)$

EXAMPLE 3.28

Factorize $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

SOLUTION

Let $f(a) = a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

$f(b) = b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2)$

$= b(b^2 - c^2) + b(c^2 - b^2)$

$f(b) = 0$

$\therefore$ By remainder theorem, $(a - b)$ is a factor of the given expression.

The given expression is cyclic, so the other two factors will also be cyclic.

$\therefore$ The other two factors are $(b - c)$ and $(c - a)$.

The given expression may have a constant factor which is non-zero. Let it be m.

$\therefore$ $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = m(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$, $c = -1$ in the above equation.

i.e., $0(1^2 - (-1)^2) + 1((-1)^2 - 0) + (-1)(0 - 1^2)$

$= m(0 - 1)(1 - (-1))(-1 - 0)$

$\Rightarrow 0 + 1 + 1 = m(-1)(2)(-1)$

$\Rightarrow m = 1$

$\therefore$ The factorization of the given cyclic expressions is $(a - b)(b - c)(c - a)$.

EXAMPLE 3.29

Factorize $a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

SOLUTION

Let $f(a) = a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

$f(b) = b(b^3 - c^3) + b(c^3 - b^3) + c(b^3 - b^3)$

$= b(b^3 - c^3) - b(b^3 - c^3) + 0$

$f(b) = 0$

$\therefore$ By remainder theorem, $a - b$ is the factor of the given expression.

The given expression is cyclic, so the other two factors are also cyclic.

$\therefore$ The factors are $(a - b)(b - c)(c - a)$.

The given expression is of degree 4 but the degree of factor is 3, hence a first degree cyclic expression is the another factor.

Let $m(a + b + c)$ be the factor $(m \neq 0)$.

$\therefore$ $a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

$= m(a + b + c)(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$ and $c = 2$ in the above equation.

$0(1^3 - (2)^3) + 1((2)^3 - 0) + 2(0 - 1^3) = m(0 + 1 + 2)(0 - 1)(1 - 2)(2 - 0)$

$\Rightarrow 0 + 1(8) + 2(-1) = m(3)(-1)(-1)(2)$

$6 = 6m$

$\Rightarrow m = 1$

$\therefore$ The factorization of the given cyclic expression is $(a - b)(b - c)(c - a)(a + b + c)$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $11x^2 - 88x^3 + 14x^4$ is called a ______ polynomial.
2. The degree of the polynomial $7x^3y^{10}z^2$ is ______.
3. The expression is a polynomial. (True/False)
4. If $A = 3x^2 + 5x - 3$ and $B = 5x^2 - 7$, then $2A - B$ is ______.
5. If $a + b + c = 0$, then $a^3 + b^3 + c^3 =$ ______.
6. Factors of $x^6 - y^6$ is ______.
7. The LCM of $\sqrt{2}x, \sqrt{8}x^7y^2$ is ______.
8. The HCF of $44a^3$ and $66b^pa^4$ is $22a^3$, then p can be ______.
9. One of the factor of $x^3 - x^2 + x - 1$ is ______ .
10. The quotient of $8x^3 - 7x^2 + 5x + 8$ when divided by $2x$ is ______.
11. The remainder obtained when $80x^3 + 55x^2 + 20x + 172$ is divided by $x + 2$ is ______.
12. Factorize $6x^2 + x - 2$.
13. Find the LCM and HCF of the polynomials $15x^2y^3z$, $3x^3yz^2$.
14. Find the remainder when x^{15} is divided by $x - 2$.
15. Find the remainder if $x^5 - 3x^3 + 5x + 1$ is divided by $2x - 1$.
16. $\sqrt{a + b - 2\sqrt{ab}}$ is ______ where $\sqrt{a} > \sqrt{b}$.
17. The product of two symmetric expressions is a/an ______ expression.
18. The square root of $a^{m^2} \cdot b^{n^2}$ is ______.
19. The value of a if $x^3 - 8x^2 + 2x + a$ is divisible by $x - 2$ is ______.
20. Factorize $a^5b - ab^5$.
21. The degree of a polynomial A is 7 and that of polynomial AB is 56, then find the degree of polynomial B.
22. If $A = x^3$, $B = 4x^2 + x - 1$, then find AB.
23. Factorize $m^7 + m^4$.
24. Factorize $\frac{1}{6}a^2 - a + \frac{4}{3}$.
25. If $3x^2 + 8ax + 3$ is a perfect square, then find the value of a.
26. The factors of $a^3 + b^3 + c^3 - 3abc$ are ______.
27. The HCF of $(a^2 + 1)(a + 11)$ and $(a^2 + 1)^2\,(a + 11)^2$ is ______.
28. The value of $81^3 - 100^3 + 19^3$ is ______.
29. If $A = 4x^3 - 8x^2$, $B = 7x^3 - 5x + 3$ and $C = 3x^3 + x - 11$, then find $(A + C) - B$.
30. $8x^2 + 11xy + by^2$ is a symmetric expression, then b = ______.

Short Answer Type Questions

31. The HCF of $(a - 1)(a^3 + m)$ and $(a + 1)(a^3 - n)$ and $(a + 1)(a^2 - n)$ is $a^2 - 1$, then the values of m and n are ______.
32. Expand $\underset{a,b,c}{\pi}\, a^2(b + c)$.
33. The factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$ is ______.
34. Expand $\sum c^2(a^2 - b^2)$.
35. If $A = 4x^3 - 8x^2$, $B = 7x^3 - 5x + 3$ and $C = 3x^3 + x - 11$, then find $2A - 3B + 4C$.
36. If $A = x^3$, $B = 4x^2 + x - 1$, $C = x + 1$, then find $(A - B)(A - C)$.
37. Find the quotient and remainder when $x^4 + 4x^3 - 31x^2 - 94x + 120$ is divided by $x^2 + 3x - 4$.
38. Factorize $a^3 + \frac{3ax}{8} + \frac{x^3}{64} - \frac{1}{8}$.
39. Find the LCM and HCF of the following polynomials.
 $36(x + 2)^2\,(x - 1)^3\,(x + 3)^5$, $45(x + 2)^5\,(x - 1)^2\,(x + 3)^5$ and $63(x - 1)^5\,(x + 2)^5\,(x + 3)^4$.
40. The LCM of the polynomials $(x^2 + x - 2)(x^2 + x - a)$ and $(x^2 + x - b)(x^2 + 5x + a)$ is $(x - 1)(x + 2)^2\,(x + 3)$, then find the values of a and b.
41. Find the remainder when x^{23} is divided by $x^2 - 3x + 2$.

42. If $lmx^2 + mnx + ln$ is a perfect square then prove that, $4l^2 = mn$.

43. Find the value of $\sqrt{(a+b+c)^2 + (a+b-c)^2 + 2(c^2 - a^2 - b^2 - 2ab)}$.

44. If $\sqrt{\dfrac{125a^6b^4c^2}{5a^4b^2}} = x$, then find $\dfrac{x^2}{abc}$.

45. Find the square root of $(x^2 + 6x + 8)(x^2 + 5x + 6)(x^2 + 7x + 12)$.

Essay Type Questions

46. Factorize $6x^4 - 5x^3 - 38x^2 - 5x + 6$.

47. For what values of p and q, the expression, $x^4 - 14x^3 + 71x^2 + px + q$ is a perfect square?

48. Find the square root of $16x^6 - 24x^5 + 25x^4 - 20x^3 + 10x^2 - 4x + 1$ by the method of division.

49. Find the quadratic polynomial when divided by x, $x - 1$ and $x - 2$ leaves remainders 1, 2 and 9 respectively.

50. Find the factors of $a^2(b + c) + b^2(c + a) + c^2(a + b) - 2b^2c$.

CONCEPT APPLICATION

Level 1

1. If the degree of a polynomial AB is 15 and the degree of polynomial B is 5, then the degree of polynomial A is
   (a) 3 (b) 8
   (c) 4 (d) 10

2. The expression $21x^2 + 11x - 2$ equals to
   (a) $(x - 2)(7x + 1)$ (b) $(7x + 1)(3x - 2)$
   (c) $(7x - 1)(3x - 2)$ (d) $(7x - 1)(3x + 2)$

3. If the LCM and HCF of two polynomials are $90\ m^5a^6b^3x^2$ and $m^3\ a^5$ respectively and also one of the monomial is $18\ m^5a^6x^2$, then the other monomial is
   (a) $5\ m^3a^5b^3$ (b) $15\ m^5a^3b^2$
   (c) $5\ m^5a^3b^5$ (d) $15\ m^3a^5b^4$

4. The remainder when $x^3 - 3x^2 + 5x - 1$ is divided by $x + 1$ is ______.
   (a) -8 (b) -12
   (c) -10 (d) -9

5. Which of the following is a homogeneous expression?
   (a) $4x^2 - 5xy + 5x^2y + 10y^2$
   (b) $5x + 10y + 100$
   (c) $14x^3 + 15x^2y + 16y^2x + 24y^3$
   (d) $x^2 + y^2 + x + y + 1$

6. $\Sigma x(y^3 - z^3) =$ ______.
   (a) $(x - y)(y - z)(z - x)(x + y + z)$
   (b) $(x - y)(y - z)(x - z)(x - y - z)$
   (c) $(x + y)(y + z)(z + x)(x + y + z)$
   (d) $(x + y)(y + z)(z + z)(z - y - z)$

7. The remainder when $f(x) = 4x^3 - 3x^2 + 2x - 1$ is divided by $2x + 1$ is ______.
   (a) 1 (b) $\dfrac{-3}{4}$
   (c) $\dfrac{-13}{4}$ (d) $\dfrac{-7}{4}$

8. The HCF of the polynomials $12a^3b^4c^2$, $18a^4b^3c^3$ and $24a^6b^2c^4$ is ______.
   (a) $12a^3b^2c^2$ (b) $6a^6b^4c^4$
   (c) $6a^3b^2c^2$ (d) $48a^6b^4c^4$

9. Find the value of a, if $(x + 2)$ is a factor of the polynomial $f(x) = x^3 + 13x^2 + ax + 20$.
   (a) -15 (b) 20
   (c) 25 (d) 32

10. The polynomial $x^3 - 4x^2 + x - 4$ on factorization gives
    (a) $(x - 4)(x^2 - 1)$
    (b) $(x - 4)(x^2 + 4)$
    (c) $(x + 4)(x^2 + 1)$
    (d) $(x - 4)(x^2 + 1)$

11. If the expression $ax^3 + 2x^2y - bxy^2 - 2y^3$ is symmetric, then $(a, b) =$

(a) (2, 2) (b) (−2, 2)
(c) (−2, −2) (d) (2, −2)

12. The square root of $y^2 + \frac{1}{y^2} + 2$ is

(a) $y + \frac{1}{y}$ (b) $y - \frac{1}{y}$
(c) $y^2 + \frac{1}{y^2}$ (d) $y^2 - \frac{1}{y^2}$

13. The product of the polynomials $2x^3 - 3x^2 + 6$ and $x^2 - x$ is ______.

(a) $2x^6 - 5x^4 + 3x^3 + 6x^2 - 6x$
(b) $2x^5 - x^4 + 3x^3 - 6x^2 + 6x$
(c) $2x^5 - 5x^4 + 3x^3 + 6x^2 - 6x$
(d) None of these

14. The LCM of $x^2 - 16$ and $2x^2 - 9x + 4$ is

(a) $(2x + 1)(x + 4)(x - 4)$
(b) $(x^2 + 16)(2x + 1)$
(c) $2(1 - 2x)(x + 4)(x - 4)$
(d) $(2x - 1)(x + 4)(x - 4)$

15. If $P = 3x^3 - 5x + 9$, $Q = 4x^3 + 5x^2 - 11$ and $R = 5x^3 + 4x^2 - 3x + 7$, then $P - 2Q + R$ is

(a) $2(3x^2 + 4x - 19)$
(b) $-6x^2 - 5x + 38$
(c) $-2(3x^2 + 4x + 19)$
(d) $-2(3x^2 + 4x - 19)$

16. If $g(x) = 3a^x + 7a^2b - 13ab^2 + 9b^y$ is a homogeneous expression in terms of a and b, then the values of x and y respectively are ______.

(a) 2, 2 (b) 2, 1
(c) 3, 2 (d) 3, 3

17. The polynomial $11a^2 - 12\sqrt{2}\,a + 2$ on factorization gives

(a) $(11a + \sqrt{2})(a - \sqrt{2})$
(b) $(a - \sqrt{2})(11a - \sqrt{2})$
(c) $(a + 11)(a + \sqrt{2})$
(d) $(11a - \sqrt{2})(a + \sqrt{2})$

18. If $x^n + 1$ is divisible by $x + 1$, n must be

(a) any natural number
(b) an odd natural number
(c) an even natural number
(d) None of these

19. What is the first degree expression to be subtracted from $x^6 + 8x^4 + 2x^3 + 16x^2 + 4x + 5$ in order to make it a perfect square?

(a) $-4x - 4$ (b) $4x + 4$
(c) $4x - 4$ (d) $-4x + 4$

20. Find the square root of $\frac{m^{n^2} n^{m^2} a^{(m+n)}}{(m+n)^{(m+n)^2}}$.

(a) $m^n n^m a^{\frac{m+n}{2}}$ (b) $\frac{m^{\frac{n^2}{2}} n^{\frac{m^2}{2}} a^{\frac{m+n}{2}}}{(m+n)^{\frac{(m+n)^2}{2}}}$
(c) $\frac{m^n n^m a^{\sqrt{m+n}}}{(m+n)^{(m+n)}}$ (d) None of these

21. What is the first degree expression to be added to $16x^6 + 8x^4 - 2x^3 + x^2 + 2x + 1$ in order to make it a perfect square?

(a) $\frac{5}{2}x + \frac{15}{16}$ (b) $-\frac{5}{2}x - \frac{15}{16}$
(c) $-\frac{5}{2}x + \frac{15}{16}$ (d) $+\frac{2}{2}x - \frac{15}{16}$

22. Factorize the polynomial $8x^3 - \frac{1}{64}$.

(a) $\left(2x - \frac{1}{4}\right)\left(4x^2 - \frac{x}{2} + \frac{1}{16}\right)$
(b) $\left(2x - \frac{1}{8}\right)\left(4x^2 + \frac{x}{2} - 16\right)$
(c) $\left(2x - \frac{1}{4}\right)\left(4x^2 + \frac{1}{16} + \frac{x}{2}\right)$
(d) $\left(2x - \frac{1}{4}\right)\left(4x^2 + \frac{x}{2} - 16\right)$

23. The product of polynomials $3x^3 - 4x^2 + 7$ and $x^2 + 1$ is

(a) $3x^5 - 4x^4 + 3x^3 + 3x^2 + 7$
(b) $x^5 + 4x^2 - 2x + 3$
(c) $3x^5 - 4x^4 - 3x^3 + 4x + 8$
(d) $3x^5 - 5x^4 + 8x^2 + 2x + 1$

24. The LCM and HCF of two monomials is $60x^4y^5a^6b^6$ and $5x^2y^3$ respectively. If one of the two monomials is $15x^4y^3a^6$, then the other monomial is

(a) $12x^2y^3a^6b^6$ (b) $20x^4y^5b^6$

(c) $20x^2y^5b^6$ (d) $15x^2y^5b^6$

25. Which of the following is a factor of the polynomial $f(x) = 2x^3 - 5x^2 + x + 2$?

(a) $x + 1$ (b) $x + 2$

(c) $2x + 1$ (d) $2x - 1$

26. If $3x - 1$ is a factor of the polynomial $81x^3 - 45x^2 + 3a - 6$, then a is ______.

(a) $\frac{8}{3}$ (b) $\frac{-7}{3}$

(c) $\frac{-10}{3}$ (d) $\frac{11}{3}$

27. If $A = 4x^3 - 5x + 7$, $B = 2x^3 - x^2 + 3$ and $C = 5x^3 - 8x^2 + 10$, then $A - 2B - C$ is

(a) $5x^3 - 2x^2 + x + 4$

(b) $-5x^3 + 10x^2 - 5x - 9$

(c) $x^3 + 10x^2 - 5x + 9$

(d) $5x^3 - 8x^2 + x - 1$

28. The square root of $x^{m^2-n^2} \cdot x^{n^2+2mn} \cdot x^{n^2}$ is

(a) x^{m+n} (b) $x^{(m+n)^2}$

(c) $x^{(m+n)/2}$ (d) $x^{\frac{1}{2}(m+n)^2}$

29. $x^{831} + y^{831}$ is always divisible by

(a) $x - y$ (b) $x^2 + y^2$

(c) $x + y$ (d) None of these

30. If $(x + 1)(x + 2)(x + 3)(x + k) + 1$ is a perfect square, then the value of k is

(a) 4 (b) 5

(c) 6 (d) 7

Level 2

31. If $A = 6x^4 + 5x^3 - 14x^2 + 2x + 2$ and $B = 3x^2 - 2x - 1$, then the remainder when $A \div B$ is

(a) x (b) $2x$

(c) $3x$ (d) $4x$

32. The polynomial $x^5 - a^2x^3 - x^2y^3 + a^2y^3$ on factorization gives

(a) $(x - y)(x - a)(x + a)(x^2 + y^2 + xy)$

(b) $(x + a)(x - y)(x - a)(x^2 - y^2 + xy)$

(c) $(x + a)(x + y)(x - a)(x^2 + y^2 + xy)$

(d) None of these

33. The HCF of the polynomials $x^4 + 6x^2 + 25$, $x^3 - 3x^2 + 7x - 5$ and $x^2 + 5 - 2x$ is

(a) $x^2 - 2x - 5$ (b) $x^2 - 2x + 5$

(c) $x - 1$ (d) $3x + 2$

34. The HCF of the polynomials $(2x - 1)(5x^2 - ax + 3)$ and $(x - 3)(2x^2 + x + b)$ is $(2x - 1)(x - 3)$. Then the values of a and b respectively are ______.

(a) 16, −1 (b) −16, 1

(c) −16, −1 (d) 16, 1

35. The remainder when x^{45} is divided by $x^2 - 1$ is

(a) $2x$ (b) $-x$

(c) 0 (d) x

36. Factorize $\sum_{a,b,c} a^2(b^4 - c^4)$.

(a) $(a - b)^2(b - c)^2(c - a)^2$

(b) $(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

(c) $(a + b)^2(b + c)^2(c + a)^2$

(d) None of these

37. The polynomial $6y^4 - 19y^3 - 23y^2 + 10y + 8$ on factorization gives

(a) $(y + 1)(y - 4)(3y + 2)(2y + 1)$

(b) $(y + 1)(y - 4)(3y - 2)(2y - 1)$

(c) $(y + 1)(y - 4)(3y - 2)(2y + 1)$

(d) $(y + 1)(y - 4)(3y + 2)(2y - 1)$

38. If the LCM of the polynomials $(y - 3)^a(2y + 1)^b(y + 13)^7$ and $(y - 3)^4(2y + 1)^9(y + 13)^c$ is $(y - 3)^6 (2y + 1)^{10}(y + 13)^7$, then the least value of $a + b + c$ is

(a) 23 (b) 3

(d) 10 (d) 16

39. The LCM of the polynomials $195(x + 3)^2(x - 2)(x + 1)^2$ and $221(x + 1)^3(x + 3)(x + 4)$ is ______.

(a) $221(x + 3)^2 (x + 1)^2(x - 2)(x - 14)$

(b) $13(x + 3)(x + 1)^2$

(c) $3315 (x + 3)^2 (x + 1)^3(x - 2)(x + 4)$

(d) None of these

40. For what value of k the HCF of $x^2 + x + (5k - 1)$ and $x^2 - 6x + (3k + 11)$ is $(x - 2)$?

(a) 2 (b) 2

(c) −2 (d) −1

41. The HCF of the polynomials $9(x + a)^p(x - b)^q(x + c)^r$ and $12(x + a)^{p+3}(x - b)^{q-3}(x + c)^{r+2}$ is $3(x + a)^6(x - b)^6(x + c)^6$, then the value of $p + q - r$ is

(a) 21 (b) 9

(c) 15 (d) 6

42. The remainders obtained when the polynomial $x^3 + x^2 - 9x - 9$ divided by x, $x + 1$ and $x + 2$ respectively are ______.

(a) −9, 0, −15 (b) −9, −16, 5

(c) 0, 0, 5 (d) −9, 0, 5

43. Find the value of

$$\frac{(a+b)^2}{(b-c)(c-a)} + \frac{(b+c)^2}{(a-b)(c-a)} + \frac{(c+a)^2}{(a-b)(b-c)}.$$

(a) −1 (b) 0

(c) 1 (d) 2

44. Find the square root of the expression $\frac{1}{xyz}(x^2 + y^2 + z^2) + 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$.

(a) $\frac{x + y + z}{xyz}$

(b) $\sqrt{\frac{yz}{x}} + \sqrt{\frac{zx}{y}} + \sqrt{\frac{xy}{z}}$

(c) $\sqrt{x} + \sqrt{y} + \sqrt{z}$

(d) $\sqrt{\frac{x}{yz}} + \sqrt{\frac{y}{xz}} + \sqrt{\frac{z}{xy}}$

45. Factorize the expression $9x^4 + \frac{1}{x^4} + 2$.

(a) $\left(3x^2 - \frac{1}{x^2} + 2\right)\left(3x^2 + \frac{1}{x^2} + 2\right)$

(b) $\left(3x^2 - \frac{1}{x^2} - 2\right)\left(3x^2 + \frac{1}{x^2} + 2\right)$

(c) $\left(3x^2 - \frac{1}{x^2} + 2\right)\left(3x^2 - \frac{1}{x^2} + 2\right)$

(d) $\left(3x^2 + \frac{1}{x^2} + 2\right)\left(3x^2 + \frac{1}{x^2} - 2\right)$

46. The following are the steps involved in factorizing $64x^6 - y^6$. Arrange them in sequential order.

(A) $\{(2x)^3 + y^3\}\ \{(2x)^3 - y^3\}$

(B) $(8x^3)^2 - (y^3)^2$

(C) $(8x^3 + y^3)(8x^3 - y^3)$

(D) $(2x + y)(4x^2 - 2xy + y^2)(2x - y)(4x^2 + 2xy + y^2)$

(a) BADC (b) BDAC

(c) BCAD (d) BACD

47. If $a + b + c = 0$, show that $a^3 + b^3 + c^3 = 3abc$. The following are the steps involved in showing the above result. Arrange them in sequential order.

(A) $a^3 + b^3 + 3ab(-c) = -c^3$

(B) $(a + b)^3 = (-c)^3$

(C) $a + b + c = 0 \Rightarrow a + b = -c$

(D) $a^3 + b^3 + 3ab(a + b) = -c^3$

(E) $a^3 + b^3 + c^3 = 3abc$

(a) ABDCE (b) BCDAE

(c) CBDAE (d) CADBE

48. If the HCF of $8x^3y^a$ and $12x^by^2$ is $4x^ay^b$, then find the maximum value of $a + b$.

(a) 2 (b) 4

(c) 6 (d) Cannot be determined

49. The polynomial $5x^5 - 3x^3 + 2x^2 - k$ gives a remainder 1, when divided by $x + 1$. Find the value of k.

(a) 5 (b) −1

(c) 2 (d) 1

50. Factorize: $a^3 + b^3 + 3ab - 1$.

(a) $(a + b - 1)(a^2 + b^2 + a + b + 1 - ab)$

(b) $(a + b - 1)(a^2 + b^2 + a + b - 1 + ab)$

(c) $(a + b - 1)(a^2 + b^2 - a - b + 1 + ab)$

(d) None of these

51. If f and g are two polynomials of degrees 3 and 4 respectively, then what is the degree of $f - g$?
(a) 1
(b) 3
(c) 4
(d) Cannot be determined

52. Find the square root of $\frac{x^2}{9}+\frac{9}{4x^2}-\frac{x}{3}-\frac{3}{2x}+\frac{5}{4}$.
(a) $\frac{2x}{3}+\frac{3}{2x}-\frac{1}{2}$ (b) $\frac{x}{3}-\frac{3}{2x}+1$
(c) $\frac{3}{x}+\frac{2}{3x}-\frac{1}{2}$ (d) $\frac{x}{3}+\frac{3}{2x}-\frac{1}{2}$

53. The square root of $(xy + xz - yz)^2 - 4xyz(x - y)$ is ______.
(a) $xy + yz - 2xyz$
(b) $(x + y - 2xy)$
(c) $(xy + 3 - y)$
(d) $(xy + yz - zx)$

54. $\left(\sum_{x,y,z}(x+1)^2\right)-\left(\sum_{x,y,z}(x)\right)^2-3=$ ______.
(a) $2\left[\sum_{x,y,z}x-\sum_{x,y,z}xy\right]$
(b) $3\left[\sum_{x,y,z}x^2-\sum_{x,y,z}x\right]$
(c) $2\left[\sum_{x,y,z}xy-\sum_{x,y,z}x^2\right]$
(d) $3\left[\sum_{x,y,z}x^2-\sum_{x,y,z}x\right]$

55. $\left(\sum_{x,y,z}x\right)^2-\left(\sum_{x,y,z}x^2\right)=$ ______.
(a) $\sum_{x,y,z}x$ (b) $2\left(\sum_{x,y,z}xy\right)$
(c) $\pi_{x,y,z}\, xy$ (d) $2\left(\sum_{x,y,z}x+y\right)$

Level 3

56. If $\sqrt{4x^4+12x^3+25x^2+24x+16} = ax^2 + bx + c$, then which of the following is true?
(a) $2b = a - c$
(b) $2a = b + c$
(c) $2b = a + c$
(d) $2b = c - a$

57. Find the square root of the algebraic expression which is the average of the following expressions
$x^2+\frac{1}{x^2}$, $-2\left(x-\frac{1}{x}\right)$ and -1.
(a) $\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}+\frac{1}{x}$
(b) $\frac{x}{\sqrt{3}}+1+x$
(c) $\frac{1}{\sqrt{3}}\left(x-1-\frac{1}{x}\right)$
(d) None of these

58. If the each of algebraic expressions $lx^2 + mx + n$, mx^2+nx+l and nx^2+lx+m are prefect squares, then $\frac{l+m}{n}=$ ______.
(a) −4 (b) 6
(c) −8 (d) None of these

59. Which of the following is to be added to make $x^6 - 6x^4 + 4x^3 + 8x^2 - 10x + 3$ a perfect square?
(a) $(x - 1)^2$ (b) $(x - 2)^2$
(c) $(2x - 3)^2$ (d) $(2x + 1)^2$

60. Resolve into factors: $\left(\sum_{x,y,z}x\right)^3-\sum_{x,y,z}x^3$.
(a) $(x + y)(y + z)(z + x)$
(b) $-(x + y)(y + z)(z + x)$
(c) $3(x + y)(y + z)(z + x)$
(d) $-3(x + y)(y + z)(z + x)$

61. Find the square root of $\frac{a^2}{4}+\frac{1}{a^2}-\frac{1}{a}+\frac{a}{2}-\frac{3}{4}$.

(a) $\frac{a}{2}-\frac{1}{a}+\frac{1}{2}$ (b) $\frac{a}{2}+\frac{2}{a}-1$

(c) $\frac{a}{2}+\frac{1}{a}-\frac{1}{2}$ (d) $\frac{a}{2}-\frac{2}{a}-\frac{1}{2}$

62. $\frac{(x+y)^3+(x-y)^3}{2}-y(3x^2+y^2)=$ ______.

(a) x^3-y^3 (b) $(x-y)^3$

(c) $2x^3-3x^2y$ (d) x^3-6xy^2

63. Find the square root of $(4a+5b+5c)^2-(5a+4b+4c)^2+9a^2$.

(a) $\sqrt{3}\,(b+c)$

(b) $3(b+c-a)$

(c) $3(b+c)$

(d) $3(b+c-a)$

64. $\frac{(a-b)^3-(a+b)^3}{2}+a(a^2+3b^2)=$ ________.

(a) a^3-b^3 (b) $(a+b)^3$

(c) a^3+b^3 (d) $(a-b)^3$

65. The square root of $(3a+2b+3c)^2-(2a+3b+2c)^2+5b^2$ is

(a) $\sqrt{5}(a+b+c)$ (b) $\sqrt{5}(a+b)$

(c) $\sqrt{5}(a+c)$ (d) $\sqrt{5}(a+c-b)$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. biquadratic
2. 15
3. False
4. $x^2 + 10x + 1$
5. $a^3 + b^3 + c^3 = 3abc$
6. $(x - y)(x + y)(x^2 + y^2 - xy)(x^2 + y^2 + xy)$
7. $\sqrt{8}x^7y^2$
8. any real number
9. $x + \dfrac{1}{x}$
10. $4x^2 - \dfrac{7}{2}x + \dfrac{5}{2}$
11. $-288.$
12. $(2x - 1)\ (3x + 2)$
13. The LCM of given polynomials $= 3 \times 5 \times x^3 \times y^3 \times z^3 = 15x^3y^3z^3$

    The HCF of given polynomials $= 3x^2yz^2$
14. $f(2) = 2^{15}$
15. $f\left(\dfrac{1}{2}\right) = \dfrac{101}{32}$
16. $\sqrt{a} - \sqrt{b}$
17. symmetric
18. $a^{\frac{m^2}{2}} \cdot b^{\frac{n^2}{2}}$
19. 20
20. $ab(a^2 + b^2)(a + b)(a - b)$
21. 49
22. $4x^5 + x^4 - x^3$
23. $m^4(m + 1)(m^2 - m + 1)$
24. $\dfrac{1}{6}(a-2)(a-4)$
25. $a = \pm\dfrac{3}{4}$
26. $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
27. $(a^2 + 1)(a + 11)$
28. -461700
29. $-8x^2 + 6x - 14$
30. 8

Short Answer Type Questions

31. 1, 1
32. $a^2(b + c)b^2(c + a)c^2(a + b)$
33. $3(a - b)(b - c)(c - a)$
34. $c^2(a^2 - b^2) + a^2(b^2 - c^2) + b^2(c^2 - a^2)$
35. $-x^3 - 16x^2 + 19x - 53$
36. $x^6 - 4x^5 - 2x^4 + 4x^3 + 5x^2 - 1$
37. $x^2 + x - 30$
38. $\left(a + \dfrac{x}{4} - \dfrac{1}{2}\right)\left(a^2 + \dfrac{x^2}{16} + \dfrac{1}{4} - \dfrac{ax}{4} + \dfrac{x}{8} + \dfrac{a}{2}\right)$
39. HCF $= 9(x + 2)^2(x - 1)^2(x + 3)^4$,

    LCM $= 1260\ (x + 2)^5(x - 1)^5(x + 3)^5$
40. $a = 6,\ b = 2$
41. $(2^{23} - 1)x + (2 - 2^{23})$
43. $2c$
44. $25abc$
45. $(x + 2)(x + 3)(x + 4)$

Essay Type Questions

46. $(x + 2)(x - 3)(2x + 1)(3x - 1)$
48. $p = -154,\ q = 121$
49. $3x^2 - 2x + 1$
50. $(a + b),\ (b + c),\ (c + a)$

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (d)
11. (c)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (b)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (a)

Level 2

31. (a)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (d)
41. (b)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (d)	**46.** (c)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (a)
51. (c)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (b)					

Level 3

56. (c)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (c)	**61.** (a)	**62.** (b)	**63.** (c)	**64.** (d)	**65.** (c)

CONCEPT APPLICATION

Level 1

1. Degree of AB = degree of A + degree of B.
2. Factorize the given expression.
3. Use the formula, HCF × LCM = product of polynomials.
4. Use remainder theorem.
5. Degree of every term should be same.
6. Use factorization concept.
7. Use remainder theorem.
8. Find the common factors with the least exponents.
9. Use factor theorem.
10. Take the terms in common and factorize.
11. An expression is symmetric if all the coefficients are equal.
12. Use $a^2 + b^2 + 2ab = (a + b)^2$ identity.
13. Use the concept of multiplication of polynomials.
14. Factorize the given polynomials.
15. Use the addition and subtraction concept of polynomials.
16. The degree of the homogeneous expression is 3.
17. (i) Factorize the given polynomials.
    (ii) Factorize the middle term such that product obtained is 22 and sum obtained is $-12\sqrt{2}$.
18. (i) Use factor theorem.
    (ii) Put $x = -1$.
    (iii) Check for what values of n, $(-1)^n + 1$ is divisible by $x + 1$.
19. Use division method to find p and q.
20. Apply the division method to find the square root.
21. Recall the concept of finding the square root of monomial.
22. Use algebraic identities.
23. Use the concept of polynomials multiplication.
24. Use the formula (LCM) (HCF) = $f(x) \cdot g(x)$
25. Use factor theorem.
26. Use factor theorem.
27. Use addition and subtraction concept of polynomials.
28. Find the square root.
29. (i) $x^n + y^n$ is always disable by $x + y$ if n is odd.
    (ii) $x^n - y^n$ is divisible by $x - y$, if n is odd number.
30. (i) The product of four consecutive numbers added to 1 is a perfect square.
    (ii) The continued product of 4 consecutive integers added by 1 is always a perfect square.

Level 2

31. (i) Divide A by B.
    (ii) Divide the polynomial A by B and then write the remainder.
32. (i) Take common terms and factorize.
    (ii) From the first two terms take x^3 common and from last two terms take y^3 common.
    (iii) Again take $x^2 - a^2$ common in the product.
    (iv) Now write the factors of $a^2 - b^2$ and $a^3 + b^3$.
33. (i) Factorize the polynomials.
    (ii) Find the HCF of the first two polynomials.
    (iii) Now find the HCF of the third polynomial and HCF of the first two polynomials.
34. (i) Factorize the polynomials and use factor theorem.
    (ii) $(x - 3)$ is a factor of $5x^2 - ax + 3$ and $2x - 1$ is a factor of $2x^2 + x + b$.
    (iii) Substitute $x = 1$ in $5x^2 - ax + 3$ and $x = \frac{1}{2}$ in $2x^2 + x + b$, then obtain the values of a and b.
35. (i) Use division algorithm.
    (ii) $f(x) = g(x)Q(x) + [ax + b]$ where $Q(x)$ is quotient and $(ax + b)$ is the remainder.
    (iii) Put $x = 1$ and $x = -1$, then find the values of a and b.
36. (i) Factorize the cyclic expression.
    (ii) Put $b = c$; the expression become zero, i.e., $b - c$ is a factor of the expression.

(iii) Similarly $(a - b)$ and $(c - a)$ are also the factors of the expression.

(iv) Since the degree of the expression is 4, the fourth factor is $k(a + b + c)$.

37. (i) Find a factor by hit and trial method and remaining factors by division method.

(ii) By observation, sum of the coefficients of odd terms = Sum of the coefficients of even terms.

(iii) Now divide the expression by $y + 1$.

(iv) Find the roots of quotient by trial and error method.

38. (i) LCM is the product of the all the factors with highest powers.

(ii) Use the LCM concept and find the values of a and b.

(iii) $a + b + c$ is least when $c = 0$.

39. (i) Find the common and uncommon factors with highest powers.

(ii) Find LCM of 195 and 221.

(iii) Find the LCM of the expressions given.

40. (i) Use factor theorem.

(ii) If $x - a$ is HCF of $f(x)$, then $f(a) = 0$.

(iii) Substitute $x = 2$ in $f(x)$ or $g(x)$ and obtained the value of k.

41. (i) HCF is the product of all common factors with least exponents.

(ii) Use the HCF concept and find the values of a, b and c.

42. (i) Use remainder theorem.

(ii) Put $x = 0$, $x = -1$, and $x = -2$ in the given expression, and obtain the corresponding reminders.

43. (i) Find the LCM of denominations and add the polynomials.

(ii) Take LCM.

(iii) Find the factors and simplify.

44. (i) Use algebraic identities to factorize the given polynomial.

(ii) Multiply the first three terms with xyz.

(iii) Now express the expression in $(a + b + c)^2$ form.

45. (i) Use algebraic identities to factorize the given polynomial.

(ii) Add and subtract 4 to the expression.

(iii) Express the expression of the form $a^2 - b^2$.

46. BCAD is the required sequential order of steps in solving the given problem.

47. CBDAE is the required sequential order.

48. Given polynomials are $8x^3y^a$ and $12x^by^2$ HCF = $4x^ay^b$

$\Rightarrow \quad a \le 3$ or $a \le b$ and $b \le a$ or $b \le 2 \Rightarrow a = b$

$\therefore$ Each of a and b is less than or equal to 2.

$\therefore$ The maximum value of $a + b$ is $2 + 2$, i.e., 4.

49. Let $f(x) = 5x^5 - 3x^3 + 2x^2 - k$

$f(-1) = 1$ (given)

$\Rightarrow 5(-1)^5 - 3(-1)^3 + 2(-1)^2 - k = 1$

$-5 + 3 + 2 - k = 1$

$\therefore k = -1$.

50. $a^3 + b^3 + 3ab - 1$

$= a^3 + b^3 + (-1)^3 - 3 \cdot a \cdot b(-1)$

$= (a + b - 1)(a^2 + b^2 + 1 - ab + b + a)$.

51. Degree of $f = 3$

Degree of $g = 4$

$\Rightarrow$ Degree of $(f - g) = 4$, since the term of degree 4 cannot be vanished.

52. $\dfrac{x^2}{9} + \dfrac{9}{4x^2} - \dfrac{x}{3} + \dfrac{3}{2x} - \dfrac{5}{4}$

$$\begin{array}{l|l|l}
\dfrac{x}{3} & \dfrac{x^2}{9} + \dfrac{9}{4x^2} - \dfrac{x}{3} - \dfrac{3}{2x} + \dfrac{5}{4} & \dfrac{x}{3} - \dfrac{1}{2} + \dfrac{3}{2x} \\
\hline
\dfrac{2x}{3} - \dfrac{1}{2} & \dfrac{9}{4x^2} - \dfrac{x}{3} - \dfrac{3}{2x} + \dfrac{5}{4} & \\
 & \quad\; -\dfrac{x}{3} \qquad\quad +\dfrac{1}{4} & \\
 & \quad\; + \qquad\qquad - & \\
\hline
\dfrac{2x}{3} - 1 + \dfrac{3}{2x} & \dfrac{9}{4x^2} - \dfrac{3}{2x} + 1 & \\
 & \dfrac{9}{4x^2} - \dfrac{3}{2x} + 1 & \\
\hline
 & 0 &
\end{array}$$

$$\therefore \sqrt{\dfrac{x^2}{9} + \dfrac{9}{4x^2} - \dfrac{x}{3} - \dfrac{3}{2x} + \dfrac{5}{4}} = \dfrac{x}{3} - \dfrac{1}{2} + \dfrac{3}{2x}.$$

53. $(xy + xz - yz)^2 - 4xyz(x - y)$

$= (xy + z(x - y))^2 - 4(xy)\,[z(x - y)]$

$= [xy - z(x - y)]^2$

$[\because (a + b)^2 - 4ab = (a - b)^2]$

$= (xy - zx + yz)^2$

$= (xy + yz - zx)^2$

$\therefore$ The square root of the given expression is $(xy + yz - zx)$.

54. $\sum\limits_{x,y,z} (x+1)^2 - \left(\sum\limits_{x,y,z} x\right)^2 - 3$

$= (x + 1)^2 + (y + 1)^2 + (z + 1)^2 - (x + y + z)^2 - 3$

$= x^2 + 2x + 1 + y^2 + 2y + 1 + z^2 + 2z + 1 - x^2 - y^2 - z^2 - 2xy - 2yz - 2zx - 3$

$= 2x + 2y + 2z - 2xy - 2yz - 2zx$

$= 2\sum\limits_{x,y,z} x - 2\sum\limits_{x,y,z} xy = 2\left(\sum\limits_{x,y,z} x - \sum\limits_{x,y,z} xy\right)$

55. $\left(\sum\limits_{x,y,z} x\right)^2 - \left(\sum\limits_{x,y,z} x^2\right)$

$= (x + y + z)^2 - (x^2 + y^2 + z^2)$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - x^2 - y^2 - z^2$

$= 2xy + 2yz + 2zx$

$= 2\left(\sum\limits_{x,y,z} xy\right).$

Level 3

56. (i) Find the square root and equate it to $ax^2 + bx + c$.

(ii) Square on both sides and obtain the values of a, b and c.

(iii) Verify the relation between a, b and c.

57. (i) Find average of the given polynomials and then apply the division method.

(ii) Average of a, b and c is $\dfrac{a+b+c}{3}$.

(iii) Express the numerator in the form of $(a + b + c)^2$.

61.

$$
\begin{array}{r|l|l}
\cline{2-3}
\dfrac{a}{2} & \dfrac{a^2}{4} + \dfrac{1}{a^2} - \dfrac{1}{a} + \dfrac{a}{2} - \dfrac{3}{4} - \dfrac{a^2}{4} & \dfrac{a}{2} + \dfrac{1}{2} - \dfrac{1}{a} \\
\cline{2-3}
\dfrac{2a}{2} + \dfrac{1}{2} & \dfrac{a^2}{4} + \dfrac{1}{a^2} - \dfrac{1}{a} + \dfrac{a}{2} - \dfrac{3}{4} - \dfrac{a^2}{4} & \\
\cline{2-2}
a + 1 - \dfrac{1}{a} & \quad \dfrac{1}{a^2} - \dfrac{1}{a} - 1 & \\
 & \quad \dfrac{1}{a^2} - \dfrac{1}{a} - 1 & \\
\cline{2-2}
 & \quad 0 & \\
\cline{2-2}
\end{array}
$$

$\therefore$ The square root of the given expression is $\dfrac{a}{2} - \dfrac{1}{a} + \dfrac{1}{2}$.

62. $\dfrac{(x+y)^3 + (x-y)^3}{2} - (y^3 + 3x^2y)$

$= \dfrac{x^3 + y^3 + 3x^2y + 3xy^2 + x^3 - y^3 - 3x^2y + 3xy^2}{2} - y^3 - 3x^2y$

$= \dfrac{2x^3 + 6xy^2}{2} - y^3 - 3x^2y$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (x - y)^3.$

63. $(4a + 5b + 5c)^2 - (5a + 4b + 4c)^2 + 9a^2$

$= (4a + 5b + 5c + 5a + 4b + 4c)\,(4a + 5b + 5c - 5a - 4b - 4c) + 9a^2$

$= (9a + 9b + 9c)(-a + b + c) + 9a^2$

$= 9(a + b + c)(-a + b + c) + 9a^2$

$= 9(b + c)^2 - 9a^2 + 9a^2$

$= 9(b + c)^2.$

$\therefore$ The square root of the given expression is $\sqrt{9(b+c)^2} = 3(b + c)$.

64. $\dfrac{(a-b)^3 - (a+b)^3}{2} + a(a^2 + 3b^2)$

$= \dfrac{a^3 - 3a^2b + 3ab^2 - b^3 - a^3 - 3a^2b - 3ab^2 - b^3}{2}$

$+ a^3 + 3ab^2$

$= \dfrac{-6a^2b - 2b^3}{2} + a^3 + 3ab^2$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$= (a - b)^3$.

65. $(3a + 2b + 3c)^2 - (2a + 3b + 2c)^2 + 5b^2$

$= (3a + 2b + 3c + 2a + 3b + 2c)\ (3a + 2b + 3c - 2a - 3b - 2c) + 5b^2$

$= (5a + 5b + 5c)(a - b + c) + 5b^2$

$= 5(a + c + b)(a + c - b) + 5b^2$

$= 5[(a + c)^2 - b^2] + 5b^2$

$= 5(a + c)^2$

$\therefore$ Square root of the given expression is

$\sqrt{5(a+c)^2} = \sqrt{5}(a+c)$.

HINTS AND EXPLANATION

Chapter

4

Linear Equations and Inequations

REMEMBER

Before beginning this chapter, you should be able to:

- Understand basic terms such as linear equation and inequations in mathematics
- Solve word problems and applications of simple algebraic equations

KEY IDEAS

After completing this chapter, you should be able to:

- Solve two simultaneous linear equations
- Learn the graphical representation of linear equations and inequations
- Understand the application of simultaneous linear equations
- Solve word problems on linear equations and inequations
- Obtain the absolute value

INTRODUCTION

In most cases, while solving the problem, first we need to frame an equation. In this chapter, we learn how to frame and solve equations. Framing an equation is more difficult than solving it. Now, let us review the basic concepts related with equations and inequations.

Algebraic Expressions

Expressions of the form $2x$, $3x + 5$, $4x - 2y$, $2x^2 + 3\sqrt{y}$, $\frac{3x^3}{2\sqrt{y}}$ are algebraic expressions. $3x$ and 5 are the terms of $3x + 5$, and $4x$ and $2y$ are the terms of $4x - 2y$. Algebraic expressions are made up of numbers, symbols and the basic arithmetical operations.

Open Sentences

Sentences which are true for some values of the variable and false for the other values of the variable are called open sentences. When a certain value is substituted for the variable, the sentence becomes a statement, either true or false.

Example: $x + 1 = 4$. This is an open sentence. When we substitute 3 for x, we get a true statement. When we substitute any other value, we get a false statement.

Example: $x + 1 < 3$. This is another open sentence. If we substitute any value less than 2 for x we get a true statement. If we substitute 2 or any number greater than 2, we get a false statement.

EQUATION

An open sentence containing the equality sign is an equation. In other words, an equation is a sentence in which there is an equality sign between two algebraic expressions.

Example: $2x + 5 = x + 3$, $3y - 4 = 20$, $5x + 6 = x + 1$ are equations.

Linear Equation

An equation, in which the highest index of the unknowns present is one, is a linear equation.

Example: $2(x + 5) = 18$, $3x - 2 = 5$, $x + y = 20$ and $3x - 2y = 5$ are some linear equations.

Simple Equation

A linear equation which has only one unknown is a simple equation.

$3x + 4 = 16$ and $2x - 5 = x + 3$ are examples of simple equations. The part of an equation which is to the left side of the equality sign is known as the left hand side, abbreviated as LHS. The part of an equation which is to the right side of the equality sign is known as the right hand side, abbreviated as RHS. The process of finding the value of an unknown in an equation is called solving the equation. The value (values) of the unknown found after solving an equation is (are) called the solution(s) or the root(s) of the equation.

Before we learn how to solve an equation, let us review the basic properties of equality.

Solving an Equation in One Variable

The following steps are involved in solving an equation:

Step 1: Always ensure that the unknown quantities are on the LHS and the known quantities or constants on the RHS.

Step 2: Add all the terms containing the unknown quantities on the LHS and all the known quantities on the RHS so that each side of the equation contains only one term.

Step 3: Divide both sides of the equation by the coefficient of the unknown.

EXAMPLE 4.1

If $2x + 10 = 40$, find the value of x.

SOLUTION

Step 1: Group the known quantities as the RHS of the equation, i.e., $2x = 40 - 10$

Step 2: Simplify the numbers on the RHS $\Rightarrow 2x = 30$.

Step 3: Since 2 is the coefficient of x, divide both sides of the equation by 2.

$$\frac{2x}{2} = \frac{30}{2} \Rightarrow x = 15.$$

EXAMPLE 4.2

Solve for x: $5x - 8 = 3x + 22$.

SOLUTION

Step 1: $5x - 3x = 22 + 8$

Step 2: $2x = 30$

Step 3: $\frac{2x}{2} = \frac{30}{2} \Rightarrow x = 15.$

EXAMPLE 4.3

A teacher has 45 chocolates. After giving two chocolates to each student, she is left with 7 chocolates. How many students are there in the class?

SOLUTION

Let the number of students in the class be x.

Total number of chocolates distributed = $2x$

Total number of chocolates left with the teacher = 7

$$2x + 7 = 45$$
$$\Rightarrow 2x = 45 - 7$$
$$\Rightarrow 2x = 38 \Rightarrow x = 19.$$

There are 19 students in the class.

SIMULTANEOUS LINEAR EQUATIONS

We have learnt how to solve an equation with one unknown quantity. Very often we come across equations involving more than one unknown quantity. In such cases we require more than one condition or equation. Generally, when there are two unknowns, we require two equations to solve the problem. When there are three unknowns, we require three equations and so on.

We need to find the values of the unknowns that satisfy all the given equations. Since the values satisfy all the given equations we call them simultaneous equations. In this chapter, we deal with simultaneous (linear) equations with two unknowns.

Let us consider the equation, $3x + 4y = 15$, which contains two unknown quantities x and y.

Here, $4y = 15 - 3x$

$$\Rightarrow y = \frac{15 - 3x}{4} \tag{1}$$

In the above equation for every value of x, there exists a corresponding value for y.

When $x = 1$, $y = 3$.

When $x = 2$, $y = \frac{9}{4}$ and so on.

If there is another equation, of the same kind, say $5x - y = 2$, from this we get,

$$y = 5x - 2 \tag{2}$$

If we need the values of x and y such that both the equations are satisfied, then

$$\frac{15 - 3x}{4} = 5x - 2 \Rightarrow 15 - 3x = 20x - 8$$

$$\Rightarrow 23x = 23 \Rightarrow x = 1.$$

On substituting the value of $x = 1$ in the first equation, we get,

$$y = \frac{15 - 3(1)}{4} \Rightarrow y = 3.$$

If both the equations are to be satisfied by the same values of x and y, there is only one solution.

Thus we can say that when two or more equations are satisfied by the same values of unknown quantities then those equations are called simultaneous equations.

Solving Two Simultaneous Equations

When two equations, each in two variables, are given, they can be solved in four ways.

1. Elimination by cancellation.
2. Elimination by substitution.
3. Adding the two equations and subtracting one equation from the other.
4. Graphical method.

Elimination by Cancellation

EXAMPLE 4.4

If $4x + 3y = 25$ and $5x + 2y = 26$, then find the values of x and y.

SOLUTION

$$4x + 3y = 25 \tag{1}$$

$$5x + 2y = 26 \tag{2}$$

Using this method, the two equations are reduced to a single variable equation by eliminating one of the variables.

Step 1: Here, let us eliminate the y term, and in order to eliminate the y term, equate the coefficient of y in both the equations.

$$(4x + 3y = 25)2 \Rightarrow 8x + 6y = 50 \tag{3}$$

$$(5x + 2y = 26)3 \Rightarrow 15x + 6y = 78 \tag{4}$$

Step 2: Subtract equation (3) from (4),

$$(15x + 6y) - (8x + 6y) = 78 - 50 \Rightarrow 7x = 28$$

$$\Rightarrow x = 4.$$

Step 3: Substitute the value of x in Eq. (1) or Eq. (2) to find the value of y. Substituting the value of x in the first equation, we have,

$$4(4) + 3y = 25$$

$$\Rightarrow 3y = 25 - 16$$

$$\Rightarrow 3y = 9 \Rightarrow y = 3.$$

$\therefore$ The solution of the given pair of equations is (x, y), i.e., $(4, 3)$.

Elimination by Substitution

EXAMPLE 4.5

If $3x - 2y = 12$ and $6x + y = 9$, then find the values of x and y.

SOLUTION

$$3x - 2y = 12 \tag{1}$$

$$6x + y = 9 \tag{2}$$

Using this method, the two equations are reduced to a single variable equation by substituting the value of one variable, obtained from one equation, in the other equation.

Step 1: Using the first equation, find x in terms of y, i.e., $3x - 2y = 12 \Rightarrow 3x = 12 + 2y$

$$\Rightarrow x = \frac{12 + 2y}{3} \tag{3}$$

Step 2: Substitute the value of x in the second equation to find the value of y, i.e., $6x + y = 9$

$$\Rightarrow 6\left(\frac{12 + 2y}{3}\right) + y = 9$$

Step 3: Simplify the equation in terms of y and find the value of y.

$$2(12 + 2y) + y = 9$$

$$\Rightarrow 24 + 4y + y = 9$$

$$\Rightarrow 5y = 9 - 24$$

$$\Rightarrow y = -3.$$

Step 4: Substituting the value of y in Eq. (3) we get $x = 2$.

$\therefore$ The solution for the given pair of equations is (x, y), i.e., $(2, -3)$.

Adding Two Equations and Subtracting One Equation from the Other

We use this method, in case of solving the system of linear equations of the form $ax + by = p$ and $bx + ay = q$.

EXAMPLE 4.6

Solve $4x + 5y = 37$ and $5x + 4y = 35$.

SOLUTION

Given, $$4x + 5y = 37 \tag{1}$$

and $$5x + 4y = 35 \tag{2}$$

Step 1: Adding both equations, we get

$$9x + 9y = 72$$

$$\Rightarrow 9(x + y) = 9 \times 8$$

$$\Rightarrow x + y = 8 \tag{3}$$

Step 2: Subtracting Eq. (2) from the Eq. (1),

$$(4x + 5y) - (5x + 4y) = 37 - 35$$

$$\Rightarrow -x + y = 2 \tag{4}$$

Step 3: Adding the Eqs. (3) and (4),

$$(x + y) + (-x + y) = 8 + 2$$

$$\Rightarrow 2y = 10 \Rightarrow y = 5.$$

Substituting $y = 5$ in any of the Eqs. (1), (2), (3) or (4), we get $x = 3$.

$\therefore$ The solution of the pair of equations is (x, y), i.e., $(3, 5)$.

Note Choosing a particular method to solve a pair of equations makes the simplification easier. One can learn which method is easiest to solve a pair of equations by becoming familiar with the different methods of solving the equation.

Graphical Method: Plotting the Points

If we consider any point in a plane, then we can determine the location of the given point, i.e., we can determine the distance of the given point from X-axis and Y-axis. Therefore, each point in the plane represents the distance from both the axes. So, each point is represented by an ordered pair and it consists of x-coordinate and y-coordinate. The first element of an ordered

pair is called x-coordinate and the second element of an ordered pair is called y-coordinate. In the first quadrant Q_1, both the x-coordinate and y-coordinate are positive real numbers. In the second quadrant Q_2, x-coordinates are negative real numbers and y-coordinates are positive real numbers. In the third quadrant Q_3 both the x-coordinate and y-coordinate are negative real numbers. In the fourth quadrant Q_4, x-coordinates are positive real numbers and y-coordinates are negative real numbers. And the origin is represented by (0, 0).

Consider the point (2, 3). Here, 2 is the x-coordinate and 3 is the y-coordinate. The point (2, 3) belongs to the first quadrant. The point (2, 3) is 2 units away from the Y-axis and 3 units away from the X-axis. If we consider the point (−3, −5), −3 is x-coordinate and −5 is y-coordinate. The point (−3, −5) belongs to Q_3 and is 3 units away from the Y-axis and 5 units away from the X-axis. The method of plotting a point in a co-ordinate plane was explained by a René Descartes, a French mathematician.

To plot a point say $P(-3, 4)$, we start from the origin and proceed 3 units towards the left hand side along the X-axis (i.e., negative direction as x-coordinate is negative), and from there we move 4 units upwards along the Y-axis (i.e., positive direction as y-coordinate is positive).

EXAMPLE 4.7

Plot the following points on the co-ordinate plane:

$A(3, 5)$, $B(2, -4)$, $C(-2, 7)$, $D(-3, -4)$, $E(0, -5)$ and $F(5, 0)$

SOLUTION

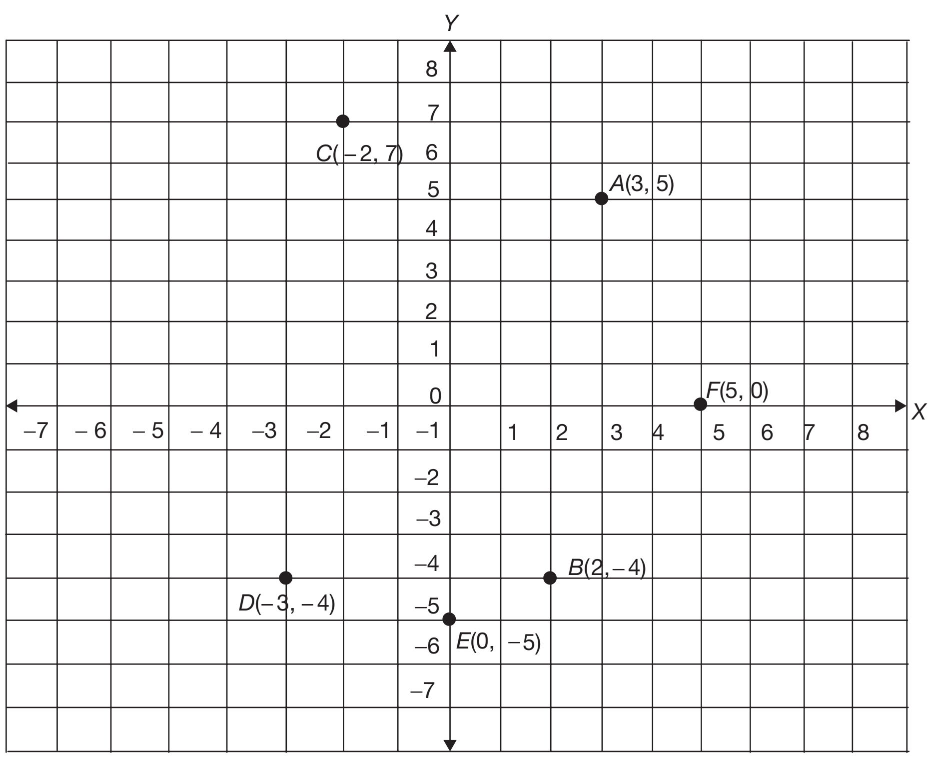


Figure 4.1

EXAMPLE 4.8

Plot the following points on the coordinate plane. What do you observe?

(a) (–2, –3), (–1, –3), (0, –3), (2, –3)

(b) (5, 3), (5, 2), (5, 1), (5, 0), (5, –1).

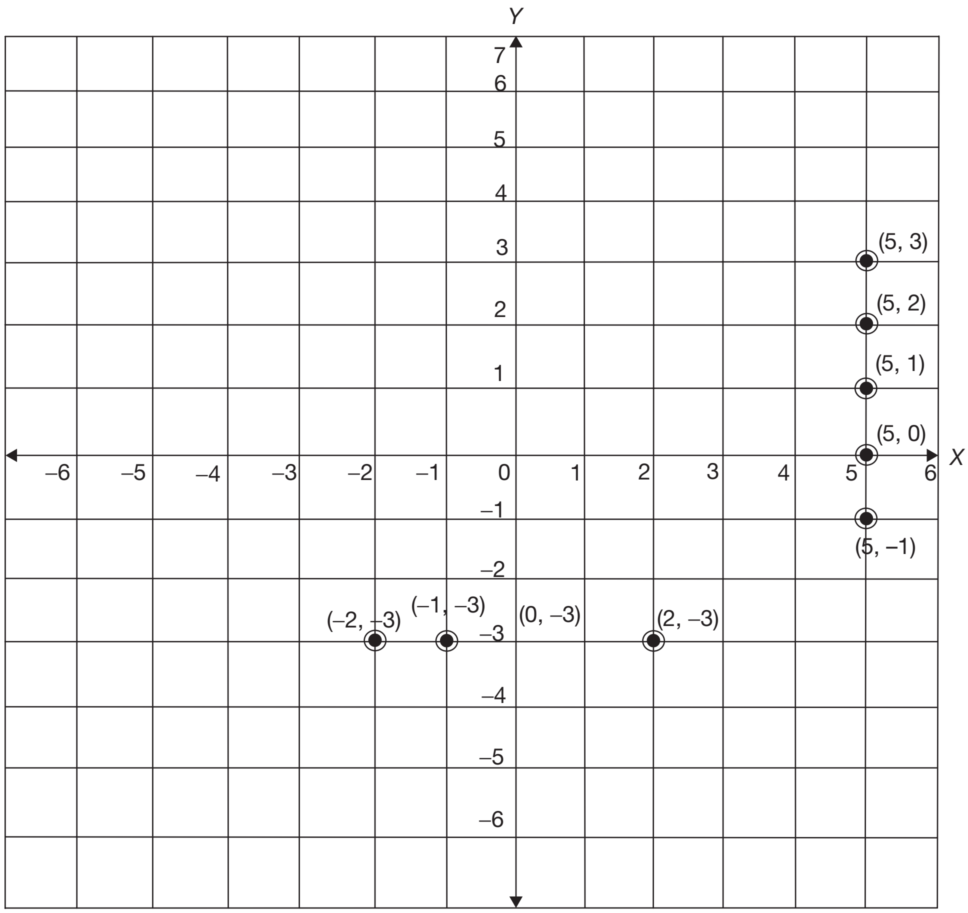


Figure 4.2

SOLUTION

(a) (–2, –3), (–1, –3), (0, –3), (2, –3)

- **(i)** The above points lie on the same straight line which is perpendicular to the *Y*-axis.
- **(ii)** The y-coordinates of all the given points are the same, i.e., $y = -3$.
- **(iii)** So, the straight line passing through the given points is represented by $y = -3$.
- **(iv)** Therefore, the line $y = -3$, is parallel to *X*-axis which intersects *Y*-axis at (0, –3).

(b) (5, 3), (5, 2), (5, 1), (5, 0), (5, –1)

- **(i)** The above points lie on the same straight line which is perpendicular to the *X*-axis.
- **(ii)** The x-coordinate of each of the given points is the same, i.e., $x = 5$.
- **(iii)** So, the straight line passing through the given points is represented by $x = 5$.
- **(iv)** Therefore, the line $x = 5$ is parallel to the *Y*-axis which intersects the *X*-axis at (5, 0).

Notes

1. The y-coordinate of every point on the X-axis is zero, i.e., $y = 0$. Therefore, the X-axis is denoted by $y = 0$.
2. The x-coordinate of every point on the Y-axis is zero, i.e., $x = 0$. Therefore, the Y-axis is denoted by $x = 0$.

Example: The line $x = 4$ is parallel to Y-axis which intersects X-axis at (4, 0).

Example: The line $y = 5$ is parallel to X-axis which intersects Y-axis at (0, 5).

EXAMPLE 4.9

Plot the following points on the coordinate plane and what do you observe?

(–3, 3), (–2, 2), (–1, 1), (0, 0), (1, –1), (2, –2), (3, –3)

SOLUTION

(a) All the given points lie on the same straight line.

(b) Every point on the straight line represents $y = -x$.

(c) The above line with the given ordered pairs is represented by the equation $y = -x$.

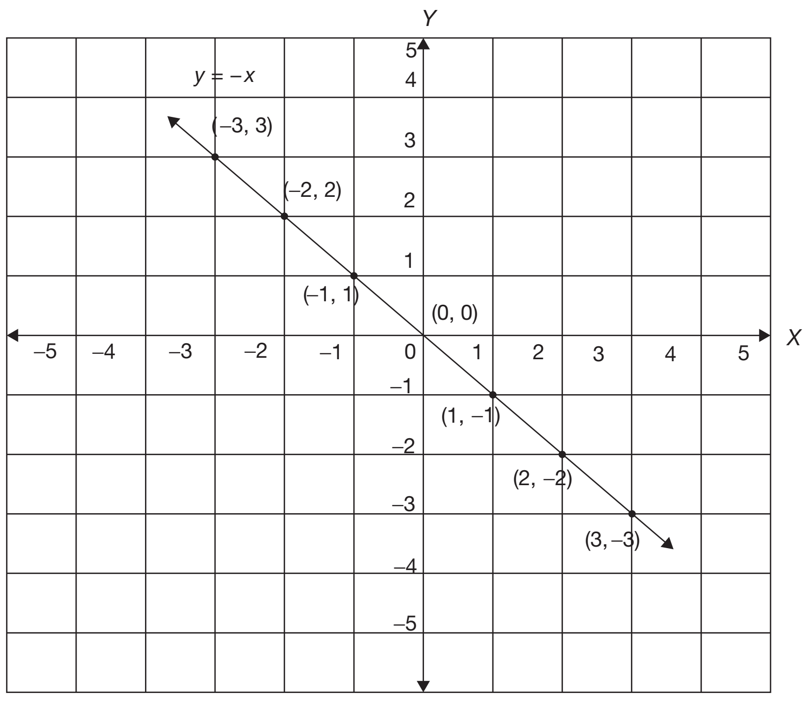


Figure 4.3

EXAMPLE 4.10

Draw the graph of the equation $y = 3x$ where R is the replacement set for both x and y.

SOLUTION

x	–2	–1	0	1	2
$y = 3x$	–6	–3	0	3	6

Some of the ordered pairs which satisfy the equation $y = 3x$ are (–2, –6), (–1, –3), (0, 0), (1, 3), (2, 6). By plotting the above points on the graph sheet, we get the following:

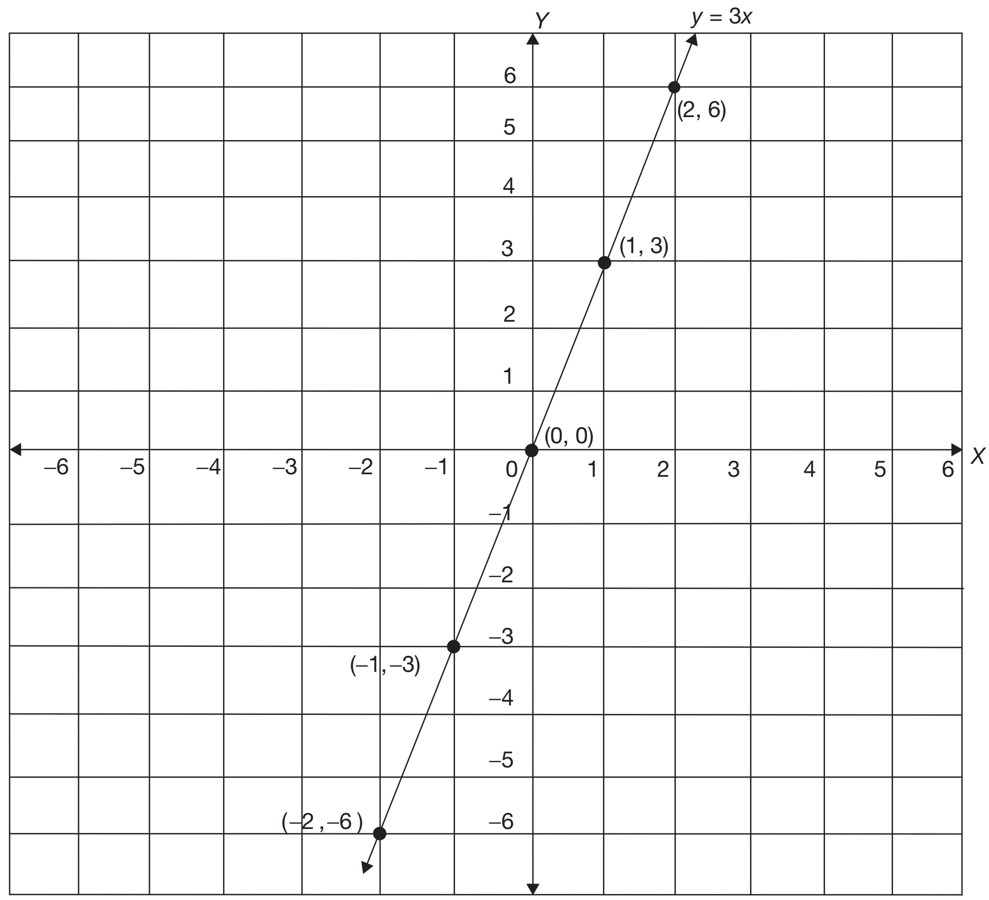


Figure 4.4

EXAMPLE 4.11

Draw the graph of the equations $x + y = 4$ and $x - y = 2$. What do you notice?

SOLUTION

(a) $x + y = 4$

x	–2	–1	0	1	2	3	4
$y = 4 - x$	6	5	4	3	2	1	0

Some of the ordered pairs which satisfy the equation $x + y = 4$ are (–2, 6), (–1, 5), (0, 4), (1, 3), (2, 2), (3, 1), (4, 0).

(b) $x - y = 2$

x	–2	–1	0	1	2	3	4	5
$y = x - 2$	–4	–3	–2	–1	0	1	2	3

∴ Some of the ordered pairs which satisfy the equation $x - y = 2$ are $(-2, -4)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(2, 0)$, $(3, 1)$, $(4, 2)$, $(5, 3)$.

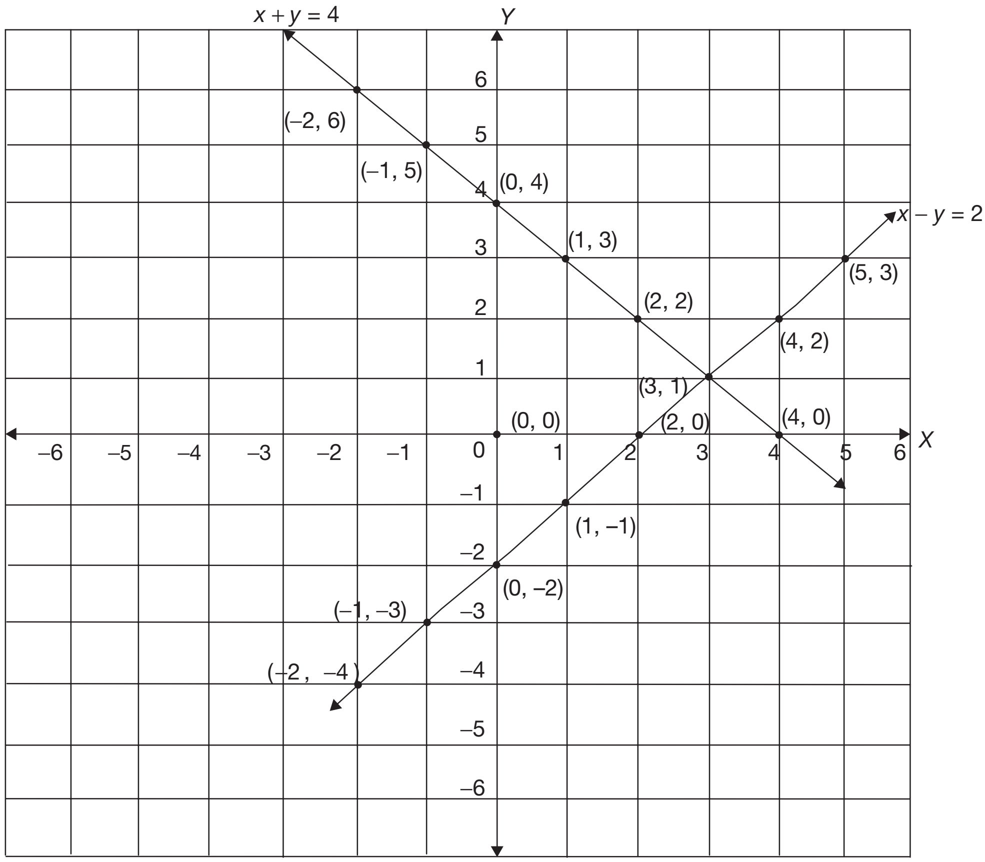


Figure 4.5

From the above graph, we notice that the two given equations intersect at the point (3, 1).

That is, $x + y = 4$ and $x - y = 2$ have a common point, (3, 1). Therefore, (3, 1) is the solution of the equations $x + y = 4$ and $x - y = 2$.

Verification:

$$x + y = 4 \quad (1)$$

$$x - y = 2 \quad (2)$$

Solving Eqs. (1) and (2), we get, $x = 3$ and $y = 1$.

∴ (3, 1) is the solution of $x + y = 4$ and $x - y = 2$.

Note From the above example, we notice that we can find the solution for simultaneous equations by representing them in a graph, i.e., by using the graphical method.

EXAMPLE 4.12

Solve the following equations $x + 4y = 2$ and $4x - y = -9$ by the graphical method and check the result.

SOLUTION

(a) $x + 4y = 2$

x	2	6	−2	−6
$y = \frac{2-x}{4}$	0	−1	1	2

Some of the ordered pairs which satisfy the equation $x + 4y = 2$ are (2, 0), (6, −1), (−2, 1), (−6, 2).

(b) $4x - y = -9$

x	−1	−2	0
$y = 4x + 9$	5	1	9

Some of the ordered pairs which satisfy the equation $4x - y = -9$ are (−1, 5), (−2, 1), (0, 9). By plotting the above points on graph sheet, we get the following graph:

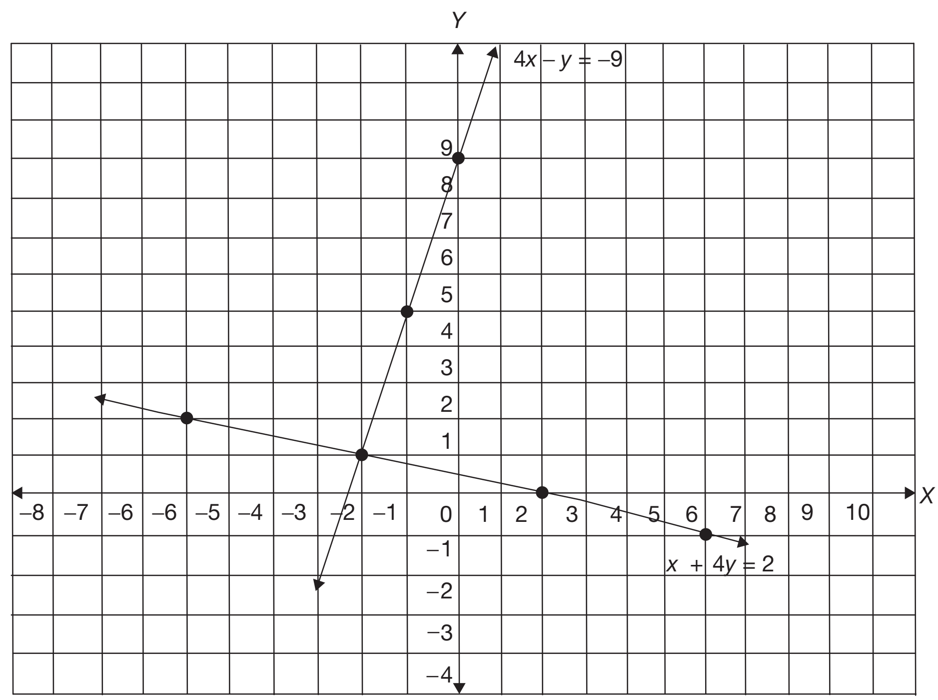


Figure 4.6

From the graph, it is clear that, the given equations intersect at (−2, 1). Therefore, it is the common solution for the equations $x + 4y = 2$ and $4x - y = -9$.

Verification:

$$x + 4y = 2 \quad (1)$$

$$4x - y = -9 \quad (2)$$

By solving Eqs. (1) and (2), we get $x = -2$ and $y = 1$.

∴ The solution is (−2, 1)

Hence, verified.

Nature of Solutions

When we try to solve a pair of equations we could arrive at three possible results. They are, having

1. A unique solution.
2. An infinite number of solutions.
3. No solution.

Let the pair of equations be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where a_1, b_1, a_2 and b_2 are the coefficients of x and y terms, while c_1 and c_2 are the known constant quantities.

A Pair of Equations Having a Unique Solution

If, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then there will be a unique solution.

We have solved such equations in the previous examples of this chapter.

A Pair of Equations Having Infinite Solutions

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have infinite number of solutions.

Note In fact this means that there are no two equations as such and one of the two equations is simply obtained by multiplying the other with a constant. These equations are known as dependent equations.

EXAMPLE 4.13

Find out the number of solutions for the following equations:

$3x + 4y = 8$

$9x + 12y = 24$

SOLUTION

For these two equations $a_1 = 3$, $a_2 = 9$, $b_1 = 4$, $b_2 = 12$, $c_1 = -8$, $c_2 = -24$.

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Since, $\frac{3}{4} = \frac{4}{12} = \frac{-8}{-24}$

$\therefore$ The above pair of equations will have infinite solutions.

A Pair of Equations Having no Solution at All

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, will have no solution. Such lines are called parallel lines.

Notes

1. In other words, the two equations will contradict each other or be inconsistent with each other.
2. A pair of equations are said to be consistent if they have a solution (finite or infinite).

EXAMPLE 4.14

Find out the number of solutions for the following equations:

$4x + 5y = 20$

$8x + 10y = 30$

SOLUTION

For these two equations, $a_1 = 4$, $a_2 = 8$, $b_1 = 5$, $b_2 = 10$, $c_1 = -20$, $c_2 = -30$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{4}{8} = \frac{5}{10} \neq \frac{-20}{-30}$$

Hence, given pair of equations have no solution at all.

Word Problems and Application of Simultaneous Equations

In this chapter, we have discussed earlier that it is essential to have as many equations as there are unknown quantities to be determined. In word problems also, it is required that there are as many independent conditions as there are unknown quantities to be determined.

Let us understand with the help of the following examples as to how word problems can be solved using simultaneous equations.

EXAMPLE 4.15

The sum of two numbers is 30 and the larger number exceeds the smaller by 6. Find the numbers.

SOLUTION

Let the numbers be a and b.

Given that $a + b = 30$ and $a - b = 6$.

By adding both the equations,

$$2a = 36 \Rightarrow a = 18$$

Substituting $a = 18$ in the first equation, $18 + b = 30$

$$\Rightarrow b = 12$$

∴ The two numbers are 18 and 12.

EXAMPLE 4.16

In a fraction, if unity is added to the numerator and subtracted from the denominator, it becomes $\frac{2}{3}$. Instead, if unity is subtracted from the numerator and added to the denominator, it becomes $\frac{1}{2}$. Find the fraction.

SOLUTION

Let the fraction be $\frac{x}{y}$, applying the first condition, we get,

$$\frac{x+1}{y-1} = \frac{2}{3}$$
$$\Rightarrow 3x + 3 = 2y - 2$$
$$\Rightarrow 3x - 2y = -5 \qquad (1)$$

Applying the second condition, we get,

$$\frac{x-1}{y+1} = \frac{1}{2}$$
$$\Rightarrow 2x - 3 = 2y + 1$$
$$\Rightarrow 2x - y = 3 \qquad (2)$$

By solving the Eqs. (1) and (2) using any of the methods discussed earlier, we get $x = 11$ and $y = 19$.

$\therefore$ The fraction is $\frac{11}{19}$.

EXAMPLE 4.17

The sum of the digits of a two digit number is 8. If 18 is added to the number, then the resultant number is equal to the number obtained by reversing the digits of the original number. Find the original number.

SOLUTION

Let the number be in the form of $10x + y$, where x and y are the tens digit and the units digit respectively. Applying the first condition, we get

$$x + y = 8 \qquad (1)$$

Applying the second condition, given in the problem,

$$10x + y + 18 = 10y + x$$
$$\Rightarrow 9x - 9y = -18$$
$$\Rightarrow x - y = -2 \qquad (2)$$

By solving Eqs. (1) and (2), we get,

$x = 3$ and $y = 5$ and the number is 35.

EXAMPLE 4.18

Four years ago, age of a person was 4 times that of his son. Six years later, the age of the person will be 10 years less than thrice the age of his son. Find the present ages of the person and his son.

SOLUTION

Let the present ages of the person and his son be x years and y years respectively. Given,

$$x - 4 = 4(y - 4)$$
$$\Rightarrow x - 4 = 4y - 16$$
$$\Rightarrow x - 4y = -12 \quad (1)$$

And also given,

$$x + 6 = 3\ (y + 6) - 10$$
$$\Rightarrow x + 6 = 3y + 8$$
$$\Rightarrow x - 3y = 2 \quad (2)$$

By solving the Eqs. (1) and (2), we get,

$x = 44$ and $y = 14$.

EXAMPLE 4.19

For what values of k is the set of equations $2x - 3(2k - 1)y = 10$ and $3x + 4(k + 1)y = 20$ are consistent?

SOLUTION

Consider the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. If this pair of equations is consistent,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ or } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

As $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2},$$

let $a_1 = 2$, $b_1 = -3(2k - 1)$, $a_2 = 3$ and $b_2 = 4(k + 1)$

$$\text{Now, } \frac{2}{3} \neq \frac{-3(2k-1)}{4(k+1)} \Rightarrow 8k + 8 \neq -18k + 9 \Rightarrow 26k \neq 1 \Rightarrow k \neq \frac{1}{26}$$

$$\therefore k \in R - \left\{\frac{1}{26}\right\}.$$

LINEAR INEQUATIONS

Introduction

If a is any real number, then a is either positive or negative or zero. When a is positive, we write $a > 0$, which is read as 'a is greater than zero'. When a is negative, we write $a < 0$, which is read as 'a is less than zero'. If a is zero, we write $a = 0$ and in this case, a is neither positive nor negative.

The two signs > and < are called the 'signs of inequalities'.

Notation

1. '>' denotes 'greater than'.
2. '<' denotes 'less than'.
3. '≥' denotes 'greater than or equal to'.
4. '≤' denotes 'less than or equal to'.

Definition

For any two non-zero real numbers a and b,

1. a is said to be greater than b when $a - b$ is positive, i.e., $a > b$ when $a - b > 0$ and
2. a is said to be less than b when $a - b$ is negative, i.e., $a < b$ when $a - b < 0$.

Listed below are some properties/results which are needed to solve problems on inequalities. The letters a, b, c, d, etc. represent real numbers.

- For any two real numbers a and b, either $a > b$ or $a < b$ or $a = b$. This property is called Law of Trichotomy.
- If $a > b$, then $b < a$.
- If $a > b$ and $b > c$, then $a > c$. This property is called transitive property.
- If $a > b$ then $a + c > b + c$.
- If $a > b$ and $c > 0$, then $ac > bc$.
- If $a > b$ and $c < 0$, then $ac < bc$.
- If $a > b$ and $c > d$, then $a + c > b + d$.
- The square of any real number is always greater than or equal to 0.
- If $a > 0$, then $-a < 0$ and if $a > b$, then $-a < -b$.
- If a and b are positive numbers and $a > b$, then $\frac{1}{a} < \frac{1}{b}$.
- If a and b are negative numbers and $a > b$, then $\frac{1}{a} < \frac{1}{b}$.
- If $a > 0$ and $b < 0$, then $\frac{1}{a} > \frac{1}{b}$.

INEQUATION

An open sentence which consists of one of the symbols, viz., $>$, $<$, $\geq$, $\leq$ is called an inequation.

Examples: **1.** $3x - 8 > 8$ **2.** $2x^2 - 3x \leq 6$

Continued Inequation

Two inequations of the same type (i.e., both consisting of $>$ or $\geq$ or both consisting of $<$ or $\leq$) can be combined into a continued inequation as explained below.

Examples:

1. If $a < b$ and $b < c$, we can write $a < b < c$.
2. If $a \geq b$ and $b > c$, we can write $a \geq b > c$.

Linear Inequation

An inequation in which the highest degree of the variables present is one is called a linear inequation.

Examples:

1. $3x + 4 \leq 8 - 3x$ and $8x - 64 \geq 8 + 5y$ are some of the examples for linear inequations.
2. $5x^2 + 6 > 7$ and $6x^3 + 6y^3 \leq 8$ are some of the examples for non-linear inequations.

Solving Linear Inequation in One Variable

We are familiar with solving linear equations. Now let us look at solving some linear inequations in one variable.

EXAMPLE 4.20

Solve the following inequations:

(a) $5x - 3 \leq 12,\ X \in N$ **(b)** $2x - 4 < 4,\ x \in R$ **(c)** $3x - 1 \geq 5,\ x \in Z$

SOLUTION

(a) $5x - 3 \leq 12,\ X \in N$

$\Rightarrow 5x \leq 15 \Rightarrow x \leq 3$

$\Rightarrow x \in \{1, 2, 3\}$.

(b) $2x - 4 < 4,\ x \in R$

$\Rightarrow 2x < 8 \Rightarrow x < 4$

The set of all the numbers which are less than 4 is the solution set of the given inequation.

(c) $3x - 1 \geq 5,\ x \in Z$

$\Rightarrow 3x \geq 6 \Rightarrow x \geq 2$

The set of all the integers which are greater than or equal to 2 is the solution set of the given in equation.

EXAMPLE 4.21

Represent the following inequations on the number line.

(a) $x \geq -2$

SOLUTION

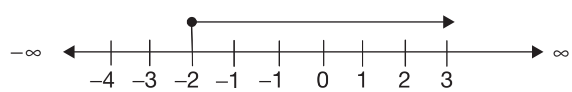


Figure 4.7

In the above figure, the ray drawn above the number line represents the solution set of the inequation. The end point of the ray is a part of the solution set. This is indicated by placing a solid dot at the end point.

(b) $x \leq 3$

SOLUTION

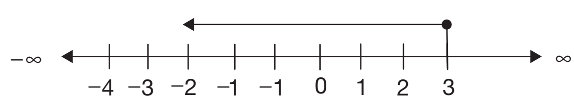


Figure 4.8

In the figure above, the ray drawn above the number line represents the solution set of the inequation. The end point of the ray is the part of the solution set. This is indicated by placing a solid dot at the end point.

EXAMPLE 4.22

Draw the graph $x \geq 1$ in the cartesian plane.

SOLUTION

We first draw the line $x = 1$ which is parallel to Y-axis and 1 unit away from the Y-axis. Then shade the region $x \geq 1$. The boundary line for the graph $x \geq 1$ is $x = 1$.

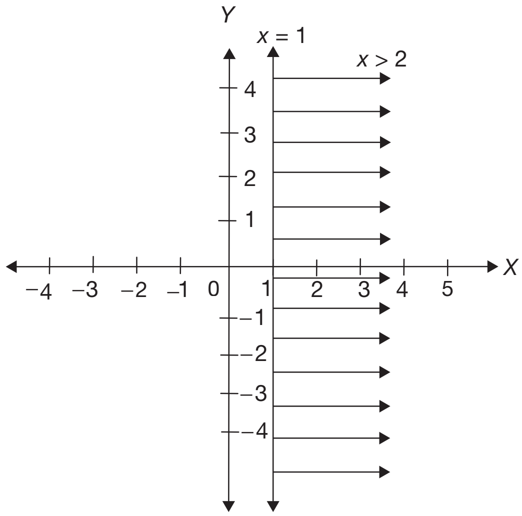


Figure 4.9

Notes

1. From the above graph, we notice that every line in a plane divides the plane into two half planes.
2. When the boundary line is included, the region is called a Closed Half Plane.
3. When the boundary line is not included, the region is called Open Half Plane.

EXAMPLE 4.23

Draw the graph of $y < 2$ in the Cartesian plane.

SOLUTION

Draw the line $y = 2$ with dots since the boundary line does not belong to the graph and then shade the region $y < 2$.

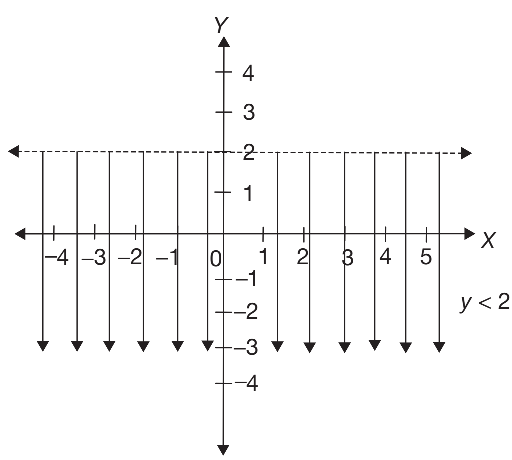


Figure 4.10

Solving Linear Inequations in Two Variables

We have already learnt about linear equations, their representation as graphs and solving linear equations in two variables with the help of graphs. These concepts are very useful in drawing the graphs representing linear inequations.

Finding Solution for the Inequation of Two Variables

We know that a linear equation of two variables of the form $ax + by + c = 0$ divides the plane into three mutually disjoint sets of points. They are as follows:

1. Points lying on the line $ax + by + c = 0$.
2. Points lying on one side of the line $ax + by + c = 0$.
3. Points lying on the other side of the line $ax + by + c = 0$.

The line $ax + by + c = 0$ divides the plane into two half planes and they are represented by $ax + by + c < 0$ and $ax + by + c > 0$.

1. The solution set of $ax + by + c > 0$ is the region that contain (0, 0) when $c > 0$ and the region does not contain (0, 0) when $c < 0$.
2. The solution set of $ax + by + c < 0$ is the region that contain (0, 0) when $c < 0$ and the region that does not contain (0, 0) when $c > 0$.
3. When $c = 0$ then origin lies on the line $ax + by = 0$. In this case, we choose any orbitary point that does not lie on $ax + by = 0$ and substitute in the given inequation. If it results a true statement then the region containing the point is the solution region otherwise the other region is the solution region of the inequation.

EXAMPLE 4.24

Draw the graph $x - y \leq 1$ in the cartesian plane.

STEPS

(a) First draw the line $x - y = 1$, i.e., $y = x - 1$

x	1	0	–1
y	0	–1	–2

As a first step, plot the ordered pairs (1, 0), (0, –1), (–1, –2) and then join them with a line.

(b) For shading the region we consider any point in one of the half planes.

Let us consider the point (0, 0) and substitute the coordinates of the point in the inequation $x - y \leq 1 \Rightarrow 0 - 0 \leq 1 \Rightarrow 0 \leq 1$,

which is true.

$\therefore$ (0, 0) belongs to the graph of the inequality $x - y \leq 1$.

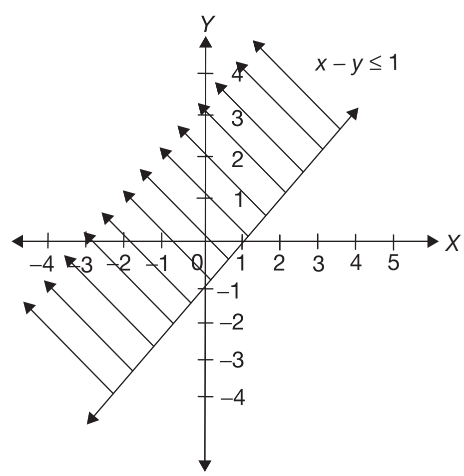


Figure 4.11

Now shade the region which includes the point (0, 0). Here, boundary line $x - y = 1$ is included.

EXAMPLE 4.25

Draw graph of $x < -y$

SOLUTION

(a) Draw $x = -y$

X	-1	0	1
Y	1	0	-1

(b) Substitute (1, 1) in $x < -y$

$\Rightarrow 1 < -1$ which is not true

$\therefore$ (1, 1) does not belong to the graph represented by $x < -y$.

(c) Now shade the region (half plane) which does not contain the point (1, 1).

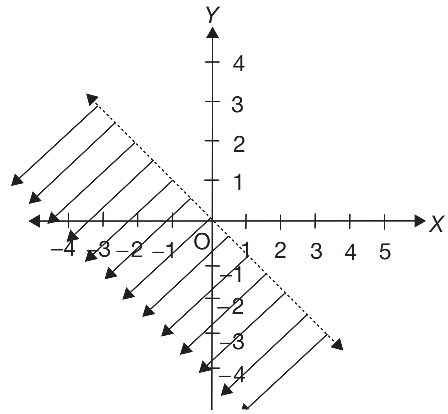


Figure 4.12

Note The dotted line indicates the non-inclusion of the line $x = -y$ in the graph.

System of Inequations

The graph of a linear inequation in two variables is one of the half planes separated by the boundary line. Any ordered pair which satisfies the inequation is a solution. Now, let us learn to solve the simultaneous inequations in two variables.

EXAMPLE 4.26

Construct the region represented by the inequations $x + 3y \geq 3$ and $3x + y \leq 3$.

SOLUTION

(a) Consider $x + 3y = 3 \Rightarrow y = \dfrac{3 - x}{3}$

x	0	3
y	1	0

Substitute (0, 0) in $x + 3y \geq 3$

$\Rightarrow 0 + 0 \geq 3$

$\Rightarrow 0 \geq 3$ which is false

$\therefore x + 3y \geq 3$ does not contain (0, 0).

(b) Consider $3x + y = 3 \Rightarrow y = 3 - 3x$

x	1	0
y	0	3

Substitute (0, 0) in the inequation $3x + y \leq 3$

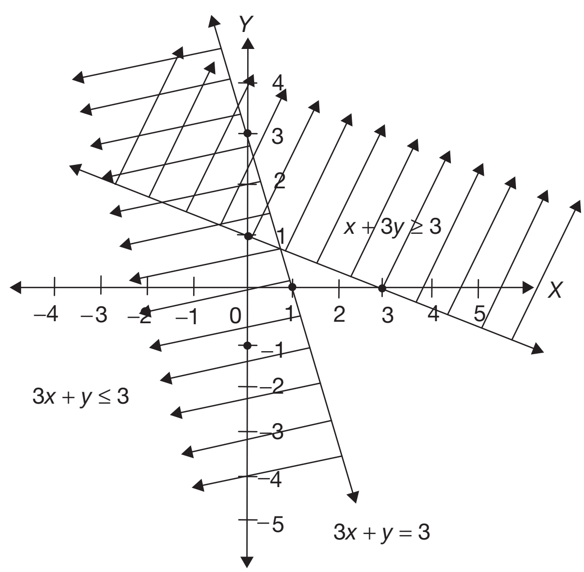


Figure 4.13

$\Rightarrow 0 + 0 \leq 3$ which is true

$\therefore 3x + y \leq 3$ contains (0, 0).

The region common to the two half planes is the graph of the given system of inequations.

EXAMPLE 4.27

Construct the region represented by the inequations $2x + y \leq 2$ and $x - 3y \leq 3$.

SOLUTION

(a) Consider $2x + y = 2 \Rightarrow y = 2 - 2x$

x	0	1
y	2	0

Substitute (0, 0) in $2x + y \leq 2$

$\Rightarrow 2(0) + 0 \leq 2 \Rightarrow 0 \leq 2$ which is true

$\therefore 2x + y \leq 2$ contains (0, 0).

(b) Consider $x - 3y = 3$

$$y = \frac{x-3}{3}$$

x	0	3
y	−1	0

Substitute (0, 0) in $x - 3y \leq 3$

$\Rightarrow 0 - 0 \leq 3 \Rightarrow 0 \leq 3$ which is true

$\therefore x - 3y \leq 3$ contains (0, 0).

The region common to the two half planes is the graph of the system of inequations.

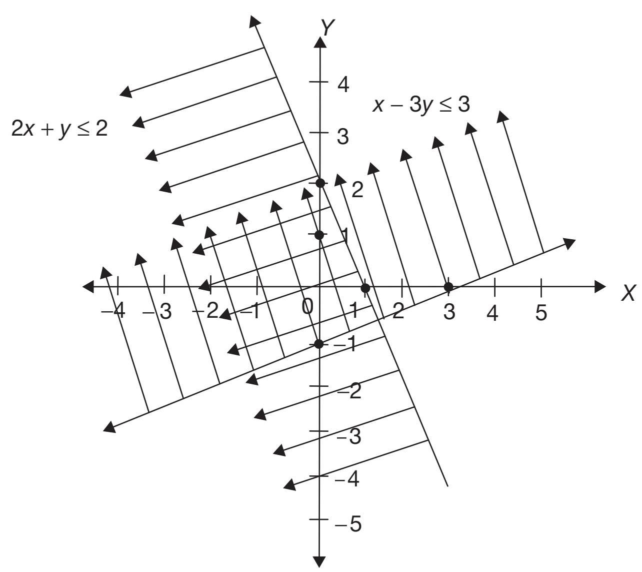


Figure 4.14

ABSOLUTE VALUE

If a is any real number, then

1. $|x| \le a \Rightarrow -a \le x \le a$
2. $|x| \ge a \Rightarrow x \ge a$ or $x \le -a$

Properties of Modulus

1. $x = 0 \Leftrightarrow |x| = 0$
2. For all values of x, $|x| \ge 0$ and $-|x| \le 0$
3. For all values of x, $|x + y| \le |x| + |y|$
4. $||x| - |y|| \le |x - y|$
5. $-|x| \le x \le |x|$
6. $|xy| = |x|\ |y|$
7. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}, y \ne 0$
8. $|x|^2 = x^2$

Interval Notation

We have seen above that the solution set or the range of values that satisfy inequalities are not discrete. Instead, we have a continuous range of values. Such ranges can be represented using the interval notation.

The set of all real numbers between a and b (where $a < b$) excluding a and b is represented as (a, b) read as 'the open interval a, b'.

$[a, b]$ read as 'the closed interval a, b' means all real numbers between a and b including a and b $(a < b)$. $[a, b)$ means all numbers between a and b, with a being included and b excluded $(a < b)$.

EXAMPLE 4.28

Solve $|2x - 3| < 5$.

SOLUTION

$|2x - 3| < 5$

$\Rightarrow -5 < 2x - 3 < 5$

$\Rightarrow -2 < 2x < 8$

$\Rightarrow -1 < x < 4$

$\therefore$ Solution set is $\{x/-1 < x < 4\}$ or $x \in (-1, 4)$.

EXAMPLE 4.29

Solve $|3x + 2| \ge 7$.

SOLUTION

$|3x + 2| \ge 7$

$3x + 2 \ge 7$ or $3x + 2 \le -7$

$3x \geq 5$ or $3x \leq -9$

$x \geq \frac{5}{3}$ or $x \leq -3$

$\therefore$ Solution set is $\left\{x/x \geq \frac{5}{3} \text{ or } x \leq -3\right\}$ or $(-\infty, -3] \cup \left[\frac{5}{3}, \infty\right)$.

EXAMPLE 4.30

Solve $|5x - 7| < -18$.

SOLUTION

The modulus of any number has to be 0 or positive. Thus, there are no values of x which satisfy the given inequality. The solution set is ϕ.

EXAMPLE 4.31

Find the solution set of $\frac{1}{2x-4} < 0$.

SOLUTION

$\frac{1}{2x-4} < 0.$

$\Rightarrow 2x - 4 < 0 \Rightarrow x < 2$

$\therefore$ The solution set is $\{x/x < 2\}$ or $(-\infty, 2)$.

EXAMPLE 4.32

Person A can assemble 10 machines per hour and Person B can assemble 15 machines per hour. Person A works for x hours per day and Person B works for y hours per day and both the persons together can assemble at the most 200 machines in a day. Frame one inequation to represent the above data.

SOLUTION

Person A can assemble $10x$ machines in a day.

Person B can assemble $15y$ machines in a day.

Both A and B can assemble at the most 200 machines.

$\Rightarrow 10x + 15y \leq 200$ is the required inequation.

EXAMPLE 4.33

Find the two whole numbers such that their sum is utmost 10 and the difference is atleast 4 and also the resultant sum is maximum.

SOLUTION

Let the two whole numbers be x and y.

The required inequations subject to the given conditions are

$$x + y \leq 10 \tag{1}$$

$$x - y \geq 4 \tag{2}$$

$$x \geq 0,\ y \geq 0 \tag{3}$$

Tracing the solution set of the above four inequations, clearly the solution set is the triangular region ABC. For the sum of the numbers to be maximum, the solution occurs at the vertices of the region ABC.

The vertices are (4, 0), (10, 0) and (7, 3). Out of these (10, 0) and (7, 3) give the maximum value for $x + y$.

$\therefore$ The solutions are (10, 0), (7, 3).

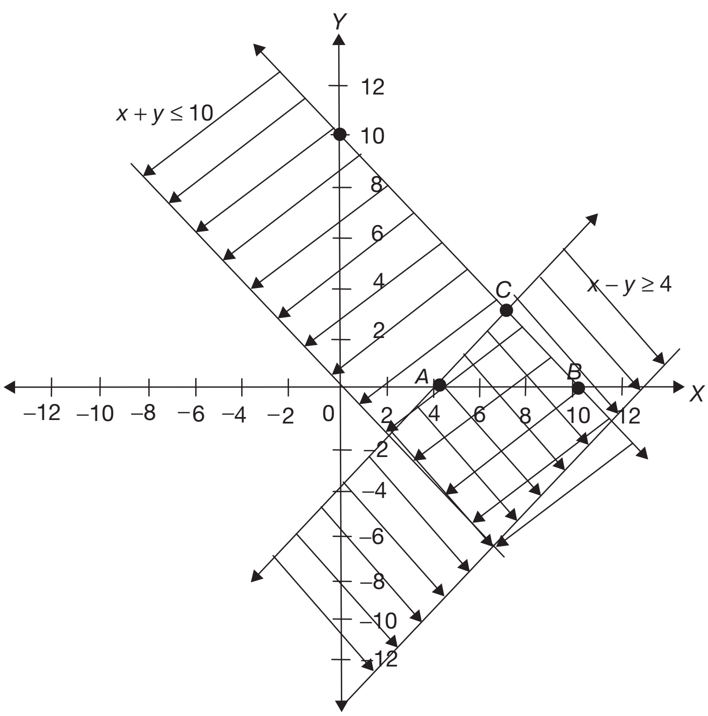


Figure 4.15

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. The system of equations $a + b = 3$ and $3a + 3b = 9$ is ______. (consistent/inconsistent)

2. The equations $px + qy + r = 0$ and $kpx + kqy + kr = 0$ are ______. (dependent/inconsistent)

3. If the equations $4x + py = 12$ and $qx + 3y = 6$ are dependent, then the values of p and q are ______ and respectively.

4. If $2x + 3y = 10$ and $3x + 2y = 5$, then the value of $x + y$ is ______.

5. If $2x - 3y = 0$, then the value of $2x + 3y$ in terms of y is ______.

6. The equations $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ are inconsistent if ______.

7. If sum of two numbers is 10 whereas their difference is 4, then the greater number is ______.

8. If $a + 2b + 3c = 20$ and $2a + 4b + c = 25$, then $c =$ ______.

9. If $2a + 3b + 4c = 35$ and $3a + 5b + 7c = 30$, then $a + b + c =$ ______.

10. If $a > b$, then $\frac{a}{c} < \frac{b}{c}$ for all a, b and $c \in R$, where $c < 0$. (True/False)

11. If $a > b$, then $ac > bc$ for all a, b and $c \in R$, where $c > 0$. (True/False)

12. The number of common integral solutions of the inequations $x > -5$ and $x < 5$ is ______.

13. The solution set of $ax + by + c < 0$, if $c < 0$ is the ______ (region that contains (0, 0)/region that does not contains (0, 0)).

14. Solution set for the inequation $\frac{1}{x+1} > 0$ is ______.

15. If $x + y \le 5$, then either $x \le 5$ or $y \le 5$ or both. (True/False)

16. If the system of linear equations is inconsistent, then the solution set is infinite. (Agree/Disagree)

17. If $\frac{1}{x+y} = \frac{1}{2}$ and $\frac{1}{x-y} = \frac{1}{3}$, then $x =$ ______ and $y =$ ______.

18. If $a + b = c$ and $a - b = d$, then the value of b is ______. (In terms of c and d).

19. Every system of dependent equations is consistent. Is the converse always true?

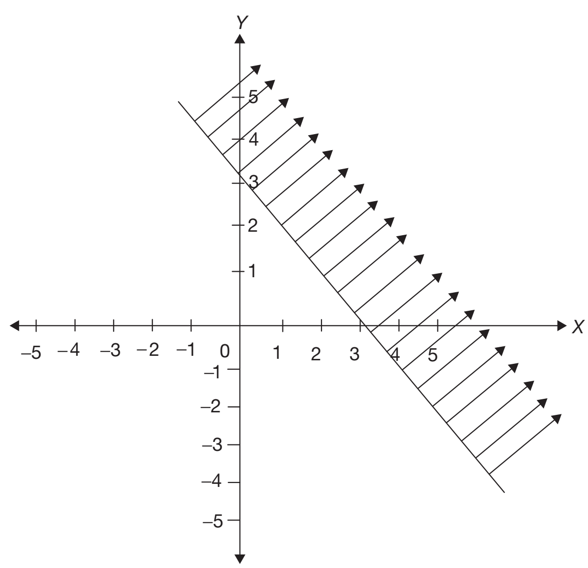
20. For how many pairs of non-zero integers, $x + y = 0$ and $x - y = 0$?

21. Any line in a plane divides the plane into three disjoint parts. (True/False)

22. If $x + y \le 7$ and $x - y \le 3$, then $x \le 5$. (True/False).

23. Boundary line for the region $y \le x + 4$ is ______.

24. Inequation that represents the following graph is ______.



25. The number of solutions for the simultaneous equations $3x + 4y = 12$ and $4x - 3y = 18$ is ______.

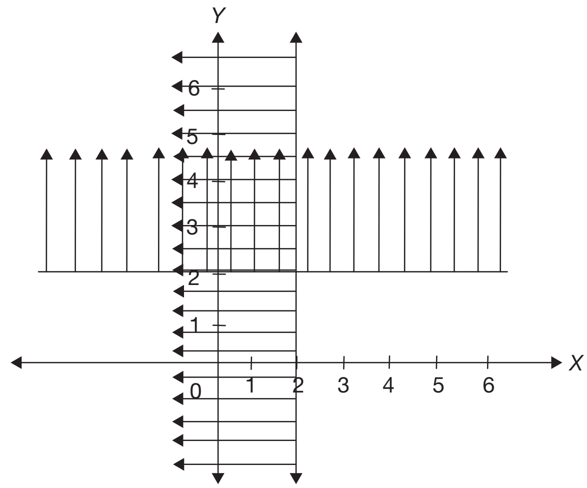
26. If the cost of 2 chocolates and 3 biscuits is ₹55 and that of 4 chocolates and 6 biscuits is ₹110, then the costs of one chocolate and one biscuit are necessarily ₹20 and ₹5 respectively. (True/False).

27. An open sentence which contains the symbol <, >, ≤ or ≥ is called an ______.

28. If $x > a$ and $x > b$ (where $a > b$), then the solution set of the inequations is ______.

29. Common region for the inequations $x \leq y$ and $y \leq x$ is ______.

30. Inequations that represents the shaded region of the following graph is ______.



Short Answer Type Questions

31. Solve: $\frac{11}{a+b} + 2(a-b) = 11$, $\frac{22}{a+b} + 3(a-b) = 17$.

32. If $px + qy + r = 0$ and $qx + py + r = 0$ $(x \neq y)$, then show that the value of $x + y$ is $\frac{-r}{p}$ or $\frac{-r}{q}$.

33. If $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ passes through the same ordered pair, then show that $a^3 + b^3 + c^3 - 3abc = 0$.

34. Find the value of x, if $2x + 3y + k = 12$ and $x + 6y + 2k = 18$.

35. In a fraction, the denominator exceeds the numerator by 8. If unity is deducted from both the numerator and the denominator, the fraction becomes $\frac{3}{7}$. Find the fraction.

36. Father's age is 3 years more than thrice the son's age. After 5 years, father's age will be 12 years more than twice the son's age. Find the father's present age.

37. Sum of successors of two numbers is 40, whereas their difference is 6. Find the two numbers.

38. If $x + y < 2$ and $y - 2x > -7$, then find the range of x.

39. Which of the following points belong to the region represented by the inequations $2x - 3y \geq 5$ and $x - 2y \leq 3$?
 (a) (3, 0)
 (b) (–4, –4)
 (c) (3, –5)
 (d) (2, –2)
 (e) (5, 1)

40. Find the number of solutions for the inequations $x + y \leq 8$ and $2x + y \leq 8$. (where x and y are positive integers).

41. If the cost of 2 pencils and 3 erasers is ₹14. Whereas the cost of 3 pencils and 5 erasers is ₹22, then find the cost of one pencil and one eraser.

42. The sum of the digits of a two digit number is 7. If 9 is added to the number, the digits interchange their places. Find the number.

43. Harry has ₹2 and ₹5 coins with him. If he has a total of 33 coins worth ₹120 with him, how many ₹5 coins does he have?

44. If $\frac{1}{x-y} < 1$, then show that x does not lie from y to $y + 1$.

45. Shade the regions that show the solution set of the following inequations:
 (a) $x > 3$
 (b) $y < 2$
 (c) $2y \leq 5$
 (d) $x \geq 0$, $y \geq 0$
 (e) $x \geq 4$, $y \leq 4$

Essay Type Questions

46. There are some chocolates with Tom and Jerry. If Tom gives certain number of chocolates to Jerry, then the number of chocolates with them will be interchanged. Instead, if Jerry gives same number of chocolates to Tom, then the number of chocolates with jerry will be one-fourth of the number of chocolates that Tom has. If the total number of chocolates with them is 100, then find the number of chocolates with Tom.

47. Solve the system of inequations graphically, $x + 2y \le 6$, $2x + y \ge 6$ and $x \le 4$.

Direction for question 48: Solve the questions graphically.

48. Find the two natural numbers so that their sum cannot exceed 6 and the difference between first and second number is positive and does not exceed 2 and also the resultant sum is maximum.

49. A test has 150 questions. A candidate gets 2 marks for each correct answer and loses 1 mark for each wrong answer and loses $\frac{1}{2}$ mark for leaving the question unattempted. A student score 165 marks. If the student left 18 questions unattempted, find the number of questions he marked wrong.

Direction for question 50: Solve the questions graphically.

50. Find the pair of whole numbers so that their sum cannot exceed 10 and difference of twice the first number and thrice the second number cannot exceed 12.

CONCEPT APPLICATION

Level 1

1. How many pairs of x and y satisfy the equations $2x + 4y = 8$ and $6x + 12y = 24$?

(a) 0 (b) 1
(c) Infinite (d) None of these

2. Find the value of 'k' for which the system of linear equations $kx + 2y = 5$ and $3x + y = 1$ has zero solutions.

(a) $k = 6$ (b) $k = 3$
(c) $k = 4$ (d) None of these

3. Find the minimum value of $|x - 3| + 11$.

(a) 8 (b) 11
(c) 0 (d) −8

4. The maximum value of $23 - |2x + 3|$ is

(a) 20 (b) 26
(c) 17 (d) 23

5. The product of a number and 72 exceeds the product of the number and 27 by 360. Find the number.

(a) 12 (b) 7
(c) 8 (d) 11

6. The total cost of 10 erasers and 5 sharpeners is at least ₹65. The cost of each eraser cannot exceed ₹4. Find the minimum possible cost of each sharpener.

(a) ₹6 (b) ₹5.50
(c) ₹5 (d) ₹6.50

7. If the system of linear equations $px + 3y = 9$ and $4x + py = 8$ has unique solution, then

(a) $p = \pm 2\sqrt{3}$ (b) $p \ne \pm 3\sqrt{2}$
(c) $p \ne \pm 2\sqrt{3}$ (d) $p = \pm 3\sqrt{2}$

8. In a group of goats and hens, the total number of legs is 12 more than twice the total number of heads. The number of goats is

(a) 8 (b) 6
(c) 2 (d) Cannot be determined

9. If $\frac{x+3}{x-3} < 1$, then which of the following cannot be the value of x?

(a) 0 (b) 1
(c) 2 (d) 4

PRACTICE QUESTIONS

10. The system of equations $px + 4y = 32$ and $2qy + 15x = 96$ has infinite solutions. The value of $p - q$ is

(a) -1. (b) 1.

(c) 0. (d) 11.

11. If x and y are two integers where $x \geq 0$ and $y \geq 0$, then the number of ordered pairs satisfying the inequation $2x + 3y \leq 1$ is ______.

(a) 1 (b) 2

(c) 3 (d) 4

12. The common solution set of the inequations $\frac{x}{2}+\frac{y}{2} \leq 1$ and $x + y > 2$ is ______.

(a) $\{(x, y)/x < 2 \text{ and } y > 2\}$

(b) $\{(x, y)/x < 1 \text{ and } y > 1\}$

(c) an empty set

(d) $\{(x, y)/x < 2 \text{ and } y < 1\}$

13. If (1, 4) is the point of intersection of the lines $2x + by = 6$ and $3y = 8 + ax$, then find the value of $a - b$.

(a) 2 (b) 3

(c) 4 (d) -3

14. If x be a negative integer, then the solution of the inequation $1 \leq 2x + 8 \leq 11$ is

(a) $\{-5, -3, -4, -2, -1\}$

(b) $\{-4, -2, -1\}$

(c) $\{-6, -3, -1\}$

(d) $\{-3, -2, -1\}$

15. If $5u + 3v = 13uv$ and $u - v = uv$, then $(u, v) =$ ______.

(a) (2, 1) (b) $\left(\frac{1}{2}, 1\right)$

(c) $\left(1, \frac{1}{2}\right)$ (d) (1, 2)

16. Solve the equations: $4(2^{x-1}) + 9(3^{y-1}) = 17$ and $3(2^x) - 2(3^y) = 6$.

(a) $(x, y) = (2, 1)$ (b) $(x, y) = (-2, -1)$

(c) $(x, y) = (1, 2)$ (d) $(x, y) = (2, -1)$

17. The solution set of $\frac{2}{x}+\frac{3}{y} = 2$ and $\frac{3}{x}+\frac{4}{y} = 20$ is

(a) $(4, -2)$ (b) $\left(-\frac{1}{2}, \frac{1}{4}\right)$

(c) $(2, -4)$ (d) $\left(\frac{1}{4}, \frac{-1}{2}\right)$

18. Cost of 5 pens and 7 note books is ₹82 and cost of 4 pens and 4 note books is ₹52. Find the cost of 2 note books and 3 pens.

(a) ₹34.50 (b) ₹30.50

(c) ₹32.50 (d) ₹36.50

19. If $(a + b, a - b)$ is the solution of the equations $3x + 2y = 20$ and $4x - 5y = 42$, then find the value of b.

(a) 8 (b) -2

(c) -4 (d) 5

20. If $0 < \frac{2x-5}{2} < 7$ and x is an integer, then the sum of the greatest and the least value of x is

(a) 9 (b) 10

(c) 6 (d) 12

21. Number of integral values of x that do not satisfy the inequation $\frac{x-7}{x-9} > 0$ is ______.

(a) 4 (b) 3

(c) 2 (d) 0

22. The solution set formed by the regions $x + y > 7$ and $x + y < 10$ in the first quadrant represents a ______.

(a) triangle (b) rectangle

(c) trapezium (d) rhombus

23. Solve $\left|3 - \frac{2x}{5}\right| \leq 4$.

(a) $\frac{5}{2} \leq x \leq \frac{35}{2}$ (b) $\frac{-5}{2} \leq x \leq \frac{35}{2}$

(c) $\frac{-35}{2} \leq x \leq \frac{-5}{2}$ (d) None of these

24. In a fraction, if numerator is increased by 2 and denominator is increased by 3, it becomes $\frac{3}{4}$ and if numerator is decreased by 3 and denominator is decreased by 6, it becomes $\frac{4}{3}$. Find the sum of the numerator and denominator.

(a) 16 (b) 18

(c) 20 (d) 14

25. If 100 cm is divided into two parts such that the sum of 2 times the smaller part and $\frac{1}{3}$ of the larger part, is less than 100 cm, then which of the following is correct?

(a) Larger portion is always less than 60.

(b) Smaller portion is always less than 60 and more than 40.

(c) Larger portion is always greater than 60.

(d) Smaller portion is always greater than 40.

26. If $2a - 3b = 1$ and $5a + 2b = 50$, then what is the value of $a - b$?

(a) 10 (b) 6

(c) 7 (d) 3

27. The fair of 3 full tickets and 2 half tickets is ₹204 and the fair of 2 full tickets and 3 half tickets is ₹186. Find the fair of a full ticket and a half ticket.

(a) ₹94 (b) ₹78

(c) ₹86 (d) ₹62

28. If $\frac{3}{2}x + 2y = \frac{x}{4} - \frac{y}{2} = 1$, then $x - y =$

(a) 1 (b) 3

(c) 2 (d) 0

29. If we add 1 to the numerator and subtract 1 from the denominator a fraction becomes 1. It also becomes $\frac{1}{2}$ if we add 1 to the denominator. Then the sum of the numerator and denominator of the fraction is

(a) 7 (b) 8

(c) 2 (d) 11

30. If $4x - 3y = 7xy$ and $3x + 2y = 18xy$, then $(x, y) =$

(a) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (b) (3, 4)

(c) (4, 3) (d) $\left(\frac{1}{3}, \frac{1}{4}\right)$

Level 2

31. Jeevesh had 92 currency notes in all, some of which were of ₹100 denomination and the remaining of ₹50 denomination. The total value of amount of all these currency notes was ₹6350. How much amount in rupees did he have in the denomination of ₹50?

(a) 3500 (b) 3350

(c) 2850 (d) 2600

32. The solution set of the inequation $\frac{1}{5+3x} \leq 0$ is

(a) $x \in \left(\frac{-5}{3}, \infty\right)$ (b) $x \in \left(-\infty, \frac{5}{3}\right)$

(c) $x \in \left(\frac{5}{3}, \infty\right)$ (d) $x \in \left(-\infty, \frac{-5}{3}\right)$

33. If $2|x| - |y| = 3$ and $4|x| + |y| = 3$, then number of possible ordered pairs of the form (x, y) is

(a) 0 (b) 1

(c) 2 (d) 4

34. The solution set formed by the inequations $x \geq -7$ and $y \geq -7$ in the third quadrant represents a

(a) trapezium (b) rectangle

(c) square (d) rhombus

35. Find the solution of the inequation $\frac{1}{|3x-5|} > 2$, where x is a positive integer.

(a) {2, 3} (b) {2, 3, 4}

(c) $x = 2$ (d) Null set

PRACTICE QUESTIONS

36. A father wants to divide ₹200 into two parts between two sons such that by adding three times the smaller part to half of the larger part, then this will always be less than ₹200. How will he divide this amount?

(a) Smaller part is always less than 50.

(b) Larger part is always greater than 160.

(c) Larger part is always less than 160.

(d) Smaller part is always greater than 40.

37. Solve $|7 - 2x| \leq 13$.

(a) $3 \leq x \leq 10$ (b) $-3 < x < 10$

(c) $-10 \leq x \leq 3$ (d) $-3 \leq x \leq 10$

38. If an ordered pair, satisfying the equations $x + y = 7$ and $3x - 2y = 11$, is also satisfies the equation $3x + py - 17 = 0$, then the value of p is ______.

(a) 2 (b) −2

(c) 1 (d) 3

39. Solve for x: $|2x + 3| < 2x + 4$.

(a) $x > -2$ (b) $x > -\frac{7}{4}$

(c) $x < -\frac{7}{4}$ (d) $x < -2$

40. Find the values of x and y, which satisfy the simultaneous equations $1010x + 1011y = 4040$ and $1011x + 1010y = 4044$.

(a) $x = 2, y = -4$ (b) $x = 0, y = 4$

(c) $x = 4, y = 4$ (d) $x = 4, y = 0$

41. A bus conductor gets a total of 220 coins of 25 paise, 50 paise and ₹1 daily. One day he got ₹110 and next day he got ₹80 in that the number of coins of 25 paise and 50 paise coins are interchanged then find the total number of 50 paise coins and 25 paise coins.

(a) 180 (b) 190

(c) 160 (d) 200

42. The common solution set of the inequations $5 \leq 2x + 7 \leq 8$ and $7 \leq 3x + 5 \leq 9$ is ______.

(a) $\frac{2}{3} \leq x \leq \frac{4}{3}$ (b) $-1 \leq x \leq \frac{4}{3}$

(c) $\frac{2}{3} \leq x \leq \frac{1}{2}$ (d) Null set

43. The solution set formed by the inequations $x + y \geq 3$, $x + y \leq 4$, $x \leq 2$ in the first quadrant represents a

(a) triangle (b) parallelogram

(c) rectangle (d) rhombus

44. Shiva's age is three times that of Ram. After 10 years Shiva's age becomes less than twice the age of Ram. What can be the maximum present age (in complete years) of Shiva?

(a) 30 (b) 10

(c) 9 (d) 29

45. In an ICC Champions trophy series, Sachin scores 68 runs and 74 runs out of three matches. A player can be placed in Grade A of ICC rankings if the average score of three matches is at least 75 and at most 85. Sachin is placed in Grade A. What is the maximum runs that he should score in the third match?

(a) 105 (b) 83

(c) 113 (d) 97

46. The sum of predecessors of two numbers is 36 and their difference is 4. Find the numbers.
The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $X - 1 + Y - 1 = 36$ and $X - Y = 4$

(B) Solve for X and Y

(C) Let $X > Y$

(D) Let the numbers be X and Y

(a) CDAB (b) CDBA

(c) DCAB (d) DCBA

47. The following are the steps involved in solving the equations $2^x + 3^y = 17$ and $2^{x+1} + 3^{y+1} = 43$ for x and y. Arrange them in sequential order.

(A) Rewrite the given equation in terms of p and q

(B) Let $p = 2^x$ and $q = 3^y$

(C) Find x and y

(D) Solve for p and q

(a) ABCD (b) ABDC

(c) BACD (d) BADC

48. There are two numbers. The predecessor of the larger number exceeds the successor of the smaller

number by 6.The sum of the numbers is 32. Find the numbers.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $M + N = 32$ and $M - 1 - (N + 1) = 6$

(B) Let $N < M$

(C) Solve for M and N

(D) Let the numbers be M and N

(a) BDCA (b) BDAC

(c) DBCA (d) DBAC

49. The following are two steps involved in finding the values of p and q from $3^p + 5^q = 52$ and $3^{p-1} + 5^{q-1} = 14$. Arrange them in sequential order.

(A) Let $x = 3^p$ and $y = 5^q$

(B) Solve for x and y

(C) Find p and q

(D) Rewrite the given equations in terms of x and y

(a) ABCD (b) ADCB

(c) ACBD (d) ADBC

50. Solve for x: $2x - 3 \le 5x + 9$.

(a) $x \ge -4$ (b) $x \ge -3$

(c) $x \ge -2$ (d) $x \ge -1$

51. X is an integer satisfying $1 \le 2X + 3 \le 7$. How many values can it take?

(a) 4 (b) 3

(c) 5 (d) 6

52. Solve for x: $5x + 4 \ge x + 12$.

(a) $x \ge 0$ (b) $x \ge 1$

(c) $x \ge 2$ (d) $x \ge 3$

53. Y is an integer satisfying $-3 \le 4Y -7 \le 5$. How many values can it take?

(a) 2 (b) 4

(c) 3 (d) 5

54. N is a three-digit number. It exceeds the number formed by reversing the digits by 792. Its hundreds digit can be

(a) 9

(b) 8

(c) Either (a) or (b)

(d) Neither (a) nor (b)

55. X is a three-digit number. The number formed by reversing the digits of X is 891 less than X. Find its units digit.

(a) 0 (b) 1

(c) 2 (d) Cannot be determined

Level 3

56. An examination consists of 160 questions. One mark is given for every correct option. If one-fourth mark is deducted for every wrong option and half mark is deducted for every question left, then one person scores 79. And if half mark is deducted for every wrong option and one-fourth mark is deducted for every left question, the person scores 76, then find the number of questions he attempted correctly.

(a) 80 (b) 100

(c) 120 (d) 140

57. Runs scored by Sachin in a charity match is 10 more than the balls faced by Lara. The number of balls faced by Sachin is 5 less than the runs scored by Lara. Together they have scored 105 runs and Sachin faced 10 balls less than the balls faced by Lara. How many runs were scored by Sachin?

(a) 45 (b) 60

(c) 50 (d) 55

58. The number of ordered pairs of different prime numbers whose sum is not exceeding 26 and difference between second number and first number cannot be less than 10.

(a) 8 (b) 9

(c) 10 (d) 11

59. The number of possible pairs of successive prime numbers, such that each of them is greater than 40 and their sum is utmost 100, is

PRACTICE QUESTIONS

(a) 3 (b) 2
(c) 4 (d) 1

60. In an election the supporters of two candidates A and B were taken to polling booth in two different vehicles, capable of carrying 10 and 15 voters respectively. If at least 90 vehicles were required to carry a total of 1200 voters, then find the maximum number of votes by which the elections could be won by the Candidate B.

(a) 900 (b) 600
(c) 300 (d) 500

61. A test has 60 questions. For each correct answer 2 marks are awarded and each wrong answer 1 mark is deducted. A candidate attempted all the questions in the test and scored 90 marks. Find the number of questions he attempted correctly.

(a) 54 (b) 48
(c) 49 (d) 50

62. Krishna and Sudheer have some marbles with them. If Sudheer gives 10 marbles to Krishna, Krishna will have 40 more marbles than Sudheer. If Sudheer gives 40 marbles to Krishna, Krishna will have 5 times as many marbles as Sudheer. Find the number of marbles with Sudheer.

(a) 65 (b) 55
(c) 70 (d) 50

63. In a test, for each correct answer 1 mark is awarded and each wrong answer half a mark is deducted. The test has 70 questions. A candidate attempted all the questions in the test and scored 40 marks. How many questions did he attempt wrongly?

(a) 15 (b) 20
(c) 25 (d) 10

64. Amar and Bhavan have a certain amount with them. If Bhavan gives ₹20 to Amar, he will have half the amount with Amar. If Amar gives ₹40 to Bhavan, he will have half the amount with Bhavan. Find the amount with Bhavan.

(a) ₹70 (b) ₹90
(c) ₹60 (4) ₹80

65. Solve for z: $4x + 5y + 9z = 36$, $6x + \frac{15}{2}y + 11z = 49$.

(a) 2 (b) 1
(c) 3 (d) Cannot be determined

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. consistent
2. dependent
3. 6, 2
4. 3
5. $6y$
6. $a + b = 0$
7. 7
8. 3
9. 40
10. True
11. True
12. 9
13. region that contains (0, 0)
14. $\{x/x > -1\}$
15. True
16. Disagree
17. $\frac{5}{2}, \frac{-1}{2}$
18. $\frac{c-d}{2}$
19. No
20. No such pairs exist
21. True
22. True
23. $y = x + 4$
24. $4x + 3y - 12 \geq 12$
25. one
26. False
27. inequation
28. $x > a$
29. $x = y$
30. $x \leq 2$ and $y \geq 2$

Shot Answer Type Questions

31. $a = 8$ and $b = 3$
34. 2
35. $\frac{7}{15}$
36. 45 years
37. 22, 16
38. $(-\infty, 3)$
39. (3, 0), (5, 1)
40. 12
41. ₹6
42. 34
43. 18

Essay Type Questions

46. 60
48. (4, 2)
49. 30

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (a)
11. (a)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (d)
21. (b)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (d)

Level 2

31. (c)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (d)	**38.** (c)	**39.** (b)	**40.** (d)
41. (d)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (c)	**46.** (c)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (a)
51. (a)	**52.** (c)	**53.** (c)	**54.** (c)	**55.** (a)					

Level 3

56. (b)	**57.** (b)	**58.** (d)	**59.** (a)	**60.** (b)	**61.** (d)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (a)

CONCEPT APPLICATION

Level 1

1. Find $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ and proceed.
2. Apply the condition $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
3. Minimum value of modulus of a number is zero.
4. Since $|x|$ is always positive.
   $\therefore a + |x| \le a$.
5. Assume the number to be x and frame the equation.
6. Frame the inequations and proceed.
7. If $ax + by = e$ and $cx + dy = f$ have unique solution then $ad - bc \neq 0$.
8. Make algebraic equation and proceed.
9. (i) Subtract one on both the sides of the given equation.
   (ii) On back substitution we can easily verify the value of x.
10. (i) System of equations has infinite solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
    (ii) If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinite solutions, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
11. Apply trial and error method.
12. Apply the concept of half planes in the coordinate system.
13. Put $x = 1$; $y = 4$ in the given lines and obtained the values of a and b by solving them.
14. If $a < x + b < c$ then $a - b < x < c - b$.
15. Divide both the equations by uv on both the sides.
16. Assume $2^x = a$ and $3^y = b$ then solve.
17. Put $\frac{1}{x} = a, \frac{1}{y} = b$ and proceed.
18. Make algebraic equations and proceed.
19. $x = a + b$ and $y = a - b$.
21. If $\frac{a}{b} > 0$ then $a > 0$, $b > 0$ (or) $a < 0$, $b < 0$.
22. Draw the regions corresponding to given inequations.
23. $|x| \le a \Rightarrow -a \le x \le a$.
24. (i) Frame equations according to the data.
    (ii) Assume the numerator and denominator as x and y respectively.
    (iii) Frame the equations in terms of x and y and solve them.
25. Let the two parts be x and $(100 - x)$ and proceed.
26. Solve the equations for a and b.
27. Let full ticket is x and half ticket be y, then frame the equations for x and y, then solve them.
28. Solve the equations for x and y.
29. Assume the fraction as $\frac{x}{y}$. Form equations in terms of x and y according to the given data and find the values of x and y.
30. Divide the two equations by xy. Assume $\frac{a}{x} = a$; $\frac{1}{y} = b$ then solve for a and b.

Level 2

31. Assume the denomination as x and y. Form the equations in terms of x and y according to the data and solve them.
32. Multiply the numerator and denominator with $5 + 3x$ and solve the inequation obtained.
33. (i) We cannot find a common solution.
    (ii) Solve the two equations for $|x|$ and $|y|$ and write the possible values of x and y.
34. Represent the inequations in the coordinate plane.

35. If $|x| < a \Rightarrow -a < x < a$.

36. Find x and y values ($x < y$) such that $x + y = 200$ and $3x + \frac{y}{2} < 200$.

37. $|x| \leq a \Rightarrow -a \leq x \leq a$.

38. (i) Solve first two equations and then substitute the values in the third equation.

(ii) Solve the equations $x + y = 7$ and $3x - 2y = 11$.

(iii) Now substitute the value of x and y in the equation $13x + py - 17 = 0$.

39. (i) $|x| < a \Rightarrow -a < x < a$.

(ii) $|x + a| = x + a$ when $x > -a$ and $-(x + a)$ when $x < -a$.

40. (i) First of all add given equations and then subtract from one another.

(ii) Subtract and add the given equations and then solve them for x and y.

41. (i) Frame equations according to the data.

(ii) Assume the number of coins of 25 paise, 50 paise and ₹1 as x, y and z respectively.

(iii) Frame the equations in terms of x, y and z according to the data and then solve.

42. (i) Solve the given inequations individually and then check.

(ii) Solve the two inequations individually, then write the common solution for x.

43. (i) Draw the graph.

(ii) Draw the region according to the inequations given, then identify the common region formed by them.

44. (i) Frame inequation according to the data.

(ii) Assume the ages of Shiva and Ram as x and y years respectively.

(iii) Frame the equation and inequation according to the data.

45. (i) As per the conditions given frame in equations and solve.

(ii) $75 \leq \dfrac{x + y + z}{3} \leq 85$

(iii) Substitute x and y and obtain the maximum value of z.

46. It can be easily seen that the correct order is DCAB.

47. It can easily be seen that the correct order is BADC.

48. It can easily be seen that the correct order is DBAC.

49. It can be easily seen that the correct order is ADBC.

50. Given $2x - 3 \leq 5x + 9$

$2x - 5x \leq 9 + 3$

$-3x \leq 12$

$x \geq -4$.

51. $1 \leq 2X + 3 \leq 7$

$\Rightarrow 1 \leq 2X + 3$ and $2X + 3 \leq 7$

$\Rightarrow -2 \leq 2X$ and $2X \leq 4$

$\Rightarrow -1 \leq X$ and $X \leq 2$.

X is an integer.

$\therefore$ $X = -1$ or 0 or 1 or 2.

$\therefore$ X has four possibilities.

52. Given $5x + 4 \geq x + 12$

$5x - x \geq 12 - 4$

$4x \geq 8$

$x \geq 2$.

53. $-3 \leq 4Y - 7 \leq 5 \Rightarrow -3 \leq 4Y - 7$ and $4Y - 7 \leq 5$

$\Rightarrow -3 + 7 \leq 4Y$ and $4Y \leq 7 + 5$

$\Rightarrow 1 \leq Y$ and $Y \leq 3$.

Y is an integer.

$\therefore$ $Y = 1$ or 2 or 3.

$\therefore$ Y has three possibilities.

54. Let N be abc.

$\therefore$ The number formed by reversing the digits $= cba$.

$abc - cba = 792$

$100a + 10b + c - (100c + 10b + a) = 792$

$99a - 99c = 792$.

$a - c = 8$, i.e., $a = c + 8$

$c \geq 0$

$\therefore$ $a \geq 8$

$\therefore$ $a = 8$ or 9.

55. Let x be abc.
The number formed by reversing the digits = cba
$cba = abc - 891$, i.e., $891 = abc - cba$
$891 = 100a + 10b + c - (100c + 10b + a)$
$= 99(a - c)$
$9 = a - c$
i.e., $a = c + 9$
$c \geq 0$
$\therefore\ a \geq 9$.
But $a \leq 9$.
$\therefore\ a = 9$.
$\therefore\ c = 0$.

Level 3

56. (i) Frame equations according to the data.
(ii) Frame the equations according to the data and then solve.

57. (i) Frame equations according to the data.
(ii) Assume runs scored by Sachin as x and balls faced by Sachin as y.
(iii) Frame the equation in terms of x and y according to the data and solve the equation.

58. Write the prime numbers up to 23, find the order pairs such that $x + y \leq 26$ and $y - x > 10$.

59. (i) Write primes as per the given conditions.
(ii) Assume the prime numbers as x, y.
(iii) Find the possible values of x and y such that $x + y < 100$, where $x > y > 40$.

60. Find the values of x and y such that $x + y > 90$ and $10x + 15y = 1200$.

61. Let the number of questions attempted correctly by the candidate = C.
He attempted all the questions.
$\therefore$ Number of questions attempted wrongly by him = $60 - C$
$2C - 1(60 - C) = 90$
$2C - 60 + C = 90$
$C = 50$.

62. Let the number of marbles with Krishna and Sudheer be M and S respectively.
$M + 10 = S - 10 + 40 \Rightarrow M = S + 20$.
$M + 40 = 5(S - 40) \Rightarrow S + 20 + 40 = 5(S - 40)$
$260 = 4S$, i.e., $S = 65$.

63. Let the number of questions attempted wrongly by the candidate be w.
$\therefore$ Number of questions be attempted correctly $= 70 - w$.
$$1(70 - w) - \frac{1}{2}w = 40$$
$$70 - \frac{3}{2}w = 40$$
$$w = 20.$$

64. Let the amounts with Amar and Bhavan be ₹A and ₹B respectively.
$$(B - 20) = \frac{1}{2}(A + 20)$$
$$\text{and } A - 40 = \frac{1}{2}(B + 40)$$
$$2B - 40 - 20 = A \text{ and } A = \frac{1}{2}B + 20 + 40$$
$$\therefore\quad A = 2B - 60 = \frac{1}{2}B + 60$$
$$\frac{3B}{2} = 120 \Rightarrow B = 80.$$

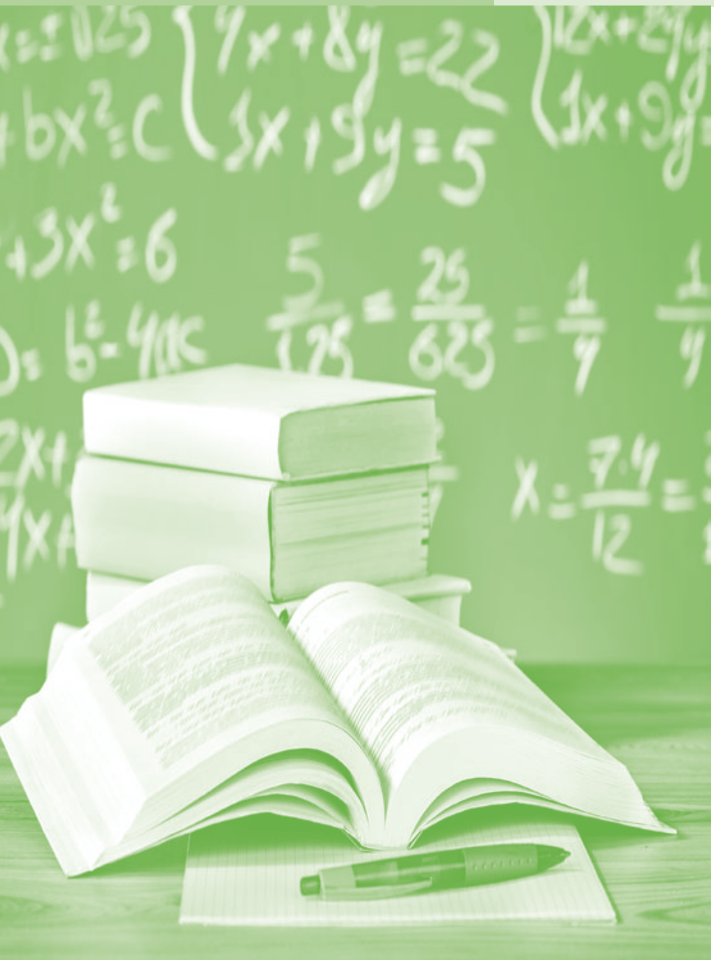
65. $3(4x + 5y + 9z) = (36)3$
$$\Rightarrow 12x + 15y + 27z = 108 \quad (1)$$
$$2\left(6x + \frac{15}{2}y + 11z\right) = (49)2$$
$$\Rightarrow 12x + 15y + 22z = 98 \quad (2)$$
$(1) - (2) \Rightarrow 5z = 10 \Rightarrow z = 2$.

HINTS AND EXPLANATION

Chapter

5

Quadratic Expressions and Equations



REMEMBER

Before beginning this chapter, you should be able to:

- Understand simple quadratic equations
- Know natural numbers, integers and fractions
- Aware of basic algebra and simple indices

KEY IDEAS

After completing this chapter, you should be able to:

- Study about quadratic expression, its zeroes and quadratic equations
- Find solutions/roots of a quadratic equation
- Understand the nature and sign of the roots of a quadratic equation
- Learn reciprocal equation and maximum or minimum value of a quadratic equation

INTRODUCTION

In previous topic, we have learnt linear expression, linear equation and linear inequation. In this topic, we shall learn quadratic expression and quadratic equation.

QUADRATIC EXPRESSION

A polynomial of second degree in one variable is termed as a quadratic polynomial or quadratic expression. The general form of a quadratic expression in x is $ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$.

Example: $2x^2 + 3x$, $x^2 - 2$, $3x^2 + 11x - 108$ are some quadratic expressions.

> **Note** The expressions $x^2 + \frac{1}{x^2} - 3$, $x^2 + \sqrt[3]{x} + 3$, $2x^2 - \frac{1}{x} + 4$ are not quadratic expressions.

Zeroes of a Quadratic Expression

If a quadratic expression, $ax^2 + bx + c$ becomes zero for $x = \alpha$, where α is a real number, then α is called a zero of the expression $ax^2 + bx + c$. A quadratic expression can have at the most two zeroes.

QUADRATIC EQUATION

The equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$ is known as a quadratic equation, or an equation of the second degree.

Example: $2x^2 + 3x + 5 = 0$, $x^2 - 5 = 0$ and $3x^2 - 4x + \sqrt{5} = 0$ are some quadratic equations.

Solutions or Roots of a Quadratic Equation

The values of x for which the equation $ax^2 + bx + c = 0$ is satisfied are called the roots of the quadratic equation. A quadratic equation cannot have more than two roots.

EXAMPLE 5.1

Verify whether $x = 2$, is a solution of $2x^2 + x - 10 = 0$.

SOLUTION

On substituting $x = 2$ in $2x^2 + x - 10$, we get

$$2(2)^2 + 2 - 10 = 10 - 10 = 0.$$

$\therefore$ 2 is a solution (or) root of $2x^2 + x - 10 = 0$.

Finding the Solutions or Roots of a Quadratic Equation

There are two ways of finding the roots of a quadratic equation.

1. Factorization method
2. Application of formula

Factorization Method to Obtain the Roots of a Quadratic Equation

The steps involved in obtaining the roots of $ax^2 + bx + c = 0$ are as follows:

1. Resolve $ax^2 + bx + c$ into factors and express $ax^2 + bx + c$ as a product of its factors.
2. For this product to be zero, one of the factors should be zero. The zeroes of the factors give the roots of the equation, $ax^2 + bx + c = 0$.

EXAMPLE 5.2

(a) Solve $x^2 - 15x + 26 = 0$.

(b) Solve $x + \frac{1}{x} = \frac{5}{2}$.

SOLUTION

(a) First, let us resolve $x^2 - 15x + 26$ into factors.

$$\Rightarrow x^2 - 15x + 26$$
$$= x^2 - 13x - 2x + 26$$
$$= x(x - 13) -2(x - 13)$$
$$= (x - 13)(x - 2).$$

The given equation, $x^2 - 15x + 26 = 0$ is reduced to $(x - 13)(x - 2) = 0$

$$\Rightarrow x - 13 = 0 \text{ (or) } x - 2 = 0$$
$$\Rightarrow x = 13 \text{ (or) } x = 2.$$

$\therefore$ $x = 2, 13$ are the roots of the given equation.

(b) $\frac{x^2 + 1}{x} = \frac{5}{2}$

$$\Rightarrow 2x^2 + 2 = 5x$$
$$\Rightarrow 2x^2 - 5x + 2 = 0$$
$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$
$$\Rightarrow 2x(x - 2) -1(x - 2) = 0$$
$$\Rightarrow (2x - 1)(x - 2) = 0$$
$$\Rightarrow 2x - 1 = 0 \text{ or } x - 2 = 0$$
$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2.$$

$\therefore$ $x = \frac{1}{2}$, 2 are the roots of the given equation.

Finding the Solutions/Roots of a Quadratic Equation by the Application of Formula

Factorization of $ax^2 + bx + c$ might not be always possible using the above method. So here, we derive a formula to find the roots of the equation, $ax^2 + bx + c = 0$, where $a \neq 0$, $b, c \in R$.

$$ax^2 + bx + c = 0$$
$$\Rightarrow ax^2 + bx = -c$$
$$\Rightarrow a\left(x^2 + \frac{b}{a}x\right) = -c$$
$$\Rightarrow x^2 + \frac{b}{a}x = \frac{-c}{a}$$
$$\Rightarrow x^2 + 2\cdot\frac{b}{2a}\cdot x = \frac{-c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides to make LHS a perfect square, we get,

$$\Rightarrow (x)^2 + 2\left(\frac{b}{2a}\right)(x) + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$
$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$\Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$\Rightarrow x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{(2a)^2}}$$
$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\therefore$ The roots of the equation $ax^2 + bx + c = 0$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Relation Between the Roots and the Coefficients of a Quadratic Equation

Let us assume that α, β are the roots of $ax^2 + bx + c = 0$.

Then, $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.

Now, the sum of the roots, $\alpha + \beta = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2b}{2a} = \dfrac{-b}{a}$.

$\therefore$ The sum of the roots, $\alpha + \beta = \dfrac{-b}{a} = \dfrac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$.

The product of the roots, $\alpha\beta = \left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$

$$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}.$$

$\therefore$ The product of the roots, $\alpha\beta = \dfrac{c}{a} = \dfrac{(\text{constant term})}{(\text{coefficient of } x^2)}$.

Nature of the Roots of a Quadratic Equation

We know that the roots of the equation $ax^2 + bx + c = 0$ are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

We call, $\Delta = b^2 - 4ac$, the discriminant of $ax^2 + bx + c = 0$.

The value of the discriminant determines the nature of the roots of the equation.

In this context, we study the following cases. We shall limit ourselves to cases where the coefficients a, b and c are real.

Case 1: If $b^2 - 4ac = 0$, i.e., $\Delta = 0$, then

$$\alpha = \frac{-b}{2a} \quad \text{and} \quad \beta = \frac{-b}{2a}.$$

So, $\alpha = \beta = \dfrac{-b}{2a}$.

Hence, when $\Delta = b^2 - 4ac = 0$, the two roots of the equation are real and equal.

Case 2: If $b^2 - 4ac > 0$, then the roots are real and distinct.

We shall consider, the nature of the roots when a, b and c are rational numbers.

1. If $b^2 - 4ac > 0$, i.e., $\Delta > 0$ and a perfect square, then the roots are rational and distinct.
2. If $b^2 - 4ac > 0$, i.e., $\Delta > 0$ and not a perfect square, then the roots are irrational and distinct.

In this case one root is a surd conjugate of the other. If one root is of the form $a + \sqrt{b}$, then the other root will be in the form of $a - \sqrt{b}$ and *vice-versa*.

Case 3: If $b^2 - 4ac < 0$, i.e., $\Delta < 0$, then the roots of the equation are imaginary.

Signs of the Roots of a Quadratic Equation

When the signs of the sum of the roots and the product of the roots of a quadratic equation are known, the signs of the roots of the quadratic equation can be determined. Let us consider the following four cases.

Case 1: When the sum and the product of the roots of a quadratic equation are both positive, then each root is positive.

For example, if the sum of the roots is 5 and the product of the roots is 6, then the roots are 2 and 3.

Case 2: When the sum of the roots is positive and the product of the roots is negative, the root having the greater magnitude is positive and the other root is negative.

For example, the sum of the roots is 6 and the product of the roots is −16, the roots are +8 and −2. Because $8 \times (-2) = -16$ and $+8 - 2 = 6$. The root with the greater magnitude, i.e., 8, has a positive sign and the root with the lesser magnitude, i.e., 2 has the negative sign.

Case 3: When the sum of the roots is negative and the product of the roots is positive, then both the roots are negative.

For example, the sum of the roots is −10 and the product of the roots is 24, the roots are −6 and −4.

$$-6 - 4 = -10 \text{ and } (-6)(-4) = 24.$$

In this case, both the roots, i.e., −6 and −4 bear the negative sign.

Case 4: When the sum of the roots is negative and the product of the roots is negative, the root having the greater magnitude has the negative sign and the other root has the positive sign.

For example, the sum of the roots is −4 and the product of the roots is −21, then the roots are −7 and +3.

$$(-7) \times 3 = -21 \text{ and } -7 + 3 = -4.$$

The root with the greater magnitude, i.e., 7, has the negative sign and the root with the lesser magnitude, i.e., 3, has the positive sign.

The above information can be tabulated as follows:

Sign of the Sum of the Roots	Sign of the Product of the Roots	Sign of the Roots
+ve	+ve	Both roots are positive
+ve	−ve	One root is positive, the other is negative. Numerically larger root is positive.
−ve	−ve	One root is positive and the other is negative. Numerically larger root is negative.
−ve	+ve	Both roots are negative.

Constructing the Quadratic Equation when its Roots are Given

Let us say that α and β are the roots of a quadratic equation.

The quadratic equation can be written as

$$(x - \alpha)(x - \beta) = 0.$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

That is, x^2 − (sum of the roots) x + (product of the roots) = 0

$$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0.$$

EXAMPLE 5.3

Solve the equation $x^2 - 11x + 30 = 0$ by using the formula.

SOLUTION

Given equation is $x^2 - 11x + 30 = 0$.

The roots of $ax^2 + bx + c = 0$, are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Here, $a = 1$, $b = -11$ and $c = 30$.

That is,

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(30)}}{2 \times 1} = \frac{11 \pm \sqrt{121 - 120}}{2}$$

$$x = \frac{11 \pm 1}{2}$$

$$x = \frac{11+1}{2} \text{ and } \frac{11-1}{2} \Rightarrow 6 \text{ and } 5.$$

∴ The roots are 5 and 6.

EXAMPLE 5.4

Find the nature of the roots of the equations given below:

(a) $x^2 - 13x + 11 = 0$

(b) $18x^2 - 14x + 17 = 0$

(c) $9x^2 - 36x + 36 = 0$

(d) $3x^2 - 5x - 8 = 0$

SOLUTION

(a) Given $x^2 - 13x + 11 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -13$ and $c = 11$.

Now, $b^2 - 4ac = (-13)^2 - 4(1)(11)$

$= 169 - 44 = 125 > 0$.

$\Rightarrow b^2 - 4ac > 0$ and is not a perfect square.

$\therefore$ The roots are distinct and irrational.

(b) Given $18x^2 - 14x + 17 = 0$.

Comparing it with $ax^2 + bx + c = 0$, $a = 18$, $b = -14$ and $c = 17$.

Now, $b^2 - 4ac = (-14)^2 - 4(18)(17)$

$= 196 - 1224$

$= -1028 < 0$

$\Rightarrow b^2 - 4ac < 0$.

$\therefore$ The roots are imaginary.

(c) Given $9x^2 - 36x + 36 = 0$.

$\Rightarrow 9(x^2 - 4x + 4) = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

Comparing the above equation, with $ax^2 + bx + c = 0$, $a = 1$, $b = -4$ and $c = 4$.

Now, $b^2 - 4ac = (-4)^2 - 4(1)(4)$

$= 16 - 16 = 0$

$\Rightarrow b^2 - 4ac = 0$

$\therefore$ The roots are real and equal.

(d) Given $3x^2 - 5x - 8 = 0$.

Comparing the above equation with $ax^2 + bx + c = 0$, we get, $a = 3$, $b = -5$ and $c = -8$.

Now, $b^2 - 4ac = (-5)^2 - 4(3)(-8)$

$= 25 + 96 = 121 > 0$

$\Rightarrow b^2 - 4ac > 0$ and is a perfect square.

$\therefore$ The roots are rational and distinct.

EXAMPLE 5.5

If α, β are the roots of the equation $x^2 - lx + m = 0$, then find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ in terms of l and m.

SOLUTION

Given α, β are the roots of $x^2 - lx + m = 0$.

$$\Rightarrow \text{Sum of the roots } = \alpha + \beta = \frac{-(-l)}{1} = l \qquad (1)$$

$$\Rightarrow \text{Product of the roots} = \alpha\beta = \frac{m}{1} = m \qquad (2)$$

Now,

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$=\frac{(\alpha+\beta)^2-2(\alpha\beta)}{(\alpha\beta)^2}$$

Substituting the values of $\alpha+\beta$ and $\alpha\beta$ in the above equation, we get,

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{l^2-2m}{m^2}$$

$\therefore$ The value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{l^2-2m}{m^2}$.

EXAMPLE 5.6

Write a quadratic equation whose roots are $\frac{5}{2}$ and $\frac{8}{3}$.

SOLUTION

Sum of the roots $=\frac{5}{2}+\frac{8}{3}=\frac{31}{6}$.

Product of the roots $=\frac{5}{2}\left(\frac{8}{3}\right)=\frac{20}{3}$.

The required quadratic equation is, x^2 − (sum of the roots)x + (product of the roots) = 0.

$$\Rightarrow x^2-\left(\frac{31}{6}\right)x+\frac{20}{3}=0$$

$$\Rightarrow 6x^2-31x+40=0$$

$\therefore$ A quadratic equation with roots $\frac{5}{2}$ and $\frac{8}{3}$ is $6x^2-31x+40=0$.

EXAMPLE 5.7

If one root of the equation, $x^2-11x+(p-3)=0$ is 3, then find the value of p and also its other root.

SOLUTION

Given that 3 is one of the roots of the equation $x^2-11x+p-3=0$.

$\Rightarrow x=3$ satisfies the given equation.

$$\Rightarrow (3)^2-11(3)+p-3=0$$

$$\Rightarrow p=33+3-9$$

$$\Rightarrow p=27$$

$\therefore$ The value of p is 27.

Since the sum of the roots of the equation is 11 and one of the roots is 3, the other root of the equation is 8.

Equations which can be Reduced to Quadratic Form

EXAMPLE 5.8

Solve $(x^2 - 2x)^2 - 23(x^2 - 2x) + 120 = 0$.

SOLUTION

Let us assume that $x^2 - 2x = y$

$\Rightarrow$ The given equation reduced to a quadratic equation in y

That is, $y^2 - 23y + 120 = 0$

$$\Rightarrow y^2 - 15y - 8y + 120 = 0$$
$$\Rightarrow y(y - 15) - 8(y - 15) = 0$$
$$\Rightarrow (y - 8)(y - 15) = 0$$
$$\Rightarrow y - 8 = 0 \text{ (or) } y - 15 = 0$$
$$\Rightarrow y = 8 \text{ (or) } y = 15$$

But $x^2 - 2x = y$

When $y = 8$, $x^2 - 2x = 8$

$$\Rightarrow x^2 - 2x - 8 = 0$$
$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$
$$\Rightarrow x(x - 4) + 2(x - 4) = 0$$
$$\Rightarrow (x + 2)(x - 4) = 0$$
$$\Rightarrow x + 2 = 0 \text{ (or) } x - 4 = 0$$
$$\Rightarrow x = -2 \text{ (or) } x = 4$$

When $y = 15$, $x^2 - 2x = 15$

$$\Rightarrow x^2 - 2x - 15 = 0$$
$$\Rightarrow x^2 - 5x + 3x - 15 = 0$$
$$\Rightarrow x(x - 5) + 3(x - 5) = 0$$
$$\Rightarrow (x - 5)(x + 3) = 0$$
$$\Rightarrow x - 5 = 0 \text{ (or) } x + 3 = 0$$
$$\Rightarrow x = 5 \text{ (or) } x = -3$$

$\therefore$ $x = -2, -3, 4$ and 5 are the required solutions of the given equation.

EXAMPLE 5.9

Solve $\sqrt{x+5} + \sqrt{5-x} = 4$.

SOLUTION

Squaring the terms on both the sides, we get

$$(\sqrt{x+5}+\sqrt{5-x})^2 = 4^2$$

$$\Rightarrow x+5+5-x+2\sqrt{(x+5)(5-x)} = 16$$

$$\Rightarrow 10+2\sqrt{25-x^2} = 16$$

$$\Rightarrow \sqrt{25-x^2} = 3$$

Squaring the terms on both the sides again, we get

$$25 - x^2 = 3^2$$

$$\Rightarrow x^2 = 25 - 9$$

$$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4.$$

$\therefore$ −4 and 4 are the required solutions of the given equation.

Reciprocal Equation

Any equation of the form $ax^4 + bx^3 + cx^2 + bx + a = 0$, in which the coefficients of terms equidistant from first and last are equal in magnitude, is called a reciprocal equation. This is one of the case of a reciprocal equation. This can be reduced to quadratic form by dividing by x^2 on both sides and with a proper substitution.

EXAMPLE 5.10

Solve $3x^4 - 8x^3 - 6x^2 + 8x + 3 = 0$.

SOLUTION

The above equation is a reciprocal equation. Dividing the equation by x^2, we get

$$\frac{3x^4 - 8x^3 - 6x^2 + 8x + 3}{x^2} = 0$$

$$\Rightarrow 3\left(x^2+\frac{1}{x^2}\right)-8\left(x-\frac{1}{x}\right)-6=0 \qquad (1)$$

Now put $x - \dfrac{1}{x} = y$

$$\therefore\ y^2 = \left(x-\frac{1}{x}\right)^2$$

$$\Rightarrow y^2 = x^2+\frac{1}{x^2}-2$$

$$\Rightarrow x^2+\frac{1}{x^2} = y^2+2$$

Substituting $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$ in terms of y in the Eq. (1) we get,

$$3(y^2 + 2) - 8(y) - 6 = 0.$$
$$\Rightarrow 3y^2 + 6 - 8y - 6 = 0$$
$$\Rightarrow 3y^2 - 8y = 0$$
$$\Rightarrow y(3y - 8) = 0$$

When $y = 0$, $x - \frac{1}{x} = 0$

$$\Rightarrow x^2 = 1$$
$$\Rightarrow x = \pm 1$$

When $y = \frac{8}{3}$, $x - \frac{1}{x} = \frac{8}{3}$

$$\Rightarrow 3x^2 - 3 = 8x$$
$$\Rightarrow 3x^2 - 8x - 3 = 0$$
$$\Rightarrow 3x^2 - 9x + x - 3 = 0$$
$$\Rightarrow 3x(x - 3) + 1(x - 3) = 0$$
$$\Rightarrow (3x + 1)(x - 3) = 0$$
$$\Rightarrow x = -\frac{1}{3} \text{ (or) } x = 3$$

$\therefore$ $x = \pm 1$, $\frac{-1}{3}$ and 3 are the required solutions of the given equation.

Constructing a New Quadratic Equation by Changing the Roots of a Given Quadratic Equation

If we are given a quadratic equation, we can build a new quadratic equation by changing the roots of the given equation as directed.

For example, consider the quadratic equation $ax^2 + bx + c = 0$, whose roots are α and β.

The new equations can be constructed in the following manner:

1. A quadratic equation whose roots are the reciprocals of the roots of the equation $ax^2 + bx + c = 0$, can be formed by substituting $\frac{1}{x}$ for x. The new equation is $a\left(\frac{1}{x}\right)^2 + b\left(\frac{1}{x}\right) + c = 0$, i.e., $cx^2 + bx + a = 0$.
2. A quadratic equation, whose roots are k more than the roots of the equation $ax^2 + bx + c = 0$, is obtained by substituting $(x - k)$ for x in the given equation.
3. A quadratic equation whose roots are k less than the roots of the equation $ax^2 + bx + c = 0$ can be obtained by substituting $(x + k)$ for x in the given equation.

4. A quadratic equation whose roots are k times the roots of the equation $ax^2 + bx + c = 0$ can be obtained by substituting $\frac{x}{k}$ for x in the given equation.

5. A quadratic equation whose roots are $\frac{1}{k}$ times the roots of the equation $ax^2 + bx + c = 0$ can be obtained by substituting kx for x in the given equation.

EXAMPLE 5.11

The roots of $x^2 - (a + 1)x + b^2 = 0$ are equal. Then choose the correct value of a, b from the following option:

(a) 5, 2 **(b)** 3, 4 **(c)** 5, −3 **(d)** 5, 4

SOLUTION

The roots of $x^2 - (a + 1)x + b^2 = 0$ are equal

$\Rightarrow (a + 1)^2 - 4b^2 = 0$

$\Rightarrow a + 1 = \pm 2b$

From the options

$a = 5$, $b = -3$ satisfies the above relation.

Maximum or Minimum Value of a Quadratic Expression

The quadratic expression $ax^2 + bx + c$ takes different values, as x takes different values.

As x varies from $-\infty$ to $+\infty$ (i.e., when x is real), the quadratic expression $ax^2 + bx + c$

1. has the minimum value, when $a > 0$.

2. has the maximum value, when $a < 0$.

The minimum or the maximum value of the quadratic expression $ax^2 + bx + c$ occurs at $x = \frac{-b}{2a}$ and is equal to $\frac{4ac - b^2}{4a}$.

When x has an imaginary value, $ax^2 + bx + c$ may have a real value or an imaginary value. For some imaginary values of x, $ax^2 + bx + c$ will be real and it may have minimum or maximum value. But such cases will be dealt in the higher stages of learning.

EXAMPLE 5.12

Find the value of x, to get the maximum value of $-3x^2 + 6x + 5$.

(a) 1 **(b)** $\frac{5}{3}$ **(c)** $\frac{-5}{6}$ **(d)** 6

SOLUTION

Maximum value of a quadratic expression occurs at $x = \frac{-b}{2a}$.

$\Rightarrow$ For $-3x^2 + 6x + 5$, maximum value occurs at $x = \frac{-6}{2(-3)}$, i.e., 1.

EXAMPLE 5.13

Choose the minimum value of $\dfrac{2x^2+12x-3}{1+18x-3x^2}$ from the following options:

(a) $\dfrac{-15}{29}$ **(b)** $\dfrac{15}{28}$ **(c)** $\dfrac{-15}{28}$ **(d)** None of these

SOLUTION

For the minimum value of $\dfrac{2x^2-12x+3}{1+18x-3x^2}$, $2x^2 - 12x + 3$ is minimum and $1 + 18x - 3x^2$ is maximum.

The minimum value of $2x^2 - 12x + 3$ occurs at $x = \dfrac{-b}{2a} = \dfrac{-(-12)}{2\times 2} = 3$.

The maximum value of $1 + 18x - 3x^2$ occurs at $x = \dfrac{-b}{2a} = \dfrac{-18}{2\times -3} = 3$.

Minimum value of given expression is $\dfrac{2(3)^2-12(3)+3}{1+18(3)-3(3)^2} = \dfrac{18-36+3}{55-27} = \dfrac{-15}{28}$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. For the expression $ax^2 + 7x + 2$ to be quadratic, the possible values of a are ______.
2. The equation, $ax^2 + bx + c = 0$ can be expressed as $a\alpha^2 + b\alpha + c = 0$, only when '$\alpha$' is ______ of the equation.
3. If -3 and 4 are the roots of the equation $(x + k)(x - 4) = 0$, then the value of k is ______.
4. The polynomial, $\sqrt{3}x^2 + 2x + 1$ is a ______ expression.
5. For the equation $2x^2 - 3x + 5 = 0$, sum of the roots is ______.
6. The quadratic equation having roots $-a$, $-b$ is ______.
7. A quadratic equation whose roots are 2 more than the roots of the quadratic equation $2x^2 + 3x + 5 = 0$ can be obtained by substituting ______ for x. [$(x - 2)/(x + 2)$]
8. For the expression $7x^2 + bx + 4$ to be quadratic, the possible values of b are ______.
9. If $(x - 2)(x + 3) = 0$, then the values of x are ______.
10. The roots of the equation $x^2 + ax + b = 0$ are ______.
11. $x = 2$ is a root of the equation $x^2 - 5x + 6 = 0$. Is the given statement true?
12. If the equation $3x^2 - 2x - 3 = 0$ has roots α and β, then $\alpha \cdot \beta =$ ______.
13. If the discriminant of the equation $ax^2 + bx + c = 0$ is greater than zero, then the roots are ______.
14. If the roots of a quadratic equation $ax^2 + bx + c$ are complex, then $b^2 <$ ______.
15. The roots of a quadratic equation $ax^2 + bx + c = 0$ are 1 and $\frac{c}{a}$, then $a + b + c =$ ______.
16. If the roots of a quadratic equation are equal, then the discriminant of the equation is ______.
17. For what values of b, the roots of $x^2 + bx + 9 = 0$ are equal?
18. If the sum of the roots of a quadratic equation is positive and product of the roots is negative, the numerically greater root has ______ sign. [positive/negative]
19. If $x = 1$ is a solution of the quadratic equation $ax^2 - bx + c = 0$, then b is equal to ______.

Short Answer Type Questions

20. Factorize the following quadratic expressions:
    (a) $x^2 + 5x + 6$
    (b) $x^2 - 5x - 36$
    (c) $2x^2 + 5x - 18$
21. Determine the nature of the roots of the following equations:
    (a) $x^2 + 2x + 4 = 0$
    (b) $3x^2 - 10x + 3 = 0$
    (c) $x^2 - 24x + 144 = 0$

Solve the following quadratic equations:

22. If $f(x) = x^2 - 5x - 36$ and $g(x) = x^2 + 9x + 20$, then for what values of x is $2f(x) = 3g(x)$?
23. Solve: $16x^4 - 28x^2 - 8 = 0$
24. For what value of m does the equation, $mx^2 + (3x - 1)m + 2x + 5 = 0$ have equal roots but of opposite sign?
25. Find the value of m for which the quadratic equation, $3x^2 - 10x + (m - 3) = 0$ has roots which are reciprocal to each other.
26. If a, b are the roots of the equation $x^2 - px + q = 0$, then find the equation which has $\frac{a}{b}$ and $\frac{b}{a}$ as its roots.
27. If one root of the equation $x^2 - mx + n = 0$ is twice the other root, then show that $2m^2 = 9n$.
28. The square of one-sixth of the number of students in a class are studying in the library and the

remaining eight students are playing in the ground. What is the total number of students of the class?

29. If 2α and 3β are the roots of the equation $x^2 + ax + b = 0$, then find the equation whose roots are a, b.

30. If α, β are the roots of the quadratic equation $lx^2 + mx + n = 0$, then evaluate the following expressions.
   (a) $\alpha^2 + \beta^2$
   (b) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$
   (c) $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3}$

31. If the price of sugar is reduced by ₹1 per kg, 5 kilograms more can be purchased for ₹1200. What was the original price of sugar per kilogram?

32. The zeroes of the quadratic polynomial $x^2 - 24x + 143$ are

33. Find the quadratic equation in x whose roots are $\dfrac{-7}{2}$ and $\dfrac{8}{3}$.

34. If k_1 and k_2 are the roots of $x^2 - 5x - 24 = 0$, then find the quadratic equation whose roots are $-k_1$ and $-k_2$.

35. The product of the roots of the equations $\dfrac{1}{x+1} + \dfrac{1}{x-2} = \dfrac{1}{x+2}$ is ______.

36. The roots of the equation $2x^2 + 3x + c = 0$ (where $c < 0$) could be ______.

37. The roots of the equation $30x^2 - 7\sqrt{3x} + 1 = 0$ are

38. If α and β are the roots of the quadratic equation $x^2 + 3x - 4 = 0$, then $\alpha^{-1} + \beta^{-1} =$ ______.

39. The roots of the equation $\dfrac{x-3}{x-2} + \dfrac{2x}{x+3} = 1$, where $x \neq 2, -3$, are

40. If the roots of the quadratic equation $4x^2 - 16x + p = 0$ are real and unequal, then find the value/s of p.

41. If one root of the quadratic equation $ax^2 + bx + c = 0$ is $15 + 2\sqrt{56}$ and a, b and c are rational, then find the quadratic equation.

42. If the roots of the equation $ax^2 + bx + 4c = 0$ are in the ratio of 3 : 4, then find the relation between a, b and c.

43. For which value of p among the following, does the quadratic equation $3x^2 + px + 1 = 0$ have real roots?

44. If the product of the roots of $ax^2 + bx + 2 = 0$ is equal to the product of the roots of $px^2 + qx - 1 = 0$, then $a + 2p =$ ______.

45. Find the roots of quadratic equation $ax^2 + (a - b + c)\,x - b + c = 0$.

46. If $(2x - 9)$ is a factor of $2x^2 + px - 9$, then $p =$ ______.

Essay Type Questions

Solve the given equations:

47. $(x + 3)(x + 4)(x + 6)(x + 7) = 1120$.

48. $(x^2 + 3x)^2 - 16(x^2 + 3x) - 36 = 0$.

49. $\sqrt{x-3} + \sqrt{3x+4} = 5$.

50. $3x^4 - 10x^3 - 3x^2 + 10x + 3 = 0$.

CONCEPT APPLICATION

Level 1

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. The solution of the equation $x^2 + x + 1 = 1$ is are
   (a) $x = 0$
   (b) $x = -1$
   (c) Both (a) and (b)
   (d) Cannot be determined

2. The discriminant of the equation $x^2 - 7x + 2 = 0$ is
   (a) 47 (b) 40
   (c) 41 (d) -41

3. Find the maximum value of the quadratic expression $-3x^2 + 7x + 4$.

PRACTICE QUESTIONS

(a) $8\frac{1}{6}$ (b) $8\frac{1}{12}$

(c) $8\frac{1}{4}$ (d) 12

4. If α and β are the roots of the equation $x^2 + 3x - 2 = 0$, then $\alpha^2\beta + \alpha\beta^2 = ?$

(a) -6 (b) -3

(c) 6 (d) 3

5. If one of the roots of an equation, $x^2 - 2x + c = 0$ is thrice the other, then $c = ?$

(a) $\frac{1}{2}$ (b) $\frac{4}{3}$

(c) $-\frac{1}{2}$ (d) $\frac{3}{4}$

6. The number of real roots of the quadratic equation $3x^2 + 4 = 0$ is

(a) 0 (b) 1

(c) 2 (d) 4

7. If α and β are the roots of the equation $x^2 + px + q = 0$ then $(\alpha - \beta)^2 =$ ______.

(a) $q^2 - 4p$ (b) $4q^2 - p$

(c) $p^2 - 4q$ (d) $p^2 + 4q$

8. Which of the following equations does not have real roots?

(a) $x^2 + 4x + 4 = 0$ (b) $x^2 + 9x + 16 = 0$

(c) $x^2 + x + 1 = 0$ (d) $x^2 + 3x + 1 = 0$

9. The sum of the roots of the equation, $ax^2 + bx + c = 0$ where a, b and c are rational and whose one of the roots is $4-\sqrt{5}$, is

(a) 8 (b) $-2\sqrt{5}$

(c) $2\sqrt{5}$ (d) 11

10. For the quadratic equation $x^2 + 3x - 4 = 0$ which of the following is a solution?

(A) $x = -4$ (B) $x = 3$

(C) $x = 1$

(a) A and B (b) B and C

(c) A and C (d) Only A

11. Find the quadratic equation whose roots are reciprocals of the roots of the equation $7x^2 - 2x + 9 = 0$.

(a) $9x^2 - 2x + 7 = 0$

(b) $9x^2 - 2x - 7 = 0$

(c) $9x^2 + 2x - 7 = 0$

(d) $9x^2 + 2x + 7 = 0$

12. The number of real roots of the quadratic equation $(x - 4)^2 + (x - 5)^2 + (x - 6)^2 = 0$ is

(a) 1 (b) 2

(c) 3 (d) None of these

13. The number of distinct real solutions of $|x|^2 - 5|x| + 6 = 0$ is

(a) 4 (b) 3

(c) 2 (d) 1

14. The number of real solutions of $|x|^2 - 5|x| + 6 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 4

15. In writing a quadratic equation of the form $x^2 + px + q = 0$, a student makes a mistake in writing the coefficient of x and gets the roots as 8 and 12. Another student makes a mistake in writing the constant term and gets the roots as 7 and 3. Find the correct quadratic equation.

(a) $x^2 - 10x + 96 = 0$

(b) $x^2 - 20x + 21 = 0$

(c) $x^2 - 21x + 20 = 0$

(d) $x^2 - 96x + 10 = 0$

16. The roots of the equation $6x^2 - 8\sqrt{2x} + 4 = 0$ are

(a) $\frac{1}{3}, \sqrt{2}$ (b) $\frac{\sqrt{2}}{3}, 1$

(c) $\frac{\sqrt{2}}{3}, \sqrt{2}$ (d) $\frac{3}{\sqrt{2}}, \sqrt{2}$

17. Which of the following equations has roots as a, b and c?

(a) $x^3 + x^2(a + b + c) + x(ab + bc + ca) + abc = 0$

(b) $x^3 + x^2(a + b + c) + x(ab + bc + ca) - abc = 0$

(c) $x^3 - x^2(a + b + c) + x(ab + bc + ca) - abc = 0$

(d) $x^3 - x^2(a + b + c) - x(ab + bc + ca) - abc = 0$

18. If the roots of the equation $2ax^2 + (3b - 9)x + 1 = 0$ are -2 and 3, then the values of a and b respectively are

PRACTICE QUESTIONS

(a) $\frac{1}{12}, \frac{5}{18}$ (b) $\frac{-1}{12}, \frac{-53}{18}$

(c) $\frac{-1}{12}, \frac{-5}{8}$ (d) $\frac{-1}{12}, \frac{55}{18}$

19. The roots of the equation $x^2 + 5x + 1 = 0$ are

(a) $\frac{5+\sqrt{21}}{2}, \frac{5-\sqrt{21}}{2}$

(b) $\frac{-5-\sqrt{21}}{2}, \frac{5+\sqrt{21}}{2}$

(c) $\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}$

(d) $\frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}$

20. If α and β are the roots of the equation $3x^2 - 2x - 8 = 0$, then $\alpha^2 - \alpha\beta + \beta^2 =$ ______.

(a) $\frac{76}{9}$ (b) $\frac{25}{3}$

(c) $\frac{16}{3}$ (d) $\frac{32}{3}$

21. If $2x^2 + (2p - 13)\, x + 2 = 0$ is exactly divisible by $x - 3$, then the value of p is

(a) $\frac{-16}{6}$ (b) $\frac{19}{6}$

(c) $\frac{16}{6}$ (d) $\frac{-19}{6}$

22. If $x^2 - ax - 6 = 0$ and $x^2 + ax - 2 = 0$ have one common root, then a can be ______.

(a) -1 (b) 2

(c) -3 (d) 0

23. The root of the equation $\frac{2}{x-1} + \frac{1}{x+2} + \frac{3x(x+1)}{(x-1)(x+2)} = 0$ among the following is ______.

(a) 2 (b) 3

(c) -1 (d) 0

24. If α and β are the roots of the equation, $2x^2 - 5x + 2 = 0$, then $(\alpha - 1)^{\beta - 1} =$ ______. where $(\alpha > \beta)$.

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 1

25. The age of a father is 25 years more than his son's age. The product of their ages is 84 in years. What will be son's age in years, after 10 years?

(a) 3 (b) 28

(c) 13 (d) 18

26. If the roots of the equation $ax^2 - bx + 5c = 0$ are in the ratio of 4 : 5, then

(a) $ab = 18c^2$

(b) $81b^2 = 4ac$

(c) $bc = a^2$

(d) $4b^2 = 81ac$

27. The speed of Uday is 5 km/h more than that of Subash. Subash reaches his home from office 2 hours earlier than Uday. If Subash and Uday stay 12 km and 48 km from their respective offices, find the speed of Uday.

(a) 10 km/h

(b) 4 km/h

(c) 9 km/h

(d) 8 km/h

28. If the roots of the quadratic equation $c(a - b)x^2 + a(b - c)x + b(c - a) = 0$ are equal, then

(a) $2b^{-1} = a^{-1} + c^{-1}$

(b) $2c^{-1} = a^{-1} + b^{-1}$

(c) $2a^{-1} = b^{-1} + c^{-1}$

(d) None of these

29. If the roots of the quadratic equation $x^2 - 3x - 304 = 0$ are α and β, then the quadratic equation with roots 3α and 3β is

(a) $x^2 + 9x - 2736 = 0$

(b) $x^2 - 9x - 2736 = 0$

(c) $x^2 - 9x + 2736 = 0$

(d) $x^2 + 9x + 2736 = 0$

30. If $x^2 + \alpha_1 x + \beta_1 = 0$ and $x^2 + \alpha_2 x + \beta_2 = 0$ have a common root $(x - k)$, then find k.

(a) $k = \frac{\alpha_2 - \alpha_1}{\beta_2 - \beta_1}$ (b) $k = \frac{\alpha_2 - \alpha_1}{\beta_2 - \beta_1}$

(c) $k = \frac{\alpha_2 - \alpha_1}{\beta_2 - \beta_1}$ (d) $k = \frac{\alpha_2 - \alpha_1}{\beta_2 - \beta_1}$

Level 2

31. If one of the roots of $x^2 + (1 + k)x + 2k = 0$ is twice the other, then $\frac{a^2+b^2}{ab}$ ______.

(a) 2 (b) 1

(c) 4 (d) 7

32. If α and β are the roots of $2x^2 - x - 2 = 0$, then $(\alpha^{-3} + \beta^{-3} + 2\alpha^{-1}\beta^{-1})$ is equal to

(a) $-\frac{17}{8}$ (b) $\frac{23}{6}$

(c) $\frac{37}{9}$ (d) $-\frac{29}{8}$

33. In a right angled triangle, one of the perpendicular sides is 4 cm greater than the other and 4 cm lesser than the hypotenuse. Find the area of triangle in cm^2.

(a) 72 (b) 48

(c) 36 (d) 96

34. In a fraction, the denominator is 1 less than the numerator. The sum of the fraction and its reciprocal is $2\frac{1}{56}$. Find the fraction.

(a) $\frac{3}{2}$ (b) $\frac{13}{12}$

(c) $\frac{18}{17}$ (d) $\frac{8}{7}$

35. The length of the rectangular surface of a table is 10 m more than its breadth. If the area of the surface is 96 m^2, its perimeter is (in m) ______.

(a) 64 (b) 44

(c) 52 (d) 48

36. If α and β are the roots of the equation $x^2 + 9x + 18 = 0$, then the quadratic equation having the roots $\alpha + \beta$ and $\alpha - \beta$ is ______, where $(\alpha > \beta)$.

(a) $x^2 + 6x - 27 = 0$ (b) $x^2 - 9x + 27 = 0$

(c) $x^2 - 9x + 7 = 0$ (d) $x^2 + 6x + 27 = 0$

37. Find the minimum value of the quadratic expression $4x^2 - 3x + 4$.

(a) $\frac{-55}{16}$ (b) $\frac{55}{16}$

(c) $\frac{16}{15}$ (d) $\frac{161}{22}$

38. If $(x + 2)$ is a common factor of the expressions $x^2 + ax - 6$, $x^2 + bx + 2$ and $kx^2 - ax - (a + b)$, then $k =$ ______.

(a) 2 (b) 3

(c) 1 (d) −2

39. The roots of a pure quadratic equation exists only if ______.

(a) $a > 0,\ c < 0$ (b) $c > 0,\ a < 0$

(c) $a > 0,\ c \le 0$ (d) Both (a) and (b)

40. The roots of the equation $\frac{x-1}{x-2}+\frac{x-3}{x-4}=2\frac{1}{3}$, (where $x \neq 2, 4$) are

(a) $6+\sqrt{10}, 6-\sqrt{10}$

(b) $6+2\sqrt{10}, 6-\sqrt{10}$

(c) $6+6\sqrt{10}, 6-6\sqrt{10}$

(d) $2+2\sqrt{10}, 6-2\sqrt{10}$

41. If $x + 3$ is the common factor of the expressions $ax^2 + bx + 1$ and $px^2 + qx - 3$, then $-\frac{(9a+3p)}{3b+q} =$ ______.

(a) −2 (b) 2

(c) 3 (d) −1

42. If the sum of the roots of an equation $x^2 + px + 1 = 0$ $(p > 0)$ is twice the difference between them, then $p =$ ______.

(a) $-\frac{1}{4}$ (b) $\frac{3}{4}$

(d) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

43. The equation $x+\frac{5}{3-x}=3+\frac{5}{3-x}$ has

(a) no real root.

(b) one real root.

(c) two equal roots.

(d) infinite roots.

44. The roots of the equation $\frac{1}{2x-3}-\frac{1}{2x+5}=8$ are

PRACTICE QUESTIONS

(a) $2-\frac{1}{\sqrt{2}}, 2-\frac{1}{\sqrt{2}}$

(b) $\frac{-1+\sqrt{17}}{2}, \frac{-1-1\sqrt{17}}{2}$

(c) $2+\frac{1}{\sqrt{2}}, 2-\frac{1}{\sqrt{2}}$

(d) $\frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}$

45. If the quadratic expression $x^2 + (a - 4)x + (a + 4)$ is a perfect square, then $a =$ _______.

(a) 0 and −4 (b) 0 and 6
(c) 0 and 12 (d) 6 and 12

46. The minimum value of $2x^2 - 3x + 2$ is _______.

(a) $\frac{7}{8}$ (b) $\frac{4}{7}$ (c) 4 (d) −3

47. The roots of $x^2 - 2x - 1 = 0$ are _______.

(a) $\sqrt{2}+1, \sqrt{2}-1$

(b) $1, \sqrt{2}$

(c) $1+\sqrt{2}, 1-\sqrt{2}$

(d) 2, 1

48. If $2x^2 + 4x - k = 0$ is same as $(x-5)\left(x+\frac{k}{10}\right) = 0$, then find the value of k.

(a) 100 (b) 90
(c) 70 (d) 35

49. If the numerically smaller root of $x^2 + mx = 2$ is 3 more than the other one, find the value of m.

(a) −1 (b) 1
(c) −2 (d) 2

Level 3

50. Two persons A and B solved a quadratic equation of the form $x^2 + bx + c = 0$. A made a mistake in noting down the coefficient of x and obtained the roots as 18 and 2, where as B obtained the roots as −9 and −3 by misreading the constant term. The correct roots of the equation are

(a) −6, −3 (b) −6, 6
(c) −6, −5 (d) −6, −6

51. If α and β are the roots of $x^2 - x + 2 = 0$, then find the value of $(\alpha^{-6} + \beta^{-6} + 2\alpha^{-3}\beta^{-3})\alpha^6\beta^6$.

(a) 16 (b) 25
(c) 30 (d) 36

52. If $b_1, b_2, b_3, \ldots, b_n$ are positive, then the least value of $(b_1 + b_2 + b_3 + \cdots + b_n)\left(\frac{1}{b_1}+\frac{1}{b_2}+\cdots+\frac{1}{b_n}\right)$ is

(a) $b_1 b_2 \cdots b_n$ (b) $n^2 + 1$
(c) $n(n + 1)$ (d) n^2

53. The equation $\sqrt{x+1}-\sqrt{4x-1}=\sqrt{x-1}$ has

(a) no solution. (b) one solution.
(c) two solutions. (d) more than two solutions.

54. Out of the group of employees, twice the square root of the number of the employees are on a trip to attend a conference held by the company, half the number are in the office and the remaining six employees are on leave. What is the number of employees in the group?

(a) 49 (b) 64
(c) 36 (d) 100

55. Find the quadratic equation whose roots are 2 times the roots of $x^2 - 12x - 13 = 0$.

(a) $x^2 - 24x - 52 = 0$

(b) $x^2 - 24x - 26 = 0$

(c) $x^2 - 14x - 15 = 0$

(d) None of these

56. If one of the roots of $ax^2 + bx + c = 0$ is thrice that of the other root, then b can be

(a) $\frac{4ac}{3}$ (b) $\frac{16ac}{9}$

(c) $4\sqrt{\frac{ac}{3}}$ (d) $\sqrt{\frac{4ac}{3}}$

57. If α, β are the roots of $px^2 + qx + r = 0$, then $\alpha^3 + \beta^3 =$ _______.

(a) $\frac{3qpr-q^3}{p^3}$ (b) $\frac{3pqr-3q}{p^3}$

(c) $\frac{pqr-3q}{p^3}$ (d) $\frac{3pqr-q}{p^3}$

PRACTICE QUESTIONS

58. If α and β are the roots of $x^2 - (a + 1)x + \frac{1}{2}(a^2 + a + 1) = 0$ then $\alpha^2 + \beta^2 =$ ______.

(a) a (b) a^2

(c) $2a$ (d) 1

59. The number of roots of the equation $2|x|^2 - 7|x| + 6 = 0$

(a) 4 (b) 3

(c) 2 (d) 1

60. In a quadratic equation $ax^2 - bx + c = 0$, a, b, c are distinct primes and the product of the sum of the roots and product of the roots is $\frac{91}{9}$. Find the difference between the sum of the roots and the product of the roots.

(a) 2 (b) 3

(c) 4 (d) Cannot be determined

61. Maximum value of $\frac{2+12x-3x^2}{2x^2-8x+9}$ is ______.

(a) 14 (b) 17

(c) 11 (d) Cannot be determined

62. If $x^2 + ax + b$ and $x^2 + bx + c$ have a common factor $(x - k)$, then $k =$ ________.

(a) $\frac{a-b}{b-c}$ (b) $\frac{b-c}{c-a}$

(c) $\frac{c-b}{b-a}$ (d) $\frac{c-b}{a-b}$

63. If $9x - 3y + z = 0$, then the value of $\frac{y}{2x} + \sqrt{\frac{y^2 - 4xz}{4x^2}}$ (where x, y, z are constants).

(a) 9 (b) 2

(c) 3 (d) 6

64. If the roots of $3x^2 - 12x + k = 0$ are complex, then find the range of k.

(a) $k < 22$ (b) $k < -10$

(c) $k > 11$ (d) $k > 12$

65. If α, β are the roots of $ax^2 + bx + c = 0$, then find the quadratic equation whose roots are $\alpha + \beta$, $\alpha\beta$.

(a) $ax^2 + (ab - ac)x - c = 0$

(b) $ax^2 + (b - c)x - bc = 0$

(c) $a^2x^2 + (b - c)x - ac = 0$

(d) $a^2x^2 + (ab - ac)x - bc = 0$

66. Ramu swims a distance of 3 km each upstream and downstream. The total time taken is one hour. If the speed of the stream is 4 km/h, then find the speed of Ramu in still water.

(a) 12 km/h (b) 9 km/h

(c) 8 km/h (d) 6 km/h

67. In solving a quadratic equation $x^2 + px + q = 0$ a student made a mistake in copying the coefficient of x and obtained the roots as 4, −3 but one of the actual roots is 2 what is the difference between the actual and wrong values of the coefficients of x?

(a) 5 (b) 4

(c) 7 (d) +6

68. The roots of $ax^2 - bx + 2c = 0$ are in the ratio of 2 : 3, then ______.

(a) $a^2 = bc$ (b) $3b^2 = 25ac$

(c) $2b^2 = 75c$ (d) $5b^2 = ac$

69. If the roots of $9x^2 - 2x + 7 = 0$ are 2 more than the roots of $ax^2 + bx + c = 0$, then $4a - 2b + c$ can be

(a) −2 (b) 7

(c) 9 (d) 10

70. If the roots of $ax^2 + bx + c = 0$ are 2 more than the roots of $px^2 + qx + r = 0$, then the value of c in terms of p, q and r is

(a) $p + q + r$ (b) $4p - 2q + r$

(c) $3p - q + 2r$ (d) $2p + q - r$

71. If the roots of $2x^2 + 7x + 5 = 0$ are the reciprocal roots of $ax^2 + bx + c = 0$, then $a - c =$ ______.

(a) 3 (b) −3

(c) −2 (d) −5

72. If the roots of the equation $ax^2 + bx + c = 0$ is $\frac{1}{k}$ times the roots of $px^2 + qx + r = 0$, then which of the following is true?

(a) $a = pk$ (b) $\frac{a}{b} = \frac{p}{q}$

(c) $aq = pbk$ (d) $ab = pqk^2$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. non-zero real numbers
2. root
3. 3
4. quadratic
5. $\frac{3}{2}$
6. $x^2 + x(a + b) + ab = 0$
7. $(x - 2)$
8. real numbers
9. 2, −3
10. $\frac{-a \pm \sqrt{a^2 - 4b}}{2}$
11. Yes
12. −1
13. real and distinct
14. $4ac$
15. Zero
16. zero
17. ±6
18. positive
19. $a + c$

Short Answer Type Questions

20. (a) $(x + 2)(x + 3)$
    (b) $(x - 9)(x + 4)$
    (c) $(x - 2)(2x + 9)$
21. (a) Complex conjugates
    (b) Real and distinct
    (c) Real and equal
22. −4 and −33
23. $\pm\sqrt{2}$
24. $\frac{-2}{3}$
25. 6
26. $qx^2 - (p^2 - 2q)x + q = 0$
27. 8 km/h
28. 12 or 24
29. $x^2 - (6\alpha\beta - 2\alpha - 3\beta)x - (2\alpha + 3\beta)(6\alpha\beta) = 0$
30. (a) $\frac{m^2 - 2ln}{l^2}$ (b) $\frac{m^2 - 2ln}{ln}$ (c) $\frac{3lmn - m^3}{n^3}$
31. ₹16
32. 11, 13
33. $6x^2 + 5x - 59 = -3$
34. $x^2 + 5x - 24 = 0$
35. zero
36. rational or irrational, but unequal
37. $\frac{1}{2\sqrt{3}}, \frac{1}{5\sqrt{3}}$
38. $\frac{3}{4}$
39. 3, $\frac{-1}{2}$
40. $p < 16$
41. $x^2 - 30x + 1 = 0$
42. $3b^2 = 49ac$
43. 4
44. 0
45. −1, $\frac{b - c}{a}$
46. −7

Essay Type Questions

47. $x = 1$, $x = -11$
48. −1, −2, 3 and −6
49. 4
50. $\frac{1 \pm \sqrt{37}}{6}, \frac{3 \pm \sqrt{13}}{2}$

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (a)
21. (b)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (a)

Level 2

31. (d)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (a)
41. (d)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (c)	**49.** (b)	

Level 3

50. (d)	**51.** (b)	**52.** (d)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (a)
60. (a)	**61.** (a)	**62.** (d)	**63.** (c)	**64.** (d)	**65.** (d)	**66.** (c)	**67.** (a)	**68.** (b)	**69.** (b)
70. (b)	**71.** (a)	**72.** (c)							

CONCEPT APPLICATION

Level 1

1. Simplify and factorize.
2. Use the formula to find the discriminant.
3. Use the formula to find the maximum value.
4. Find the sum and product of the roots. Let $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$.
5. Let the roots be α and 3α.
6. Solve for x.
7. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.
8. Find the value of the discriminant for each of the equations.
9. If one root is $4-\sqrt{5}$, then the other root is $4 + \sqrt{5}$, because the coefficients of the x^n terms are rational.
10. Solve for x.
11. The quadratic equation with reciprocals of the roots of the equation $f(x) = 0$ is $f\left(\frac{1}{x}\right) = 0$.
12. (i) Use the concept of perfect square of a number.
    (ii) If $a^2 + b^2 + c^2 = 0$ is true only when $a = b = c = 0$.
13. (i) Solve the equation to find the number of real roots.
    (ii) Replace $|x|$ by y and solve for y.
    (iii) Now, $x = \pm y$.
14. (i) Use the concept $|x|$ and find the roots.
    (ii) Replace $|x|$ by y and solve for y.
    (iii) Now, $x = \pm y$.
15. (i) Use the concept of sum and product of the roots of a quadratic equation.
    (ii) The product of the roots obtained by the first student is product of the roots of the required quadratic equation.
    (iii) The sum of the roots obtained by the second student is sum of the roots of the required quadratic equation.
16. Take 2 as common and then factorize.
17. Let $(x - a)(x - b)(x - c) = 0$ and expand.
18. Substitute $x = -2$ and $x = 3$ to get to equation in a and b and then solve for a and b.
19. Apply formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
20. Find $\alpha\beta$ and $(\alpha + \beta)$ and use, $\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 3\alpha\beta$.
21. Substitute $x = 3$ in the given equation and simplify.
22. Find x in terms of a and substitute $x = a$ in either of the equations.
23. Simplify and solve for x.
24. Factorize LHS of the given equation to find α and β.
25. (i) Form a quadratic equation by assuming the age of son as r years. The roots of quadratic equation are 1 and $\frac{c}{a}$ only if sum of all the coefficient $s = 0$.
    (ii) Assume the ages of father and son as x and $(25 - x)$ years.
    (iii) Write the relation in terms of x according to the data and then solve the equation.
26. (i) Use the concept of sum and product of the roots of a quadratic equation.
    (ii) Take the roots as 4α, 5α.
    (iii) Using the sum of the roots and product of the roots eliminate α.
27. (i) Frame the quadratic equation from the given data.
    (ii) Assume the speed of Subhash as x and Subhash reaches his home in t hours and Uday reaches his home in $(t + 2)$ hours.
    (iii) Now, use time $= \frac{\text{distance}}{\text{speed}}$, then solve the equation for x.
28. (i) If sum of all coefficients is zero, then 1, $\frac{c}{a}$ are the roots of $ax^2 + bx + c = 0$.

(ii) If the sum of the coefficients is 0, then 1 and $\frac{c}{a}$ are the roots of the equation.

(iii) Use product of the roots concept.

29. (i) The quadratic equation with thrice the roots of $f(x) = 0$ as roots is $f\left(\frac{1}{x}\right) = 0$.

(ii) The quadratic equation whose roots are m times of the roots of the equation $f(x) = 0$ is $\frac{-1}{12}, \frac{55}{18} = 0$.

30. (i) Use the concept of common root of given equations.

(ii) Put $x = k$ in the given equations and solve for k.

Level 2

31. (i) Use the concept of sum and product of the roots of a quadratic equation.

(ii) Assume the roots as α and 2α.

(iii) Find the sum of the roots and product of the roots.

(iv) From the above equation eliminate 'α'.

(v) Then obtain the value of $\frac{k^2+1}{k}$.

32. (i) Simplify the required expression and find $\alpha + \beta$ and $\alpha\beta$.

(ii) Find the sum of the roots and product of the roots.

(iii) Use relation, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.

33. (i) Use Pythagorean theorem to find the sides of the triangle.

(ii) Assume the sides as x, $x - 4$ and hypotenuse as $x + 4$.

(iii) Find the value of x using the relation $(\text{hypotenuse})^2$ = sum of the squares of the other two sides.

(iv) The area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$.

34. (i) Form the quadratic equation and solve for x.

(ii) Assume the fraction as $\frac{x}{x-1}$.

(iii) $\frac{x}{x-1} + \frac{x-1}{x} = \frac{21}{16}$.

(iv) Solve the above equation.

35. (i) Use the formula to find the area of the rectangle.

(ii) Assume the length and breadth as lm and $(l - 10)$ m.

(iii) Find the value of 'l', using $l(l - 10) = 96$.

(iv) Calculate the perimeter of the rectangle using $2(l + l - 10)$.

36. Find $\alpha + \beta$, $\alpha\beta$ and using these values find $\alpha - \beta$.

37. Use the formula to find the minimum value.

38. Substitute $x = -2$ in the first two expressions, equated to zero.

39. (i) A pure quadratic equation is $ax^2 + c = 0$.

(ii) Pure quadratic equation is $ax^2 + c = 0$.

40. (i) Simplify the equation 1.

(ii) Take the LCM of the equation.

(iii) Convert it into quadratic equation.

(iv) Solve the equation for x.

41. (i) If $x + k$ is the common root, then $x = -k$ satisfies both the equations.

(ii) If $x + a$ is factor of $f(x)$, then $f(-a) = 0$.

(iii) Write p in terms of q and b in terms of a.

(iv) Now substitute these values in the given expression and simplify.

42. Find the sum and product of the roots and form the equation as per the condition given in the problem.

43. (i) Simplify the equation.

(ii) A rational function $\frac{f(x)}{g(x)}$ is defined only when $g(x) > 0$.

44. (i) Simplify the equation.

(ii) Take the LCM.

(iii) Convert it into quadratic equation.

(iv) Solve the equation by using formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

45. (i) If the equation is a perfect square, then it has equal roots.

(ii) Quadratic equation is a perfect square, if $b^2 - 4ac = 0$.

(iii) Substitute the value of b and c in the above equation and obtained the value of a.

46. The minimum value of $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$. $(a > 0)$

The minimum value of $2x^2 - 3x + 2 = \frac{4 \times 2 \times 2 - (-3)^2}{4 \times 2} = \frac{7}{8}$.

47. Given $x^2 - 2x - 1 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$$x = 1 + \sqrt{2} \text{ or } 1 - \sqrt{2}.$$

48. $2x^2 + 4x - k = 0$ (1)

$\therefore \Rightarrow (x - 5)$ is a factor of Eq. (1).

$\Rightarrow x - 5 = 0 \Rightarrow x = 5$

$\therefore 2(5)^2 + 4(5) - k = 0$

$50 + 20 - k = 0$

$\therefore k = 70.$

49. The difference of the roots of $ax^2 + bx + c = 0$ is $\frac{\sqrt{b^2 - 4ac}}{a}$ for $a > 0$.

$\therefore$ For $x^2 + mx - 2 = 0$. It is $\sqrt{m^2 + 8} = 3$
$\Rightarrow m = \pm 1$.

That is, the equation could be $x^2 + x - 2 = 0$ or $x^2 - x - 2 = 0$.

That is, $(x + 2)(x - 1) = 0$ or $(x - 2)(x + 1) = 0$

The roots are -2, 1 or -1, 2.

As the numerically smaller root is greater, the roots are -2, 1 and $m = 1$.

Level 3

50. (i) Use the concept of sum of the roots and product of the roots of a quadratic equation.

(ii) The product of the roots obtained by A and sum of the roots obtained by B is equal to the product and sum of the roots of the required equation respectively.

51. (i) Simplify the expression and find $(\alpha + \beta)$ and $\alpha\beta$.

(ii) First find the sum of the roots and product of the roots.

(iii) $\alpha^{-6} + \beta^{-6} + \frac{2}{\alpha^3\beta^3} = \frac{(\alpha^3 + \beta^3)}{\alpha^6\beta^6}$

52. (i) Take $b_1 = b_2 = b_3 \ldots b_n = k$ and find the value.

(ii) AM $(a_1, a_2, \ldots, a_n) \geq$ HM $(a_1, a_2, \ldots, a_n)$.

(iii) $\frac{a_1 + a_2 + \cdots + a_n}{n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_2}}$.

53. (i) Simplify the equation.

(ii) Square the given expression twice and then solve for x.

54. (i) Form the quadratic equation and solve for x.

(ii) Assume number of employees in the group as x. Then write the quadratic equation in x according to the data and solve it.

55. $f(x) = x^2 - 12x - 13 = 0$

If the roots of $g(x)$ are 2 times the roots of $f(x)$, then

$$g(x) = f\left(\frac{x}{2}\right) = 0.$$

$$\Rightarrow \left(\frac{x}{2}\right)^2 - 12\left(\frac{x}{2}\right) - 13 = 0$$

$$\Rightarrow \frac{x^2}{4} - \frac{12x}{2} - 13 = 0$$

$$\Rightarrow x^2 - 24x - 52 = 0.$$

56. Let the roots be k and $3k$.

Sum of the roots $= k + 3k$

$\Rightarrow 4k = \frac{-b}{a} \Rightarrow k = \frac{-b}{4a}$.

Product of the roots = $k \times 3k$

$\Rightarrow 3k^2 = \frac{c}{a}$

$\Rightarrow 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$

$\Rightarrow \frac{3b^2}{16a^2} = \frac{c}{a}$

$\Rightarrow 3b^2 = 16ac$

$\Rightarrow b = \pm 4\sqrt{\frac{ac}{3}}$.

57. $\alpha + \beta = \frac{-q}{p}$

$\alpha \cdot \beta = \frac{r}{p}$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(\frac{-q}{p}\right)^3 - 3\left(\frac{r}{p}\right)\left(\frac{-q}{p}\right)$

$= \frac{-q^3 + 3pqr}{p^3}$

$\therefore \alpha^3 + \beta^3 = \frac{3pqr - q^3}{p^3}$.

58. $x^2 - (a + 1)x + \frac{1}{2}(a^2 + a + 1) = 0$

$\alpha + \beta = a + 1$

$\alpha\beta = \frac{1}{2}(a^2 + a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a + 1)^2 - 2\left[\frac{1}{2}(a^2 + a + 1)\right]$

$= a^2 + 2a + 1 - a^2 - a - 1 = a$.

59. $2|x|^2 - 7|x| + 6 = 0$

Let $|x| = y$

$2y^2 - 7y + 6 = 0$

$2y^2 - 3y - 4y + 6 = 0$

$y(2y - 3) - 2(2y - 3) = 0$

$(2y - 3)(y - 2) = 0$

$2y - 3 = 0$ or $y - 2 = 0$

$y = \frac{3}{2}$ or $y = 2$

$|x| = \frac{3}{2}$ or $|x| = 2$

$x = \pm\frac{3}{2}$ or $x = \pm 2$.

$\therefore$ x has 4 real solutions.

60. Product of the sum of the roots and product of the roots is $\frac{91}{9}$, i.e.,

$\left(\frac{b}{a} \times \frac{c}{a}\right) = \frac{91}{9}$

$\frac{bc}{a^2} = \frac{91}{9}$

$\frac{bc}{a \times a} = \frac{13 \times 7}{3 \times 3}$

$\Rightarrow \frac{b}{a} = \frac{13}{3}, \frac{c}{a} = \frac{7}{3}$ or $\frac{b}{a} = \frac{7}{3}, \frac{c}{a} = \frac{13}{3}$

The required difference is $\frac{13}{3} - \frac{7}{3} = 2$.

61. For the maximum value of $\frac{2 + 12x - 3x^2}{2x^2 - 8x + 9}$, $2 + 12x - 3x^2$ is maximum and $2x^2 - 8x + 9$ is minimum.

The maximum value of $2 + 12x - 3x^2$ and minimum value of $2x^2 - 8x + 9$ occurs at $x = \frac{-b}{2a}$, i.e., 2.

When $x = 2$,

$\frac{2 + 12x - 3x^2}{2x^2 - 8x + 9} = \frac{2 + 24 - 12}{8 - 16 + 9} = 14$.

62. $x^2 + ax + b$ and $x^2 + bx + c$ have a common factor $(x - k)$

$\Rightarrow k^2 + ak + b = 0$ and $k^2 + bk + c = 0$

$\Rightarrow k^2 + ak + b = k^2 + bk + c$

$ak + b = bk + c$

$k = \frac{c - b}{a - b}$.

63. $9x - 3y + z = 0$ consider $xa^2 - ya + z = 0$, a quadratic equation in a. Where x, y, z are constants.

Let $a = 3 \Rightarrow (3)^2\, x - 3y + z = 0$ is a quadratic equation in 3.

$$3 = \frac{-(-y) + \sqrt{y^2 - 4 \cdot x \cdot z}}{2.x}$$

$$\Rightarrow 3 = \frac{y}{2x} + \sqrt{\frac{y^2 - 4xz}{4x^2}}.$$

64. Given the roots of the given equation are complex

$\Rightarrow b^2 - 4ac < 0$

$\Rightarrow (-12)^2 - 4(3)\ k < 0$

$144 - 12\ k < 0$

$-12k < -144$

$12\ k > 144$

$k > 12.$

65. α, β are the roots of $ax^2 + bx + c = 0$

$$\Rightarrow \alpha + \beta = \frac{-b}{a}$$

$$\alpha \cdot \beta = \frac{c}{a}.$$

Quadratic equation whose roots are $\alpha + \beta$, and $\alpha\beta$

is $x^2 - \left(\frac{-b}{c} + \frac{c}{a}\right)x + \frac{-b}{a} \times \frac{c}{a} = 0.$

$$x^2 + \left(\frac{b-c}{a}\right)x - \frac{bc}{a^2} = 0$$

$a^2\, x^2 + (ab - ac)x - bc = 0.$

66. Let the speed of Ramu = x km/h

Total time taken is = 1 hour

That is,

$$\frac{3}{x-4} + \frac{3}{x+4} = 1$$

$$\frac{3x + 12 + 3x - 12}{x^2 - 16} = 1$$

$6x = x^2 - 16$

$x^2 - 6x - 16 = 0$

$x^2 - 8x + 2x - 16 = 0$

$x(x - 8) + 2(x - 8) = 0$

$(x - 8)\ (x + 2) = 0$

$x = 8$ km/h ($\because$ speed cannot be -2 km/h).

67. Quadratic equation with 4, -3 as roots is $x^2 - 1x - 12 = 0$, quadratic equation whose product of the roots is -12. (1)

As one of the actual roots is 2, the other root is -6. (from (1))

The quadratic equation is $x^2 - (-6 + 2)\ x - 12 = 0$

$x^2 + 4x - 12 = 0.$

$\therefore$ The difference between the coefficients of x = $4 - (-1) = 5$.

68. Let α, β be the roots of $ax^2 - bx + 2c = 0$

$$\text{Given } = \frac{\alpha}{\beta} = \frac{2}{3}$$

$$\Rightarrow \alpha = \frac{2\beta}{3}$$

Product of the roots

$$\alpha \cdot \beta = \frac{2c}{a}$$

$$\frac{2\beta}{3} \times \beta = \frac{2c}{a}$$

$$\beta^2 = \frac{3c}{a} \qquad (1)$$

Sum of the roots = $\alpha + \beta$

$$\Rightarrow \frac{2\beta}{3} + \beta = \frac{b}{a}$$

$$\Rightarrow \frac{5\beta}{3} = \frac{b}{a}$$

$$\beta = \frac{3b}{5a}$$

$$\beta^2 = \frac{9b^2}{25a^2} \qquad (2)$$

From Eqs. (1) and (2), $\frac{3c}{a} = \frac{9b^2}{25a^2}$ $3b^2 = 25ac.$

69. $f(x) = ax^2 + bx + c = 0$

$f(x - 2) = 9x^2 - 2x + 7 = 0$

$\because\ a(x - 2)^2 + b(x - 2) + c = 9(x)^2 - 2x + 7$

$a(x^2 - 4x + 4) + bx - 2b + c = 9x^2 - 2x + 7$

$ax^2 - (4a - b)x + 4a - 2b + c = 9x^2 - 2x + 7$

$\Rightarrow 4a - 2b + c = 7.$

70. Let $f(x) \equiv px^2 + qx + r = 0$

Given the quadratic equation whose roots are 2 more than the roots of $f(x)$ as $ax^2 + bx + c = 0$.

$\Rightarrow f(x-2) = ax^2 + bx + c$

$\Rightarrow p(x-2)^2 + q(x-2) + r = ax^2 + bx + c$

$\Rightarrow px^2 - 4px + 4p + qx - 2q + r = ax^2 + bx + c$

$\Rightarrow px^2 + (q-4p)x + (4p - 2q + r) = ax^2 + bx + c$

$\Rightarrow c = 4p - 2q + r.$

71. If the roots of $2x^2 + 7x + 5 = 0$ (1) are the reciprocal roots of $ax^2 + bx + c = 0$, then $ax^2 + bx + c = 0$ is obtained by substituting $\frac{1}{x}$ in Eq. (1).

That is, $2\left(\frac{1}{x}\right)^2 + 7\left(\frac{1}{x}\right) + 5 = 0$

$\Rightarrow 2 + 7x + 5x^2 = 0 \Rightarrow 5x^2 + 7x + 2 = 0$

$a = 5,\ b = 7,\ c = 2$

$a - c = 5 - 2 = 3.$

72. $ax^2 + bx + c$

$= p(kx)^2 + q(kx) + r = 0$

$\Rightarrow a = pk^2,\ b = qk,\ c = r$

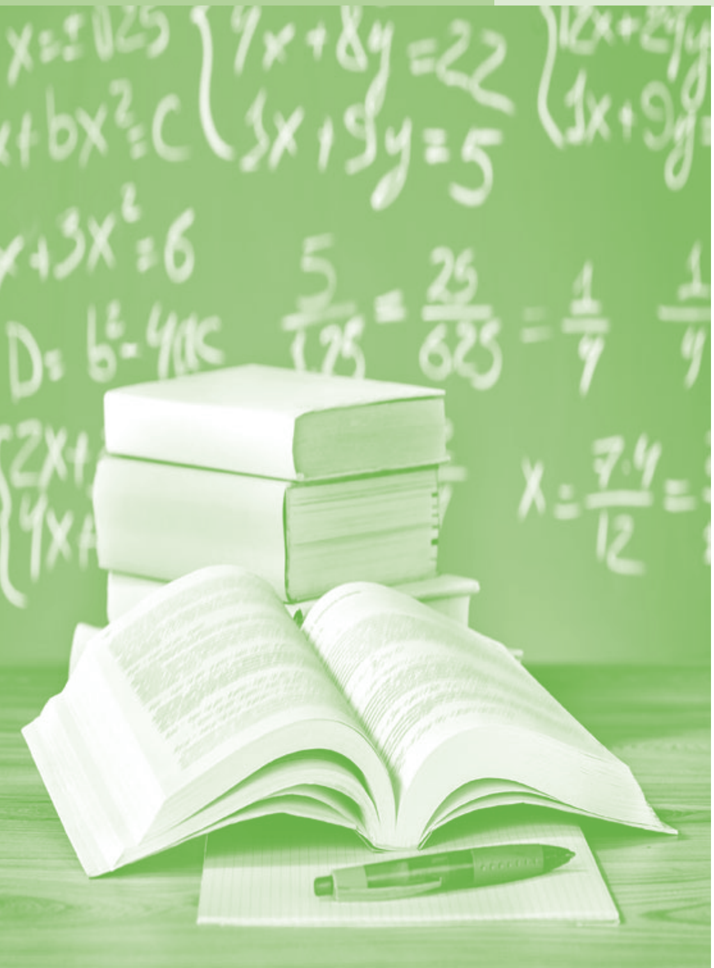
$\frac{a}{b} = \frac{pk^2}{qk}$

$\Rightarrow aq = pbk.$

Chapter

6

Sets and Relations



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the basic definitions of sets
- Apply basic operations on sets
- Understand basic concept of Venn diagrams

KEY IDEAS

After completing this chapter, you should be able to:

- learn about sets, representation of sets and some definitions related to sets
- Apply operations on sets
- Understand the Venn diagrams
- Obtain cartesian products of sets
- Know about relation of sets, its representation, types, properties and to find its domain and range
- Understand the functions of sets

INTRODUCTION

In everyday life we come across different collections of objects. For example, A herd of sheep, a cluster of stars, a posse of policemen, etc. In mathematics, we call such collections as sets. The objects are referred to as the elements of the sets.

SET

A set is a well-defined collection of objects.

Let us understand what we mean by a well-defined collection of objects.

We say that a collection of objects is well-defined if there is some reason or rule by which we can say whether a given object of the universe belongs to or does not belong to the collection.

Elements of a Set

The objects in a set are called elements or members of the set. We usually denote the sets by capital letters A, B, C or X, Y, Z, etc.

If a is an element of a set A, then we say that a belongs to A and we write, $a \in A$.

If a is not an element of A, then we say that a does not belong to A and we write, $a \notin A$.

To understand the concept of a set, let us look at some examples.

Examples:

1. Let us consider the collection of odd natural numbers less than or equal to 15.
   In this example, we can definitely say what the collection is. The collection comprises the numbers 1, 3, 5, 7, 9, 11, 13 and 15.
2. Let us consider the collection of students in a class who are good at painting. In this example, we cannot say precisely which students of the class belong to our collection. So, this collection is not well-defined.

Hence, the first collection is a set where as the second collection is not a set. In the first example given, the set of odd natural numbers less than or equal to 15 can be represented as set $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$.

Some Sets of Numbers and Their Notations

N = Set of all natural numbers = $\{1, 2, 3, 4, 5, \ldots\}$.
W = Set of all whole numbers = $\{0, 1, 2, 3, 4, 5, \ldots\}$.
Z or I = Set of all integers = $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$.

Q = Set of all rational numbers = $\left\{\dfrac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$.

Cardinal Number of a Set

The number of elements in a set A is called its cardinal number. It is denoted by $n(A)$. A set which has finite number of elements is a finite set and a set which has infinite number of elements is an infinite set.

Examples:

1. Set of English alphabets is a finite set.
2. Set of number of days in a month is a finite set.

3. The set of all even natural numbers is an infinite set.
4. Set of all the lines passing through a point is an infinite set.
5. The cardinal number of the set $X = \{a, c, c, a, b, a\}$ is $n(X) = 3$ as in sets only distinct elements are counted.

Representation of Sets

We represent sets by the following methods:

Roster or List Method

In this method, a set is described by listing out all the elements in the set.

Examples:

1. Let W be the set of all letters in the word JANUARY. Then we represent W as, $W = \{A, J, N, R, U, Y\}$.
2. Let M be the set of all multiples of 3 less than 20. Then we represent the set M as, $M = \{3, 6, 9, 12, 15, 18\}$.

Set Builder Method

In this method, a set is described by using a representative and stating the property or properties which the elements of the set satisfy, through the representative.

Examples:

1. Let D be the set of all days in a week. Then we represent D as, $D = \{x: x \text{ is a day in a week}\}$.
2. Let N be the set of all natural numbers between 10 and 20, then we represent the set N as, $N = \{x: 10 < x < 20 \text{ and } x \in N\}$.

Some Simple Definitions of Sets

Empty Set or Null Set or Void Set

A set with no elements in it is called an empty set (or) void set (or) null set. It is denoted by { } or ϕ. (read as 'phi')

Examples:

1. Set of all positive integers less than 1 is an empty set.
2. Set of all mango trees with apples is an empty set.

Singleton Set

A set consisting of only one element is called a singleton set.

Examples:

1. The set of all vowels in the word MARCH is a singleton, as A is the only vowel in the word.
2. The set of whole numbers which are not natural numbers is a singleton, as 0 is the only whole number which is not a natural number.

Equivalent Sets

Two sets A and B are said to be equivalent if their cardinal numbers are equal. We write this symbolically as $A \sim B$ or $A \leftrightarrow B$.

Examples:

1. Sets, $X = \{2, 4, 6, 8\}$ and

   $Y = \{a, b, c, d\}$ are equivalent as $n(X) = n(Y) = 3$.

2. Sets, $X = \{\text{Dog, Cat, Rat}\}$ and

   $Y = \{\triangle, \bigcirc, \square\}$ are equivalent.

3. Sets, $X = \{-1, -7, -5\}$ and

   $Y = \{\text{Delhi, Hyderabad}\}$ are not equivalent as $n(X) \neq n(Y)$.

Note If the sets A and B are equivalent, we can establish a one-to-one correspondence between the two sets. That is, we can pair up elements in A and B such that every element of set A is paired with a distinct element of set B and every element of B is paired with a distinct element of A.

Equal Sets

Two sets A and B are said to be equal if they have the same elements.

Examples:

1. Set, $A = \{a, e, i, o, u\}$ and $B = \{x: x \text{ is a vowel in the English alphabet}\}$ are equal sets.
2. Set, $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$ are not equal sets.
3. Set, $A = \{1, 2, 3, 4, \ldots\}$ and $B = \{x: x \text{ is a natural number}\}$ are equal sets.

Note If A and B are equal sets, then they are equivalent but the converse need not be true.

Disjoint Sets

Two sets A and B are said to be disjoint, if they have no elements in common.

Examples:

1. Sets $X = \{3, 6, 9, 12\}$ and $Y = \{5, 10, 15, 20\}$ are disjoint as they have no elements in common.
2. Sets $A = \{a, e, i, o, u\}$ and $B = \{e, i, j\}$ are not disjoint as they have elements e and i in common.

Subset

Let A and B be two sets. If every elements of set A is also an element of set B, then A is said to be a subset of B or B is said to be a superset of A. If A is a subset of B, then we write $A \subseteq B$ or $B \supseteq A$.

Examples:

1. Set $A = \{2, 4, 6, 8\}$ is a subset of set $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
2. Set of all primes except 2 is a subset of the set of all odd natural numbers.
3. Set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ is a superset of set $B = \{1, 3, 5, 7\}$.

Notes

1. Empty set is a subset of every set.
2. Every set is a subset of itself.
3. If a set A has n elements, then the number of subsets of A is 2^n.
4. If a set A has n elements, then the number of non-empty subsets of A is $2^n - 1$.

Proper Subset and Superset

If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B and is denoted by $A \subset B$. If $A \subset B$, then B is called a superset of A and is denoted as $B \supset A$.

Power Set

The set of all subsets of a set A is called power set of A. It is of A denoted by $P(A)$.

Example: Let $A = \{x, y, z\}$. Then the subsets of A are ϕ, $\{x\}$, $\{y\}$, $\{z\}$, $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x, y, z\}$.

So, $P(A) = \{\phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.

We observe that the cardinality of $P(A)$ is $8 = 2^3$.

Notes

1. If a set A has n elements, then the number of subsets of A is 2^n, i.e., the cardinality of the power set is 2^n.
2. If a set A has n elements, then the number of proper subsets of A is $2^n - 1$.
3. If a set A has n elements, then the number of non-empty proper subsets is $2^n - 2$.

Universal Set

A set which consists of all the sets under consideration or discussion is called the universal set. It is usually denoted by $\cup$ or μ.

Example:

Let $A = \{a, b, c\}$, $B = \{c, d, e\}$ and $C = \{a, e, f, g, h\}$. Then, the set $\{a, b, c, d, e, f, g, h\}$ can be taken as the universal set here.

$\therefore\ \mu = \{a, b, c, d, e, f, g, h\}$.

Complement of a Set

Let μ be the universal set and $A \subseteq \mu$. Then, the set of all those elements of μ which are not in set A is called the complement of the set A. It is denoted by A' or A^c.
$A' = \{x : x \in \mu \text{ and } x \notin A\}$.

Examples:

1. Let $\mu = \{3, 6, 9, 12, 15, 18, 21, 24\}$ and $A = \{6, 12, 18, 24\}$. Then, $A' = \{3, 9, 15, 21\}$.
2. Let $\mu = \{x\text{: } x \text{ is a student and } x \in \text{ class X}\}$.

   And $B = \{x\text{: } x \text{ is a boy and } x \in \text{ class X}\}$.

   Then, $B' = \{x\text{: } x \text{ is a girl and } x \in \text{ class X}\}$.

Notes

1. A and A' are disjoint sets.
2. $\mu' = \phi$ and $\phi' = \mu$.

Operations on Sets

Union of Sets

Let A and B be two sets. Then, the union of A and B, denoted by $A \cup B$, is the set of all those elements which are either in A or in B or in both A and B.
That is, $A \cup B = \{x: x \in A \text{ or } x \in B\}$.

Examples:

1. Let $A = \{-1, -3, -5, 0\}$ and $B = \{-1, 0, 3, 5\}$ then, $A \cup B = \{-5, -3, -1, 0, 3, 5\}$.
2. Let $A = \{x: 5 \leq 5x < 25 \text{ and } x \in N\}$ and $B = \{x: 5 \leq (10x) \leq 20 \text{ and } x \in N\}$, then, $A \cup B = \{x: 5 \leq 5x \leq 20 \text{ and } x \in N\}$.

Notes

1. If $A \subseteq B$, then $A \cup B = B$.
2. $A \cup \mu = \mu$ and $A \cup \phi = A$.
3. $A \cup A' = \mu$.

Intersection of Sets

Let A and B be two sets. Then the intersection of A and B, denoted by $A \cap B$, is the set of all those elements which are common to both A and B.
That is, $A \cap B = \{x: x \in A \text{ and } x \in B\}$.

Examples:

1. Let $A = \{1, 2, 3, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 7\}$. $A \cap B = \{1, 3, 5, 7\}$.
2. Let A be the set of all English alphabet and B be the set of all consonants. $A \cap B$ is the set of all consonants in the English alphabet.
3. Let E be the set of all even natural numbers and O be the set of all odd natural numbers. $E \cap O = \{\ \}$ or ϕ.

Notes

1. If A and B are disjoint sets, then $A \cap B = \phi$ and $n(A \cap B) = 0$.
2. If $A \subseteq B$, then $A \cap B = A$.
3. $3A \cap \mu = A$ and $A \cap \phi = \phi$.
4. $A \cap A' = \phi$

Difference of Sets

Let A and B be two sets. Then the difference $A - B$ is the set of all those elements which are in A but not in B. That is, $A - B = \{x: x \in A \text{ and } x \notin B\}$.

Example: Let $A = \{3, 6, 9, 12, 15, 18\}$ and $B = \{2, 6, 8, 10, 14, 18\}$.
$A - B = \{3, 9, 12, 15\}$ and $B - A = \{2, 8, 10, 14\}$.

Notes

1. $A - B \neq B - A$ unless $A = B$.
2. For any set $A \cdot A' = \mu - A$.

Symmetric Difference of Sets

Let A and B be two sets. Then the symmetric difference of A and B, denoted by $A \Delta B$, is the set of all those elements which are either in A or in B but not in both, i.e., $A \Delta B = \{x: x \in A$ and $x \notin B$ or $x \in B$ and $x \notin A\}$.

Note $A \Delta B = (A - B) \cup (B - A)$ (or) $A \Delta B = (A \cup B) - (A \cap B)$.

Example: Let $A = \{1, 2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 12\}$. $A \Delta B = (A - B) \cup (B - A) = \{1, 2, 4, 8, 10\} \cup \{3\} = \{1, 2, 3, 4, 8, 10\}$.

EXAMPLE 6.1

If $A = \{3, 5, 7, 8\}$ and $B = \{7, 8, 9, 10\}$, then find the value of $(A \cup B) - (A \cap B)$. The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $A \cup B = \{3, 5, 7, 8, 9, 10\}$ and $A \cap B = \{7, 8\}$.

(B) Given $A = \{3, 5, 7, 8\}$ and $B = \{7, 8, 9, 10\}$.

(C) $(A \cup B) - (A \cap B) = \{3, 5, 9, 10\}$.

(D) $(A \cup B) - (A \cap B) = \{3, 5, 7, 8, 9, 10\} - \{7, 8\}$.

SOLUTION

The sequential order is BADC.

Some Results For any three sets A, B and C, we have the following results:

1. **Commutative law:**
   - **(i)** $A \cup B = B \cup A$
   - **(ii)** $A \cap B = B \cap A$
   - **(iii)** $A \Delta B = B \Delta A$
2. **Associative law:**
   - **(i)** $(A \cup B) \cup C = A \cup (B \cup C)$
   - **(ii)** $(A \cap B) \cap C = A \cap (B \cap C)$
   - **(iii)** $(A \Delta B) \Delta C = A \Delta (B \Delta C)$
3. **Distributive law:**
   - **(i)** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   - **(ii)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. **De-Morgan's law:**
   (i) $(A \cup B)' = A' \cap B'$
   (ii) $(A \cap B)' = A' \cup B'$
   (iii) $A - (B \cup C) = (A - B) \cap (A - C)$
   (iv) $A - (B \cap C) = (A - B) \cup (A - C)$
5. **Identity law:**
   (i) $A \cup \phi = \phi \cup A = A$
   (ii) $A \cap \mu = \mu \cap A = A$
6. **Idempotent law:**
   (i) $A \cup A = A$
   (ii) $A \cap A = A$
7. **Complement law:**
   (i) $(A')' = A$
   (ii) $A \cup A' = \mu$
   (iii) $A \cap A' = \phi$

Dual of an Identity

An identity obtained by interchanging $\cup$ and $\cap$, and ϕ and μ in the given identity is called the dual of that identity.

Examples:
1. Consider the identity, $A \cup B = B \cup A$. Dual of the identity is, $A \cap B = B \cap A$.
2. Consider the identity, $A \cup \mu = \mu$. Dual of the identity is, $A \cap \phi = \phi$.

Venn Diagrams

We also represent sets pictorially by means of diagrams called Venn diagrams. In Venn diagrams, the universal set is usually represented by a rectangular region and its subsets by closed regions inside the rectangular region. The elements of the set are written in the closed regions and the elements which belong to the universal set are written in the rectangular region.

Example: Let $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 4, 6, 7, 8\}$ and $B = \{2, 3, 4, 5, 9\}$. We represent these sets in the form of Venn diagram as in Fig. 6.1.

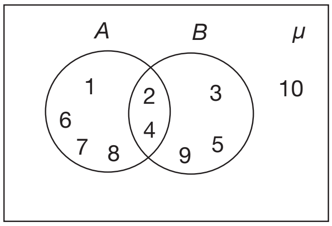


Figure 6.1

We can also represent the sets in Venn diagrams by shaded regions.

Examples:
1. Venn diagram of $A \cup B$, where A and B are two overlapping sets and it is shown in Fig. 6.2:

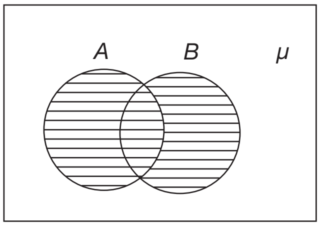


Figure 6.2

2. Let A and B be two overlapping sets. Then, the Venn diagram of $A \cap B$ is as shown in Fig. 6.3:

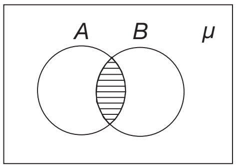


Figure 6.3

3. For a non-empty set A, Venn diagram of A' is as shown in Fig. 6.4:

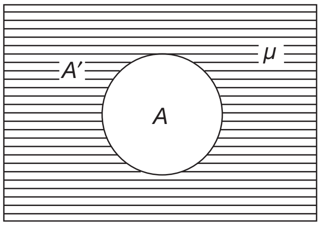


Figure 6.4

4. Let A and B be two overlapping sets. Then, the Venn diagram of $A - B$ is as shown in Fig. 6.5:

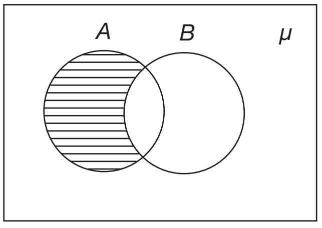


Figure 6.5

5. Let A and B be two sets such $A \subseteq B$. We can represent this relation using Venn diagram as shown in Fig. 6.6:

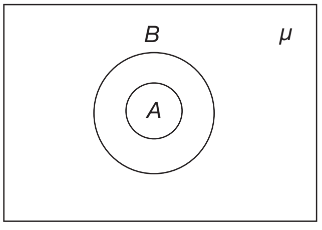


Figure 6.6

Some Formulae on the Cardinality of Sets

Let $A = \{1, 2, 3, 5, 6, 7\}$ and $B = \{3, 4, 5, 8, 10, 11\}$. Then, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11\}$ and $A \cap B = \{3, 5\}$.

In terms of the cardinal numbers, $n(A) = 6$, $n(B) = 6$, $n(A \cap B) = 2$ and $n(A \cup B) = 10$.

So, $n(A) + n(B) - n(A \cap B) = 6 + 6 - 2 = 10 = n(A \cup B)$.

We have the following formulae:

For any three sets A, B and C.

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
2. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

EXAMPLE 6.2

If $n(A) = 7$, $n(B) = 5$ and $n(A \cup B) = 10$, then find $n(A \cap B)$.

SOLUTION

Given, $n(A) = 7$, $n(B) = 5$ and $n(A \cup B) = 10$.

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

So, $10 = 7 + 5 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 2$.

EXAMPLE 6.3

If $n(A) = 8$ and $n(B) = 6$ and the sets A and B are disjoint, then find $n(A \cup B)$.

SOLUTION

Given, $n(A) = 8$ and $n(B) = 6$. A and B are disjoint.

$\Rightarrow A \cap B = \phi \Rightarrow n(A \cap B) = 0$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 6 - 0 = 14$.

Note If A and B are two disjoint sets, then $n(A \cup B) = n(A) + n(B)$.

EXAMPLE 6.4

In a locality, the number of residents who read only The Hindu, only The Times of India, both the newspapers and neither of the newspapers are in the ratio 2 : 3 : 4 : 1. The number of residents who read at least one of these newspapers is 160 more than those who read neither of these newspapers. Find the number of residents in the locality.

SOLUTION

Let the number of residents who read only The Hindu be $2x$.

$\therefore$ Number of residents who read The Times of India, both the newspapers and neither of the newspapers are $3x$, $4x$ and x respectively.

$2x + 3x + 4x = x + 160$

$8x = 160 \Rightarrow x = 20$

Number of residents in the locality

$= 2x + 3x + 4x + x = 10x = 200$.

Ordered Pair

Let A be a non-empty set and $a, b \in A$. The elements a and b written in the form (a, b) is called an ordered pair. In the ordered pair (a, b), a is called the first coordinate and b is called the second coordinate.

Note Two ordered pairs are said to be equal only when their first as well as the second coordinates are equal, i.e., $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.
So, $(1, 2) \neq (2, 1)$ and if $(a, 5) = (3, b) \Rightarrow a = 3$ and $b = 5$.

CARTESIAN PRODUCT OF SETS

Let A and B be two non-empty sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), such that $a \in A$ and $b \in B$.
That is, $A \times B = \{(a, b): a \in A, b \in B\}$.

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$. $A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\}$ and $B \times A = \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}$.

We observe that, $A \times B \neq B \times A$ and $n(A \times B) = 6 = n(B \times A)$.

Notes

1. $A \times B \neq B \times A$, unless $A = B$.
2. For any two sets A and B, $n(A \times B) = n(B \times A)$.
3. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

Some Results on Cartesian Product

1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (or) $(A \cup B) \times (A \cup C)$.
2. $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (or) $(A \cap B) \times (A \cap C)$.
3. If $A \times B = \phi$, then either
   (i) $A = \phi$
   (ii) $B = \phi$
   (iii) both $A = \phi$ and $B = \phi$.

   Cartesian product of sets can be represented in following ways:

1. Arrow diagram
2. Tree diagram
3. Graphical representation

Representation of $A \times B$ Using Arrow Diagram

EXAMPLE 6.5

If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then find $A \times B$.

SOLUTION

In order to find $A \times B$, represent the elements of A and B as shown in the diagram; 6.8.

Now draw the arrows from each element of A to each element of B.

Now, represent all the elements related by arrows in ordered pairs in a set, which is the required $A \times B$.

That is, $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$.

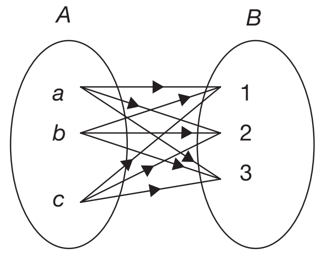


Figure 6.8

Representation of $A \times B$ Using a Tree Diagram

EXAMPLE 6.6

If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then find $A \times B$.

SOLUTION

To represent $A \times B$ using tree diagram, write all the elements of A vertically and then for each element of A, write all the elements of B and draw arrows as shown in the diagram,

$\therefore$ $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c\ 1), (c, 2), (c, 3)\}$.

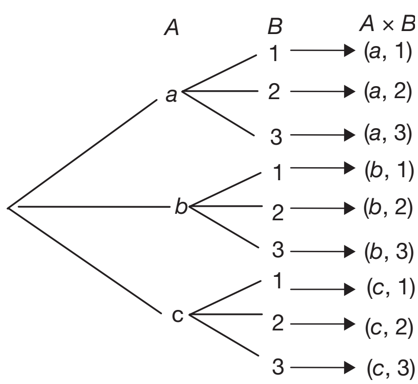


Figure 6.9

Graphical Representation of $A \times B$

EXAMPLE 6.7

If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then find $A \times B$.

SOLUTION

Consider the elements of A on the X-axis and the elements of B on the Y-axis and mark the points (see Fig. 6.10).

$\therefore$ $A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$.

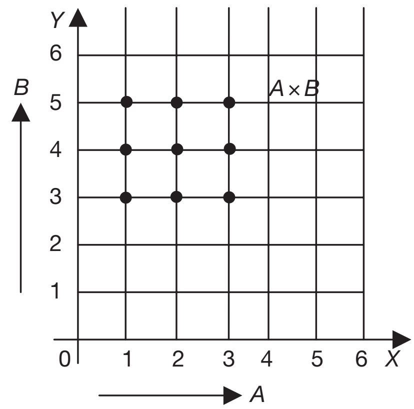


Figure 6.10

RELATION

We come across certain relations in real life and also in basic geometry, like *is father of*, *is a student of*, *is parallel to*, *is similar to*, etc.

Definition

Let A and B be two non-empty sets and $R \subseteq A \times B$. R is called a relation from the set A to B. (Any subset of $A \times B$ is called a relation from A to B).

$\therefore$ A relation contains ordered pairs as elements. Hence, 'A relation is a set of ordered pairs'.

Examples:

Let $A = \{1, 2, 4\}$ and $B = \{2, 3\}$.

Then, $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$.

1. Let $R_1 = \{(1, 2), (1, 3), (2, 3)\}$

   Clearly, $R_1 \subseteq A \times B$ and we also notice that, for every ordered pair $(a, b) \in R_1$, $a < b$.

   So, R_1 is the relation *is less than* from A to B.

2. Let $R_2 = \{(4, 2), (4, 3)\}$.

   Clearly, $R_2 \subseteq A \times B$ and we also notice that, for every ordered pair $(a, b) \in R_2$, $a > b$.

   So, R_2 is the relation *is greater than* from A to B.

Notes

1. If $n(A) = p$ and $n(B) = q$, then the number of relations possible from A to B is 2^{pq}.
2. If $(x, y) \in R$, then we write xRy and read as x is related to y.

Domain and Range of a Relation

Let A and B be two non-empty sets and R be a relation from A to B, we note the followings:

1. The set of first coordinates of all ordered pairs in R is called the domain of R.
2. The set of second coordinates of all ordered pairs in R is called the range of R.

Example: Let $A = \{1, 2, 4\}$, $B = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (4, 3)\}$ be a relation from A *to* B. Then, domain of $R = \{1, 2, 4\}$ and range of $R = \{1, 2, 3\}$.

Representation of Relations

We represent the relations by the following methods:

Roster-Method (or) List Method

In this method, we list all the ordered pairs that satisfy the rule or property given in the relation.

Example: Let $A = \{1, 2, 3\}$. If R is a relation on the set A having the property *is less than*, then the roster form of R is, $R = \{(1, 2), (1, 3), (2, 3)\}$.

Set-builder Method

In this method, a relation is described by using a representative and stating the property or properties, which the first and second coordinates of every ordered pair of the relation satisfy, through the representative.

Example: Let $A = \{1, 2, 3\}$. If R is a relation on the set A having the property *is greater than or equal to*, then the set builder form of R is, $R = \{(x, y)/x, y \in A \text{ and } x \geq y\}$.

Arrow Diagram

In this method, a relation is described by drawing arrows between the elements which satisfy the property or properties given in the relation.

Example: Let $A = \{1, 2, 4\}$ and $B = \{2, 3\}$. Let R be a relation from A to B with the property *is less than*. Then, the arrow diagram of R is as shown in Fig. 6.11.

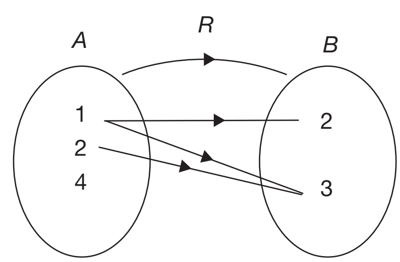


Figure 6.11

Inverse of a Relation

Let R be a relation from A to B. The inverse relation of R, denoted by R^{-1}, is defined as, $R^{-1} = \{(y, x)/(x, y) \in R\}$.

Example: Let $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (4, 3)\}$ be a relation from A to B, where $A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$. Then, $R^{-1} = \{(1, 1), (1, 2), (2, 1), (3, 2), (3, 4)\}$.

Notes

1. Domain of R^{-1} = Range of R.
2. Range of R^{-1} = Domain of R.
3. If R is a relation from A to B, then R^{-1} is a relation from B to A.
4. If $R \subseteq A \times A$, then R is called a binary relation or simply a relation on the set A.
5. For any relation R, $(R^{-1})^{-1} = R$.

Types of Relations

1. **One-one relation:** A relation $R: A \to B$ is said to be one-one relation if different elements of A are paired with different elements of B, i.e., $x \neq y$ in $A \Rightarrow f(x) \neq f(y)$ in B.

   Example:

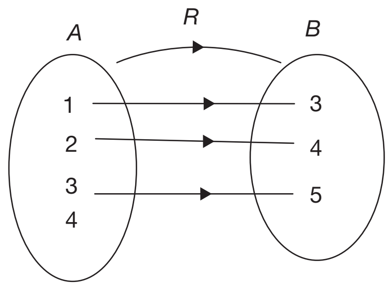


Figure 6.12

   Here, the relation $R = \{(1, 3), (2, 4), (3, 5)\}$.

2. **One-many relation:** A relation $R: A \to B$ is said to be one-many relation if at least one element of A is paired with two or more elements of B.

   Example:

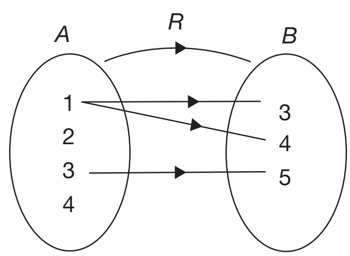


Figure 6.13

   Here, the relation $R = \{(1, 3), (1, 4), (3, 5)\}$.

3. **Many-one relation:** A relation $R: A \to B$ is said to be many-one relation if two or more elements of A are paired with an element of B.

   Example:

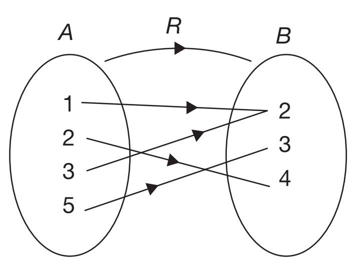


Figure 6.14

   Here, the relation $R = \{(1, 2), (2, 4), (3, 2), (5, 3)\}$.

4. **Many-many relation:** A relation $R: A \to B$ is said to be many-many relation if two or more elements of A are paired with two or more elements of B.

Example:

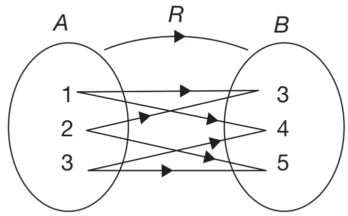


Figure 6.15

Here, the relation $R = \{(1, 3), (1, 4), (2, 3), (2, 5), (3, 4), (3, 5)\}$.

Properties of Relations

Reflexive Relation

A relation R on a set A is said to be reflexive if for every $x \in A$, $(x, x) \in R$.

Examples:

1. Let $A = \{1, 2, 3\}$ then, $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (2, 3)\}$ is a reflexive on A.
2. Let $A = \{1, 2, 3\}$ then, $R = \{(1, 1), (2, 3), (1, 2), (1, 3), (2, 2)\}$ is not a reflexive relation as $3 \in A$ but $(3, 3) \notin R$.

Note Number of reflexive relations defined on a set having n elements is 2^{n^2-n}.

Symmetric Relation

A relation R on a set A is said to be symmetric, if for every $(x, y) \in R$, $(y, x) \in R$.

Examples:

1. Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$ is a symmetric relation on A.
2. Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (3, 1), (2, 2), (3, 3)\}$ is not a symmetric relation as, $(1, 2) \in R$ but $(2, 1) \notin R$.

Note A relation R on a set A is symmetric iff $R = R^{-1}$, i.e., R is symmetric if $R = R^{-1}$.

Transitive Relation

A relation R on a set A is said to be transitive if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.

That is, R is said to be transitive, whenever $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Examples:

1. If $A = \{1, 2, 3\}$, then, $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (3, 1), (2, 2), (3, 3)\}$ is a transitive relation.
2. Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 2), (2, 2), (2, 1), (3, 3), (1, 3)\}$ is not a transitive relation as $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

Anti-symmetric Relation

A relation R on a set A is said to be anti-symmetric if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$., i.e., R is said to be anti-symmetric if for $x \neq y$, $(x, y) \in R \Rightarrow (y, x) \notin R$.

Examples:

1. Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (3, 3)\}$ is an anti-symmetric relation.
2. Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (3, 3)\}$ is not an anti-symmetric relation as $(1, 2) \in R$ and $(2, 1) \in R$ but $1 \neq 2$.

Equivalence Relation

A relation R on a set A is said to be an equivalence relation if it is,

1. Reflexive
2. Symmetric
3. Transitive

Note For any set A, $A \times A$ is an equivalence relation. In fact it is the largest equivalence relation.

Identity Relation

A relation R on a set A defined as, $R = \{(x, x)/x \in A\}$ is called an identity relation on A. It is denoted by I_A.

Example: *Let* $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on A.

Note Identity relation is the smallest equivalence relation on a set A.

FUNCTION

Let A and B be two non-empty sets and f is a relation from A to B.

If f is such that

1. for every $a \in A$, there is $b \in B$ such that $(a, b) \in f$ and
2. no two ordered pairs in f have the same first element, then f is called a function from set A to set B and is denoted as $f: A \rightarrow B$.

Notes

1. If $(a, b) \in f$ then $f(a) = b$ and b is called the f image of a and a is called the preimage of b.
2. If $f: A \rightarrow B$ is a function, then A is called the domain of f and B is called the co-domain of f.
3. The set $f(A)$ which is all the images of elements of A under the mapping f is called the range of f. Few examples of functions are listed below in the Fig. 6.16:

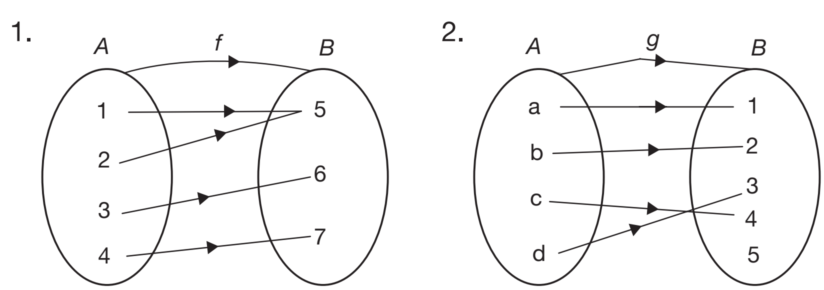


Figure 6.16

EXAMPLE 6.8

If $R = \{(x, y): x \in W, y \in W \text{ and } (x + 2y)^2 = 36\}$, then R^{-1} is ______.

(a) $\{(0, 3), (2, 2), (1, 4), (0, 6)\}$

(b) $\{(0, 6), (0, 3), (2, 2), (4, 1)\}$

(c) $\{(3, 0), (2, 2), (1, 4), (0, 6)\}$

(d) $\{(3, 0), (2, 2), (1, 4), (6, 0)\}$

SOLUTION

Given $(x + 2y)^2 = 36$ and $(x, y) \in W$.

$\Rightarrow x + 2y = 6$

$\therefore$ The possible values of (x, y) are $(0, 3)$, $(2, 2)$, $(4, 1)$, $(6, 0)$.

$\Rightarrow R = \{(0, 3), (2, 2), (4, 1), (6, 0)\}$

$\therefore R^{-1} = \{(3, 0), (2, 2), (1, 4), (0, 6)\}$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If $P = \{1, 2, 3, 4, 5, 6, 7\}$ and $Q = \{2, 5, 8, 9\}$, then find $P \cup Q$.

2. If $P = \{1, 2, 3, 4, 5, 6, 7\}$ and $Q = \{2, 5, 8, 9\}$, then find $P - Q$.

3. If $P = \{1, 2, 3, 4, 5, 6, 7\}$ and $Q = \{2, 5, 8, 9\}$, then find $P \cap Q$.

4. If $X \subset Y$ and $Y \subset X$, then ________.

5. The complement of ϕ' is ________.

6. $A \cap A' =$ ________.

7. The symmetric difference of A and B is commutative. (True/False)

8. The cardinal number of a set is 5. Find the cardinal number of the power set.

9. The order in which the elements are placed plays an important role in sets. (True/False)

10. If $n(A \cup B) = 16$ and $n(A \cap B) = 4$, then the number of elements in the symmetric difference of A and B is ________.

11. If P and Q are disjoint, then $(P \cap Q)'$ is ________.

12. If P and Q are disjoint, then $P - Q =$ ________ and $Q - P =$ ________.

13. If $V = \{a, e, i, o, u\}$, then find the number of non-empty proper subsets of V.

14. If a set has 510 non-empty proper subsets, then find the cardinal number of the set.

15. If A is universal set, then $((A')')'$ is ________.

16. If $n(P) = 2$ and $n(Q) = 5000$, then $n(P \times Q) =$ ________.

17. If $n(P) = 2439$ and $Q = \phi$ then $n(P \times Q) =$ ________.

18. If $P = \{a, b, c, d\}$ and $Q = \{1, 2, 3, 4, 5\}$ then $n(P \times Q) =$ ________.

19. Let $A = \{a, b, c\}$ and $B = \{p, q\}$. Draw the arrow diagram of $A \times B$.

20. If $n(A) = 6$ and $n(B) = 3$, then find the number of subsets of $A \times B$.

21. Let $A = \{x, y, z\}$ and $B = \{p, q\}$, then draw the tree diagram of $A \times B$ and $B \times A$.

22. If $n(P) = 17$, $n(Q) = 10$ and $n(P \cap Q) = 7$, then $n(P \Delta Q)$ is ________.

23. If $(x, 2p + q) = (y, p + 2q)$ then $p - q =$ ________.

24. If $n(A) = 40$ and $n(B) = 23$, then find $n(A - B)$ and $n(B - A)$ when $B \subset A$.

25. Find $n(P \times Q)$, if $n(Q - P) = 10$ and $n(P - Q) = 13$ and $n(P \cap Q) = 8$.

26. Find the number of relations from A to A, where $A = \{1, 2, 3, 4\}$.

27. A relation $R = \{(a, b), (a, a), (a, c), (x, x), (x, y), (y, y), (d, d), (d, c)\}$. Write a relation $E \subset R$ such that x is equal to y.

28. $A = \{1, 2, 3, 4\}$, $B = \{3, 4\}$ and $R = \{(3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$ is a relation from B into A. Write R in set-builder form.

29. $P = \{3, 5, 6, 8, 9\}$, $Q = \{6, 10, 12, 16, 17\}$ and $R = \{(x, y)/(x, y) \in P \times Q, 2x = y\}$ is a relation from P into Q, write R in list form.

30. If $n(X \cap Y') = 9$, $n(Y \cap X') = 10$ and $n(X \cup Y) = 25$, then find $n(X \times Y)$.

Short Answer Type Questions

31. If $A = \{2, 3, 4, 6, 7, 9, 10, 12\}$, $B = \{1, 3, 5, 8, 9, 10, 11, 15\}$, $C = \{3, 4, 7, 10, 11, 13, 15\}$ and $\mu = \{1, 2, 3, \ldots, 15\}$. Then, find $(A \cup B)'$.

32. If $A = \{2, 3, 4, 6, 7, 9, 10, 12\}$, $B = \{1, 3, 5, 8, 9, 10, 11, 15\}$, $C = \{3, 4, 7, 10, 11, 13, 15\}$ and $\mu = \{1, 2, 3, \ldots, 15\}$. Then, find $(A \cup B \cup C)'$.

33. If $n(X - Y) = 30 + a$, $n(Y - X) = 20 + 2a$, $n(X \cup Y) = 100$ and $n(X \cap Y) = 15 + 2a$, then find a.

34. If $n(P \Delta Q) = n(P \cup Q)$, then P and Q are ________.

35. If $n(A - B) = 25$, $n(B - A) = 15$ and $n(A \cup B) = 60$, then $n(A \cap B) =$ ________.

PRACTICE QUESTIONS

36. If $n(P \cap Q) = 12$ and $n(Q) = 37$, then find the value of $n(P' \cap Q)$

37. In a colony of 170 members, 70 subscribe Deccan Chronicle and 120 subscribe Times of India. How many subscribe only Deccan Chronicle? (Each subscribes at least one.)

38. $A = \{1, 2, 3, 4\}$ and $f(x) = 2x^2$, $x \in A$. If $f(x) = 18$, then find x.

39. Write the following sets in the roster form.

(i) $P = \{x/x \in W$ and $x \notin N\}$ (ii) $S = \{f/f$ is a factor of $13\}$

40. A, B and C are three different sets and $A \times (B \cap C) = (A \times B) \cap (A \times C)$. Judge the given statements by taking any three non empty sets A, B and C. (True/False).

41. Given that $R = \{(1, 1), (3, 3), (2, 3), (3, 2), (2, 2)\}$ on the set $A = \{1, 2, 3\}$. Which property is not satisfied by R on A?

42. What type of relation does R define on the set of integers, if $x + y = 8$?

43. In a class of 50 students 20 take Sanskrit but not Hindi and 37 take Sanskrit. How many students take Hindi but not Sanskrit? (Each student takes either Sanskrit or Hindi.)

44. In a class of 60 students, 25 speak Hindi, 45 speak English. How many of them speak both English and Hindi, if each student speaks either English or Hindi?

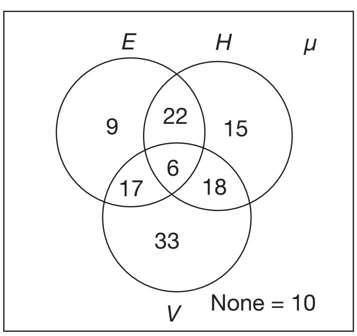
45. In term examination 40% students failed in English, 32% failed in Physical Science. What is the pass percentage, if 10% failed in both?

Essay Type Questions

46. The following figure depicts the number of families subscribed for three different newspapers, i.e., Eenadu (E), Hindu (H) and Vaartha (V).

Find the number of people who read

(i) atleast two papers
(ii) atmost two papers
(iii) atleast three papers
(iv) atmost three papers
(v) atleast one paper.
(vi) atmost one paper.



47. On the set of all colleges in a state, a relation R is defined such that 'two colleges are related if they belong to the same district'. Find the properties satisfied by R.

48. In a set of students studying in the same class, two students are related 'if their weights are not equal'. Find the properties satisfied by it.

49. If a set A has 4 elements and a reflexive relation R defined in set A has x elements, then what is the range of x?

50. In a club 45% plays cricket, 20% plays only football. Find the percentage of members who plays only cricket if 10% play both. (Each plays at least one.)

CONCEPT APPLICATION

Level 1

1. Which of the following cannot be the cardinal number of the power set of any finite set?

(a) 26 (b) 32
(d) 8 (d) 16

2. Consider the following statements

p: $3 \in \{1, \{3\}, 5, 7\}$
q: $2 \in \{1, \{2, 4\}, 5\}$

Which of the following is true?

PRACTICE QUESTIONS

(a) p alone

(b) q alone

(c) Both (p) and (q)

(d) Neither (p) nor (q)

3. If $A = \{1, 2, 3\}$ and $B = \{2, 6, 7\}$, then $(A - B) \cup (B - A) =$

(a) ϕ (b) μ

(c) $\{1, 2, 3, 6, 7\}$ (d) $\{1, 3, 6, 7\}$

4. If $(x - y, x + y) = (2, 8)$, then the values of x and y are respectively

(a) 5, 3 (b) 7, 5

(c) 4, 2 (d) 10, 8

5. If $X = \{x\text{: } x^2 - 12x + 20 = 0\}$ and $Y = \{x\text{: } x^2 + 5x - 14 = 0\}$, then $X - Y =$

(a) $\{2\}$ (b) $\{10\}$

(c) $\{-7\}$ (d) $\{ \}$

6. The number of subsets of $A \times B$ if $n(A) = 3$ and $n(B) = 3$ is

(a) 512 (b) 256

(c) 511 (d) 255

7. If $A = \{1, 2, 3, 4\}$, then how many subsets of A contain the element 3?

(a) 24 (b) 28

(c) 8 (d) 16

8. In 'aRb' if 'a and b have the same teacher', then R is ________.

(a) reflexive (b) symmetric

(c) transitive (d) equivalence

9. A relation R: $Z \to Z$ is such that $R = \{(x, y)/y = 2x + 1\}$ is a

(a) one to one relation.

(b) many to one relation.

(c) one to many relation.

(d) many to many relation.

10. If P_n is the set of first n prime numbers, then $\bigcup_{n=2}^{10} P_n =$

(a) $\{2, 3, 5, 7, 11, 13, 17, 19\}$

(b) $\{3, 5\}$

(c) $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

(d) $\{2, 3\}$

11. If P_n is the set of first n prime numbers, then $\bigcap_{n=3}^{10} P_n$ is

(a) $\{3, 5, 7, 11, 13, 17, 19\}$

(b) $\{2, 3, 5\}$

(c) $\{2, 3, 5, 7, 11, 13, 17\}$

(d) $\{3, 5, 7\}$

12. If $n(\mu) = 100$, $n(A) = 50$, $n(B) = 20$ and $n(A \cap B) = 10$, then $n[(A \cup B)^c] =$

(a) 60 (b) 30

(c) 40 (d) 20

13. Let Z denote the set of integers, then $\{x \in Z\text{: } |x - 3| < 4 \cap \{x \in Z\text{: } |x - 4| < 5\} =$

(a) $\{-1, 0, 1, 2, 3, 4\}$

(b) $\{-1, 0, 1, 2, 3, 4, 5\}$

(c) $\{0, 1, 2, 3, 4, 5, 6\}$

(d) $\{-1, 0, 1, 2, 3, 5, 6, 7, 8, 9\}$

14. If $A = \left\{n : \dfrac{n^3 + 5n^2 + 2}{n} \text{ is an integer and } n \text{ itself is an integer}\right\}$, then the number of elements in the set A is

(a) 1 (b) 2

(c) 3 (d) 4

15. If $A = \{1, 2, 3\}$, then the relation $R = \{(1, 1) (2, 2), (3, 1), (1, 3)\}$ is

(a) reflexive. (b) symmetric.

(c) transitive. (d) equivalence.

16. If $n(A \times B) = 45$, then $n(A)$ cannot be

(a) 15 (b) 17

(c) 5 (d) 9

17. If A, B and C are three non-empty sets such that A and B are disjoint and the number of elements contained in A is equal to those contained in the set of elements common to the sets A and C, then $(A \cup B \cup C)$ is necessarily equal to

PRACTICE QUESTIONS

(a) $n(B \cup C)$.

(b) $n(A \cup C)$.

(c) Both (a) and (b)

(d) None of these

18. R and S are two sets such that $n(R) = 7$ and $R \cap S \neq \phi$. Further $n(S) = 6$ and $S \,\Delta\, R$. The greatest possible value of $n(R \,\Delta\, S)$ is ________.

(a) 11 (b) 12

(c) 13 (d) 10

19. Consider the following statements:

(i) Every reflexive relation is anti-symmetric.

(ii) Every symmetric relation is anti-symmetric. Which among (i) and (ii) is true?

(a) (i) alone is true

(b) (ii) alone is true

(c) Both (i) and (ii) are true

(d) Neither (i) nor (ii) is true

20. The relation 'is not equal to' is defined on the set of real numbers is satisfies which of the following?

(a) Reflexive only (b) Symmetric only

(c) Transitive only (d) Equivalence

21. If $R = \{(a, b)/|a+b| = |a|+|b|\}$ is a relation on a set $\{-1, 0, 1\}$ then R is ________.

(a) reflexive (b) symmetric

(c) anti symmetric (d) equivalence

22. For all p, such that $1 \leq p \leq 100$, if $n(A_p) = p + 2$ and $A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{100}$ and $\bigcap_{p=3}^{100} A_p = A$, then $n(A) =$

(a) 3 (b) 4

(c) 5 (d) 6

23. If $R = \{(a, b)/a + b = 4\}$ is a relation on N, then R is ______.

(a) reflexive (b) symmetric

(c) anti symmetric (d) transitive

24. Let R be a relation defined on S, the set of squares on a chess board, such that xRy, for $x, y \in S$, if x and y share a common side. Then, which of the following is false for R?

(a) Reflexive (b) Symmetric

(c) Transitive (d) All the above

25. A = {ONGC, BHEL, SAIL, GAIL, IOCL} and R is a relation defined as 'two elements of A are related if they share exactly one common letter'. For instance, BHEL and SAIL are related as they have a common letter L. The relation R is ______.

(a) anti-symmetric (b) only transitive

(c) only symmetric (d) equivalence

26. X is the set of all engineering colleges in the state of $A.P$ and R is a relation on X defined as two colleges are related iff they are affiliated to the same university then R is

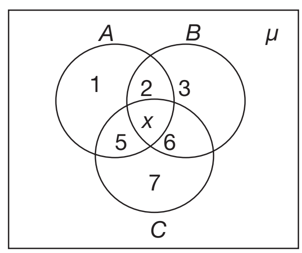
(a) only reflexive. (b) only symmetric.

(c) only transitive. (d) equivalence.

27. In the following figure, which of the following can be the value of $n(A \cup B \cup C)$? In the figure, 1, 2, 3, … represents the number of elements in the respective regions.

(a) 22 (b) 23

(c) 24 (d) 25



28. In a class, each student likes either cricket or football, 40% of the students like football, 80% of the students like cricket. The number of students who like only cricket is 40 more than the number of students who like only football. What is the strength of the class?

(a) 80 (b) 100

(c) 120 (d) 150

29. For all p, such that $1 \leq p \leq 100$, $n(A_p) = p + 1$ and $A_1 \subset A_2 \subset \ldots \subset A_{100}$. Then $\bigcup_{p=1}^{100} A_p$ contains ______ elements.

(a) 99 (b) 100

(c) 101 (d) 102

Level 2

30. In a class, 70 students wrote two tests, *viz.*, Test-I and Test-II. 50% of the students failed in Test-I and 40% of the students failed in Test-II. How many students passed in both the tests?

(a) 21 (b) 7

(c) 28 (d) 14

31. Every man in a group of 20 men likes either mangoes or an apple. Every man who likes apples also likes mangoes. 9 men like mangoes but not apples. How many like mangoes and apples?

(a) 9 (b) 11

(c) 10 (d) 12

32. In an election, two contestants A and B contested. x% of the total voters voted for A and $(x + 20)$% for B. If 20% of the voters did not vote, then find x.

(a) 30 (b) 25

(c) 40 (d) 35

33. If $A = \{1, 2, 3, 4\}$, then how many subsets of A contain the element 1 but not 4?

(a) 16 (b) 4

(c) 8 (d) 24

34. A relation R: $Z \to Z$ defined by $R = \{(x, y)/y = x^2 - 1\}$ is

(a) one to one relation.

(b) many to one relation.

(c) one to many relation.

(d) many to many relation.

35. If a set A has 13 elements and R is a reflexive relation on A with n elements, $n \in Z^+$, then

(a) $13 \le n \le 26$ (b) $0 \le n \le 26$

(c) $13 \le n \le 169$ (d) $0 \le n \le 169$

36. Example of an equivalence relation among the following is

(a) is a father of (b) is less than

(c) is congruent to (d) is an uncle of

37. If $A = \{p \in N$; p is a prime and $p = \dfrac{7n^2 + 3n + 3}{n}$ for some $n \in N\}$, then the number of elements in the set A is

(a) 1 (b) 2

(c) 3 (d) 4

38. 'aRb' if 'a is the father of b'. Then R is ________.

(a) reflexive (b) symmetric

(c) transitive (d) None of these

39. Let A be a set of compartments of a train. Then the relation R is defined on A as 'aRb if and only if a and b have the link between them'. Then which of the following is true for R?

(a) Reflexive (b) Symmetric

(c) Transitive (d) All of these

40. A relation R: $N \to N$ defined by $R = \{(x, y)/y = x^2 + 1\}$ is

(a) one to one. (b) one to many.

(c) many to one. (d) many to many.

41. If $R = \{(a, b)/|a + b| = a + b\}$ is a relation defined on a set $\{-1, 0, 1\}$ then R is ________.

(a) reflexive (c) symmetric

(c) anti symmetric (d) transitive

42. Set-builder form of the relation $R = \{(-2, -7), (-1, -4), (0, -1), (1, 2), (2, 5)\}$ is

(a) $R = \{(x, y)/y = 2x - 3; x, y \in Z\}$

(b) $R = \{(x, y)/y = 3x - 1; x, y \in Z\}$

(c) $R = \{(x, y)/y = 3x - 1; x, y \in N\}$

(d) $R = \{(x, y)/y = 3x - 1; -2 \le x < 3 \text{ and } x \in Z\}$

43. A group of 30 men participate in a survey on language skills. The number of men who know both English and Hindi was equal to the number of men who know neither of these languages. The number of men who know English is 4 more than those who know Hindi. How many know Hindi?

(a) 11 (b) 12

(c) 13 (d) 14

44. In a locality, the number of people buying only The Times of India is 80% of the number of people buying both The Times of India and The Hindu. The number of people buying only The Hindu is 60% less than the number who buy both.

The number of people buying neither of these is 22,000 less than the number of people in the locality. How many people buy both newspapers?

(a) 10,000 (b) 20,000
(c) 25,000 (d) 30,000

45. Find the number of subsets of $A \times B$, if $n(A) = 2$ and $n(B) = 4$.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) The number of elements in $A \times B$ is $4 \times 2 = 8$.
(B) The number of subsets of a set with n elements $= 2^n$.
(C) Given $n(A) = 2$ and $n(B) = 4$.
(D) $\therefore$ Required number of subsets is $2^8 = 256$.

(a) CBAD (b) CABD
(c) CDAB (d) CBDA

46. If $\mu = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then find $A^c \cap B^c$. The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $A^c = \{5, 6\}$ and $B^c = \{1, 2\}$
(B) $A^c \cap B^c = \{5, 6\} \cap \{1, 2\}$
(C) We know that $A^c = \mu - A$ and $B^c = \mu - B$.
(D) $A^c \cap B^c = \phi$

(a) CBAD (b) CDBA
(c) CABD (d) CADB

47. If $A = \{1, 2\}$ and $B = \{2, 3\}$, then find the number of elements in $(A \times B) \cap (B \times A)$. The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $(A \times B) \cap (B \times A) = \{(2, 2)\}$
(B) Given $A = \{1, 2\}$ and $B = \{2, 3\}$
(C) $n[(A \times B) \cap (B \times A)] = 1$
(D) $A \times B = \{(1, 2)\ (1, 3)\ (2, 2)\ (2, 3)\}$ and $B \times A = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

(a) BADC (b) BDCA
(c) BCAD (d) BDAC

48. The number of subsets of $\{\{a\}, \{b, c\}, d, e\}$ is _______.

(a) 32 (b) 16
(c) 8 (d) 20

49. If $R = \{(a, a), (a, c), (b, c), (b, b), (c, c), (a, b)\}$ on the set $X = \{a, b, c\}$, then how many subsets of R are reflexive relations?

(a) 15 (b) 16
(c) 8 (d) 9

50. The relation $R = \{(2, 2), (1, 1), (1, 3), (3, 1)\}$ on the set $A = \{1, 2, 3\}$ is _______.

(a) reflexive (b) symmetric
(c) transitive (d) Both (b) and (c)

51. Which of the following statement(s) is/are true?

(A) Every subset of an infinite set is infinite.
(B) Every set has a proper subset.
(C) Number of subsets of every set is even.
(D) Every subset of a finite set is finite.

(a) A and B (b) A, B and C
(c) B, C and D (d) D

52. If A and B are two non empty sets and $n(A \times B) = 36$, then which of the following cannot be equal to $n(B)$?

(a) 9 (b) 6
(c) 8 (d) 12

53. If the number of reflexive relations defined on a set A is 64, then the number of elements in A is _______.

(a) 3 (b) 2
(c) 6 (d) 5

Level 3

54. In a rehabilitation programme, a group of 50 families were assured new houses and compensation by the government. Number of families who got both is equal to the number of families who got neither of the two. The number of families who got new houses is 6 greater than the number of families who got compensation. How many families got houses?

(a) 22 (b) 28

(c) 23 (d) 25

55. In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to half the number of employees who like all the three. Then a possible value of the number of employees in the office is _________.

(a) 65 (b) 90

(c) 77 (d) 84

56. In a school, on the Republic day, three dramas A, B and C are performed on the dais. In a group of people, who attended the function and who like at least one of the three dramas, 16 people like A, 20 people like B, 15 people like C, 4 people like both A and B, 3 people like both A and C, 3 people like both B and C and 2 people like all the three. Then how many people like at most two?

(a) 59 (b) 41

(c) 4 (d) 6

57. The students of a class like at least one of the games out of Chess, Caroms and Judo. The number of students who like only Chess and Caroms, only Caroms and Judo, only Chess and Judo and the number of those who like all the three are equal. The number of students who like only Chess, only Caroms, only Judo and the number of those who like all the three are equal. A possible value of the number of students in the class is

(a) 30 (b) 40

(c) 50 (d) 70

58. There are a total of 70 ladies who watch at least one of the channels, i.e., Zee TV, Sony TV and Star Plus. The total number of ladies who watch Zee or Sony but not Star plus, the number of ladies who watch Sony or Star Plus but not Zee and the number of ladies who watch Star Plus or Zee but not Sony is 90. How many ladies watch at least two of these channels if 10 ladies watch all the three channels?

(a) 25

(b) 30

(c) 40

(d) 35

59. If $R = \{(x, y)/x \in W, y \in W, (2x + y)^2 = 49\}$, then R^{-1} is _______.

(a) $\{(5, 1), (3, 2), (1, 3)\}$

(b) $\{(7, 0), (5, 1), (3, 2), (1, 3)\}$

(c) $\{(7, 0), (1, 5), (2, 3), (1, 3)\}$

(d) $\{(0, 7), (5, 1), (3, 2), (1, 3)\}$

60. Which of the following cannot be the number of reflexive relations defined on a set A?

(a) 1 (b) 4

(c) 4096 (d) 512

61. In a class, the number of students who like only Chess, only Caroms, both the games and neither of the games are in the ratio 2 : 4 : 1 : 3. The number of students who like at least one of these games is 120 more than those who like neither of the games. Find the number of students in the class.

(a) 300 (b) 240

(c) 270 (d) 360

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. {1, 2, 3, 4, 5, 6, 7, 8, 9}
2. {1, 3, 4, 6, 7}
3. {2, 5}
4. $X = Y$
5. ϕ
6. ϕ
7. True
8. 32
9. False
10. 12
11. μ
12. P, Q
13. 30
14. 9
15. ϕ
16. 10000
17. 0
18. 20
20. 2^{18}
22. 13
23. 0
24. 17, 0
25. 378
26. 2^{16}
27. $E = \{(a, a), (x, x), (y, y), (d, d)\}$
28. $R = \{(x, y)/(x, y) \in B \times A, x > y\}$
29. $R = \{(3, 6), (5, 10), (6, 12), (8, 16)\}$
30. 240

Short Answer Type Questions

31. {13, 14}
32. {14}
33. $a = 7$
34. disjoint
35. 20
36. 25
37. 50
38. $x = 3$
39. (i) $P = \{0\}$
    (ii) $S = \{1, 13\}$
40. True
41. anti-symmetric property
42. one to one type of relations
43. 13
44. 10
45. 38%

Essay Type Questions

46. (i) 63 (ii) 124
    (iii) 6 (iv) 130
    (v) 120 (vi) 67
47. reflexive, symmetric, transitive
48. only symmetric property
49. $4 \le x \le 2^{16}$
50. 35%

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (d)	**20.** (b)
21. (d)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (c)	

Level 2

30. (b)	**31.** (b)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (b)
40. (a)	**41.** (b)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (b)	**49.** (c)
50. (b)	**51.** (d)	**52.** (c)	**53.** (a)						

Level 3

54. (b)	**55.** (c)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (b)	**60** (d)	**61.** (a)

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Required answer cannot be expressed in the form of 2^n, $n \in w$.
2. Recall that x and $\{x\}$ are different.
3. $x \in A - B \Rightarrow x \in A$ and $x \notin B$.
4. If $(x, y) = (a, b)$, then $x = a$ and $y = b$.
5. Solve the two equations for x then write X and Y. Now find $X - Y$.
6. Number of subsets $= 2^{n(A \times B)}$.
7. The number of subsets of A containing one particular element is 2^{n-1}.
8. Use the definition of reflexive, symmetric and transitive.
9. Recall one-one relation.
10. (i) Recall the definition of union of sets and subsets.

    (ii) If $A \subset B$, then $A \cup B = B$.

    (iii) $\bigcup_{n=2}^{10} P_n = P_2 \cup P_3 \cup \ldots \cup P_{10} = P_{10}$.
11. (i) Recall the definition of intersection of sets and subsets.

    (ii) If $A \subset B$, then $A \cap B = A$.

    (iii) $\bigcap_{n=3}^{10} P_n = P_3 \cap P_4 \cap \ldots \cap P_{10} = P_3$.
12. (i) $n(A \cup B)' = n(\mu) - n(A \cup B)$.

    (ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

    (iii) $n[(A \cup B)^C] = n(\mu) - n(A \cup B)$.
13. $|x| < a \Rightarrow -a < x < a$.
14. List out value of n, such that $2/n$ is an integer.
15. Use the definition of reflexive, symmetric and transitive.
16. $n(A)$ is a factor of $n(A \times B)$.
17. (i) $n(A \cap B) = 0$, $n(A) = n(A \cap C)$ and $n(A \cap B \cap C) = 0$.

    (ii) $n(A \cap B) = 0$; $n(A) = n(A \cap C)$ and $n(A \cap B \cap C) = 0$.

    (iii) If $A \subseteq C$, $A \cup C = C$.

    (iv) Substitute the above values in $n(A \cup B \cup C)$.
18. (i) Recall all the concept of symmetrical difference.

    (ii) For the greatest possible value of $n(R \,\Delta\, S)$, $n(R \cap S)$ is minimum.

    (iii) As $R \cap S \neq \phi$, $n(R \cap S) = 1$.

    (iv) $n(R \,\Delta\, S) = n(R \cup S) - n(R \cap S)$.
19. Recall the definitions of reflexive, symmetric, antisymmetric, transitive relations.
20. Recall the definitions of reflexive, symmetric and transitive relations.
21. (i) Write all the elements of R.

    (ii) $R = \{(0, 0), (1, 1), (-1, -1), (0, -1), (-1, 0), (1, 0), (0, 1)\}$.

    (iii) Check the properties, which R satisfy.
22. (i) When $A_1 \subset A_2 \subset \ldots \subset A_n$ then $\bigcap_{i=1}^{n} A_i = A_1$

    (ii) If $A \subset B$, then $A \cap B = A$.

    (iii) $\bigcap_{p=3}^{100} A_p = A_3$.

    (iv) Find $n(A_3)$ by using $n(A_p) = p + 2$.
23. (i) Write all possibilities for (a, b).

    (ii) $R = \{(1, 3), (2, 2), (3, 1)\}$.

    (iii) Check the properties, which R satisfy.
24. (i) Check by taking examples.

    (ii) Consider the three squares of the board as follows.

1	2	3

    Squares 1 and 2 share a common side and squares 2 and 3 share a common side, but squares 1 and 3 does not share a common side.
25. (i) Write the elements of R.

    (ii) Write all the elements of R and verify.
26. (i) Verify the reflexive, symmetric and transitive properties of relations.

    (ii) Take an example and proceed.

27. Adding all the elements

28. (i) $n(F \cup C) = n(F) + n(C) - n(F \cap C)$.

(ii) Let the total number of students be x.

(iii) 80% of x – 40% of $x = 40$. Find x.

29. (i) Recall the definition of union of sets and subsets.

(ii) If $A \subset B$, then $A \cup B = B$.

(iii) $\bigcup_{n=1}^{100} A_n = A_{100}$.

(iv) Now evaluate $n(A_{100})$ by using the given condition.

Level 2

30. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

31. Apply concept of subsets.

32. Apply the formula of $A \cup B$.

33. The number of subsets of A that contain a particular element and does not contain another particular element is 2^{n-2}.

34. (i) Write some possibilities of (x, y).

(ii) $(2, 3)$ and $(-2, 3)$ are the two elements of R.

(iii) Now check which type of relation is R.

35. (i) Recall the properties of identity relation.

(ii) If $n(A) = p$ and R is reflexive defined on A, then $p \le n(R) \le p^2$.

36. Use the definition of reflexive, symmetric and transitive.

37. Substitute $n = 1$ and $n = 3$.

38. Use the definition of reflexive, symmetric and transitive.

39. (i) Verify the reflexive, symmetric and transitive properties of relations.

(ii)

1	2	3

Compartments 1 and 2 are linked and compartments 2 and 3 are linked but compartments 1 and 3 are not linked.

40. (i) Write some elements of R and then check.

(ii) $(2, 3)$ and $(-2, 3)$ are the two elements of R.

(iii) Now check which type of relation is R.

41. (i) Verify the reflexive, symmetric and transitive properties of relations.

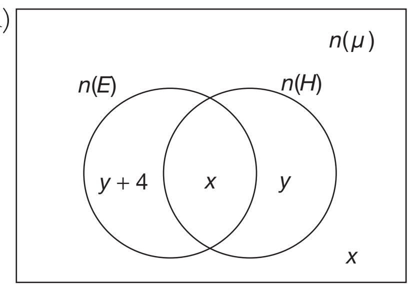
(ii) $R = \{(0, 0), (1, 1), (1, -1), (-1, 1), (0, 1), (1, 0)\}$.

(iii) Check which properties does R satisfy.

42. Select from options.

43. (i) $n(E \cup H) = n(E) + n(H) - n(E \cap H)$

(ii)



(iii) $(4 + y) + (x) + (y) + x = 30$.

(iv) The number of men who knows Hindi is $x + y$.

44. (i) Use Venn diagrams.

(ii) Take the number of families who buy both the news papers, The Times of India and The Hindu as x.

(iii) $n(A \cup B) = 0.8x + x + 0.4x$.

(iv) Use $2.2x = 22000$, then find x.

45. The sequential order is CABD.

46. The sequential order is CABD.

47. The sequential order is BDAC.

48. Let $A = \{\{a\}, \{b, c\}, d, e\} \Rightarrow n(A) = 4$

$\therefore$ The number of subsets of $A = 2^4 = 16$.

49. Given $R = \{(a, a), (a, c), (b, c), (b, b), (c, c), (a, b)\}$

$\therefore$ The number of reflexive relations = The number of subsets formed by the elements(a, c), (b, c) and $(a, b) = 2^3 = 8$.

50. Given, $A = \{1, 2, 3\}$ and $R = \{(2, 2), (1, 1), (1, 3), (3, 1)\}$

Here, $3 \in A$ but $(3, 3) \notin R$. Hence R is not reflexive.

Also, $(3, 1) \in R$, and $(1, 3) \in R$ but $(3, 3) \notin R$. R is not transitive. But $\forall (x, y) \in R$, there exist $(y, x) \in R$. Hence R is symmetric. $(3, 1), (1, 3), \in R$ but $(3, 3) \notin R$. Hence, R is not transitive.

51. (a) is false since the set N of all natural numbers is an infinite set having a finite subset, i.e., $\{1\}$

(b) is false since $\varnothing$ has no proper subsets.

(c) is false since number of subsets of an empty set is odd (i.e., $2^{\circ} = 1$).

(d) is clearly true.

52. $n(A \times B) = n(A) \times n(B) \Rightarrow n(A)$ and $n(B)$ should be the factors of $n(A \times B)$.

$\therefore n(B)$ cannot be 8.

53. We have number of reflexive relations defined on A is 2^{n^2-n}, where n is the cardinal number of A.

Given, $2^{n^2-n} = 64 \Rightarrow 2^{n^2-n} = 2^6$

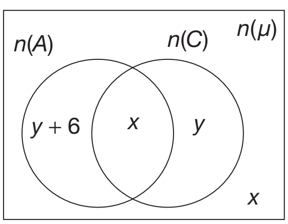
$\Rightarrow n^2 - n = 6 \Rightarrow n(n-1) = 3 \times 2$

$\therefore n = 3$.

Level 2

54. (i) Use Venn diagram.

(ii) Given data can be expressed as follows:

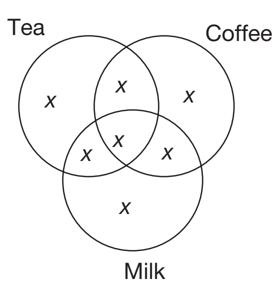


(iii) $(y + 6) + (x) + (y) + (x) = 50$.

(iv) Number of families who got houses $= x + y + 6$.

55. (i) Use Venn diagrams.

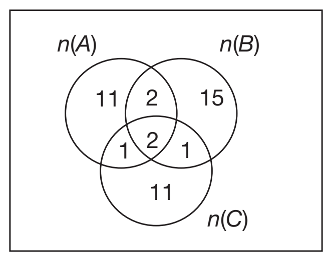
(ii) The given information can be expressed as



(iii) Total number of employees is $7x$, i.e., a multiple of 7.

56. (i) Use Venn diagrams.

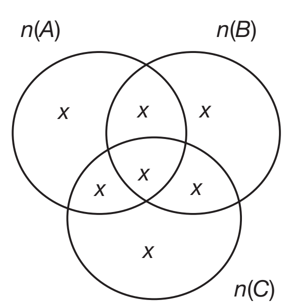
(ii) Given data can be expressed as follows:



(iii) Number of people who like at most two dramas = Number of people who like only one drama + Number of people who like only two of the dramas.

57. (i) Use Venn diagram.

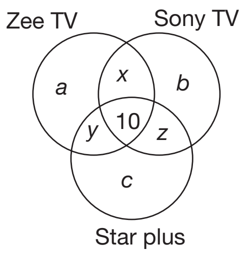
(ii) Given data can be expressed as



(iii) Total number of students is $7x$.

58. (i) Use Venn diagram.

(ii)



(iii) $(a + b + c) + (x + y + z) + 10 = 70$.

(iv) $(a + x + b) + (b + z + c) + (a + y + c) = 90$.

(v) Using the above information find $(x + y + z + 10)$.

59. Given $(2x + y)^2 = 49$ and $(x, y) \in W$.

$\Rightarrow 2x + y = 7$

$\therefore$ The possible values of (x, y) are $(0, 7)$, $(1, 5)$, $(3, 1)$ and $(2, 3)$ $\Rightarrow R = \{(0, 7),(1, 5), (2, 3), (3, 1)\}$.

$\therefore R^{-1} = \{(7, 0), (5, 1), (3, 2), (1, 3)\}$.

60. We have the number of reflexive relations defined on A is 2^{n^2-n}

Option (a) $\Rightarrow 2^{n^2-n} = 2^0$

$\Rightarrow n(n - 1) = 1 \times 0 \Rightarrow n = 1$.

Option (b) $\Rightarrow 2^{n^2-n} = 2^2$

$\Rightarrow n(n - 1) = 2 \times 1 \Rightarrow n = 2$.

Option (c) $\Rightarrow 2^{n^2-n} = 2^{12}$

$\Rightarrow n(n -1) = 4 \times 3 \Rightarrow n = 4$.

Option (d) $\Rightarrow 2^{n^2-n} = 2^9 \Rightarrow n(n - 1) = 9$.

No integer value of n satisfies the above condition.

$\therefore$ 512 cannot be the number of reflexive relations defined on a set A.

61. Let the number of students who like only Chess be $2x$. The number of students who like only Carroms, both the games and neither of the games are $4x$, x and $3x$ respectively.

Given, $2x + 4x + x = 3x + 120$

$\Rightarrow 4x = 120 \Rightarrow x = 30$

$\therefore$ The total number of students $= 2x + 4x + x + 3x = 10x = 300$.

HINTS AND EXPLANATION

Chapter 7

Matrices

REMEMBER

Before beginning this chapter, you should be able to:

- Understand basic terms related to matrix
- Find order of a matrix

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the order of a matrix, types of matrices and comparable matrices
- Apply the operations on matrices
- Find transpose of a matrix
- Learn symmetric and skew-symmetric matrices

INTRODUCTION

A matrix is a rectangular arrangement of elements in the form of rows and columns. The elements can be numbers (real or complex) or variables. Matrices is the plural of matrix.

Horizontal line of elements is called a row and the vertical line of elements is called a column. The rectangular array of elements in a matrix are enclosed by brackets [ ] or parentheses ( ). Generally we use capital letters to denote matrices.

Examples:

1. $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \end{bmatrix}$ is a matrix having 2 rows and 3 columns.

2. $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a matrix having 2 rows and 2 columns.

Order of a Matrix

If a matrix A has m rows and n columns, then $m \times n$ is called the order (or type) of matrix, and is denoted as $A_{m \times n}$.

Examples:

1. $A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 4 & 1 \end{bmatrix}$ is a matrix consisting of 2 rows and 3 columns. So its order is 2×3.

2. $B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ is a matrix consisting of 3 rows and 1 column. So its order is 3×1.

So in general, a set of mn elements can be arranged as a matrix having m rows and n columns as shown below.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2n} \\ a_{31} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_{ij} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdot & \cdot & \cdot & \cdot & a_{mn} \end{bmatrix}_{m \times n} \quad \text{or } A = [a_{ij}]_{m \times n}$$

Where a_{ij} represents the element in the ith row and jth column.

Example:

$$\text{Let } P = \begin{bmatrix} 2 & 3 & 51 \\ 4 & -2 & -3 \\ 5 & -31 & 1 \end{bmatrix}$$

In the above matrix

$$a_{11} = 2,\ a_{12} = 3,\ a_{13} = 51$$
$$a_{21} = 4,\ a_{22} = -2,\ a_{23} = -3$$
$$a_{31} = 5,\ a_{32} = -31,\ a_{33} = 1.$$

In compact form, any matrix A can be represented as

$$A = [a_{ij}]_{m \times n} \text{ where } 1 \le i \le m,\ 1 \le j \le n.$$

Types of Matrices

Rectangular Matrix

In a matrix if the number of rows is not equal to the number of columns, then the matrix is called a rectangular matrix.

Example:

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \\ 1 & 3 \end{bmatrix}_{3\times 2}$$

A has 3 rows and 2 columns.

Row Matrix

A matrix which has only one row is called a row matrix.

Examples:

1. $[5 \quad -3 \quad 2 \quad 1]$ is a row matrix of order 1×4.

2. $[4 \quad 3 \quad 13 \quad -4 \quad -31]$ is a row matrix of order 1×5.

In general, order of a row matrix is $1 \times n$, where n is the number of columns.

Column Matrix

A matrix which has only one column is called a column matrix.

Examples:

1. $\begin{bmatrix} 5 \\ -3 \\ 2 \\ -1 \end{bmatrix}_{4\times 1}$ is a column matrix of order 4×1.

2. $\begin{bmatrix} -3 \\ 5 \\ 20 \\ -2006 \\ 2 \\ -1 \end{bmatrix}_{6\times 1}$ is a column matrix of order 6×1.

In general, order of a column matrix is $m \times 1$, where m is the number of rows of the matrix.

Null Matrix or Zero Matrix

If every element of a matrix is zero, then the matrix is called a null matrix or zero matrix. A zero matrix of order $m \times n$ is denoted by $O_{m\times n}$.

Example:

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{2\times 4} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

Square Matrix

In a matrix, if the number of rows is equal to the number of columns, then the matrix is called a square matrix.

A matrix of order $n \times n$ is termed as a square matrix of order n.

Examples:

1. $\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$ is a square matrix of order 2.

2. $\begin{bmatrix} a & b & -3 \\ 4 & c & -2 \\ y & x & z \end{bmatrix}$ is a square matrix of order 3.

Principal Diagonal of a Square Matrix In a square matrix A of order n, the elements a_{ii}((i.e., a_{11}, a_{22}, …, a_{mn}) constitute the principal diagonal. The elements a_{ii} are called elements of principal diagonal.

Examples:

1. $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

The elements 2, 5 (i.e., a_{11} and a_{22}) constitute the principal diagonal of A.

2. $P = \begin{bmatrix} -3 & 4 & 5 \\ 6 & -2 & 1 \\ a & 3 & b \end{bmatrix}$

The elements −3, −2, b constitute principal diagonal of P.

Trace of a Matrix The sum of the principal diagonal elements of a square matrix A is the trace of that matrix. It is represented as Trace A or Tr(A).

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & -2 \end{bmatrix}$$

Trace $A = 1 + 5 - 2 = 4$.

Diagonal Matrix

In a square matrix, if all the non-principal diagonal elements are zeroes and at least one of the principal diagonal elements is non-zero, then the matrix is called a diagonal matrix.

Examples:

1. $A = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$ is a diagonal matrix of order 2.

2. $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ is a diagonal matrix of order 3.

In compact the above matrices can be written as diagonal $(-3 \quad -2)$ and diagonal $(0 \quad 2 \quad -3)$.

A diagonal matrix $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ can be written as diagonal $(a\ b\ c)$.

Scalar Matrix

In a square matrix if all the principal diagonal elements are equal ($\neq 0$) and rest of the elements are zeroes, then the matrix is called a scalar matrix.

Examples:

1. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is a scalar matrix of order 2.
2. $\begin{bmatrix} 31 & 0 & 0 \\ 0 & 31 & 0 \\ 0 & 0 & 31 \end{bmatrix}$ is a scalar matrix of order 3.

Identity Matrix (or) Unit Matrix

In a square matrix if all the principal diagonal elements are unity and rest of the elements are zeroes, then the matrix is called an identity matrix or unit matrix. It is denoted by I_n.

Examples:

1. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$ is an identity matrix of order 2.
2. $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is an identity matrix of order 4.

Comparable Matrices

Two matrices A and B can be compared, only when they are of same order.

Example:

Consider two matrices A and B given by

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 21 \\ 10 & -4 \end{bmatrix}_{3\times 2} \quad \text{and} \quad B = \begin{bmatrix} -3 & 10 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}_{3\times 2}.$$

Matrices A and B can be compared as both of them are of order 3×2.

Equality of Two Matrices

Two matrices are said to be equal only when

1. they are of same order.
2. the corresponding elements of both the matrices are equal.

Example:

If $\begin{bmatrix} 2 & -3 \\ 1 & 2 \\ 6 & -2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -3 \\ a & 2 \\ 6 & b \end{bmatrix}$ are equal matrices, then $a = 1$ and $b = -2$.

ADDITION OF MATRICES

1. Two matrices A and B can be added only when they are of same order.

   Example:

   Let $A = \begin{bmatrix} -3 & 2 & 1 \\ 5 & 6 & -5 \end{bmatrix}$, $B = \begin{bmatrix} -13 & 21 & 33 \\ -52 & 4 & 49 \end{bmatrix}$.

   Here both the matrices A and B are of order 2×3. So, they can be added.

2. The sum matrix of two matrices A and B is obtained by adding the corresponding elements of A and B.

3. Here $A + B = \begin{bmatrix} -3+(-13) & 2+21 & 1+33 \\ 5+(-52) & 6+4 & -5+49 \end{bmatrix} = \begin{bmatrix} -16 & 23 & 34 \\ -47 & 10 & 44 \end{bmatrix}_{2\times 3}$.

Properties of Matrix Addition

1. Matrix addition is closed, i.e., sum of two matrices is also a matrix.
2. Matrix addition is commutative, i.e., if A and B are two matrices of same order, then $A + B = B + A$.
3. Matrix addition is associative, i.e., if A, B and C are three matrices of same order, then, $A + (B + C) = (A + B) + C$.
4. **Identity matrix:** If $O_{m\times n}$ is a null matrix of order $m \times n$ and A is any matrix of order $m \times n$, then $A + O = O + A = A$. So, O is called an identity matrix under addition.
5. **Additive inverse:** If $A_{m\times n}$ is any matrix of order $m \times n$, then $A + (-A) = (-A) + A = O$. So, $-A$ is called the additive inverse of the matrix A.
6. If k is a scalar and A and B are two matrices of same order, then $k(A + B) = kA + kB$.

MATRIX SUBTRACTION

1. Matrix subtraction is possible only when both the matrices are of same order.
2. The difference of two matrices of same type (i.e., order) A and B, i.e., $A - B$, is obtained by subtracting the corresponding elements of B from that of A.

EXAMPLE 7.1

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}$, then find $A - B$.

SOLUTION

$$A-B=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}-\begin{bmatrix}-3 & 1\\ 4 & -2\end{bmatrix}$$

$$A-B=\begin{bmatrix}2-(-3) & 3-1\\ -1-4 & 4-(-2)\end{bmatrix}=\begin{bmatrix}5 & 2\\ -5 & 6\end{bmatrix}.$$

Note Trace $(A \pm B)$ = Trace $(A) \pm$ Trace (B).

TRANSPOSE OF A MATRIX

For a given matrix A, the matrix obtained by interchanging its rows or columns is called transpose of the matrix A and is denoted by A^T or A^1.

Examples:

1. If $A=\begin{bmatrix}2 & -1 & 3 & 4\\ 5 & -6 & 11 & -1\end{bmatrix}$, then transpose of A, i.e., $A^T=\begin{bmatrix}2 & 5\\ -1 & -6\\ 3 & 11\\ 4 & -1\end{bmatrix}$

   Here we can note that order of A is 2×4, while that of A^T is 4×2.

2. $A=\begin{bmatrix}-1 & 3\\ 4 & 2\end{bmatrix}$, then $A^T=\begin{bmatrix}-1 & 4\\ 3 & 2\end{bmatrix}$

3. If $A = [5003]$, then $A^T = [5003]$

Notes

1. If the order of a matrix is $m \times n$, then the order of its transpose matrix is $n \times m$.
2. $(A^T)^T = A$.
3. $[(A^T)^T]^T = A^T$.

Example:

Let $A=\begin{bmatrix}-2 & 5\\ 3 & -1\\ -4 & 0\end{bmatrix}$

Now,

$$A^T=\begin{bmatrix}-2 & 5\\ 3 & -1\\ -4 & 0\end{bmatrix}^T=\begin{bmatrix}-2 & 3 & -4\\ 5 & -1 & 0\end{bmatrix}$$

$$(A^T)^T=\begin{bmatrix}-2 & 3 & -4\\ 5 & -1 & 0\end{bmatrix}^T=\begin{bmatrix}-2 & 5\\ 3 & -1\\ -4 & 0\end{bmatrix}=A.$$

3. If A and B are two matrices of same order, then $(A+B)^T = A^T + B^T$.
4. If $k(\neq 0)$ is a scalar and A is any matrix, then $(kA)^T = kA^T$.

Symmetric Matrix

A square matrix is said to be symmetric $\Leftrightarrow A = A^T$, i.e., transpose of the given matrix is itself.

Examples:

1. If $A = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}$, then $A^T = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix} = A.$

Thus we can observe that $A^T = A$. So, A is a symmetric matrix.

2. Similarly for $P = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & -1 \\ 43 & -1 & 2 \end{bmatrix}$,

$$P^T = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & -1 \\ 43 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & -1 \\ 43 & -1 & 2 \end{bmatrix} = P.$$

So, P is a symmetric matrix.

Note If A is a square matrix, then $\frac{1}{2}(A + A^T)$ is always a symmetric matrix.

Skew-symmetric Matrix

A square matrix A is said to be skew symmetric if $A^T = -A$, i.e., transpose of the matrix is equal to its additive inverse.

Example:

$$\text{If } A = \begin{bmatrix} 0 & 2006 \\ -2006 & 0 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 0 & -2006 \\ 2006 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2006 \\ -2006 & 0 \end{bmatrix} = -A.$$

So, A is a skew-symmetric matrix.

Note If A is a square matrix, then $\frac{1}{2}(A - A^T)$ is always skew-symmetric.

MULTIPLICATION OF MATRICES

Multiplication of a Matrix by Scalar

If every element of a matrix A is multiplied by a scalar (real or complex), i.e., k, the matrix obtained is k times A and is denoted by kA and the operation is called scalar multiplication.

EXAMPLE 7.2

If $A = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix}$, then find

(a) $-A$ **(b)** $3A$ **(c)** $\frac{1}{4}A$.

SOLUTION

(a) $-A = -\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1\times 2 & -1\times 3 & -1\times(-1) \\ -1\times 5 & -1\times 6 & -1\times 1 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 1 \\ -5 & -6 & -1 \end{bmatrix}.$$

(b) $3A = 3\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3\times 2 & 3\times 3 & 3(-1) \\ 3\times 5 & 3\times 6 & 3\times 1 \end{bmatrix} = \begin{bmatrix} 6 & 9 & -3 \\ 15 & 18 & 3 \end{bmatrix}.$$

(c) $\frac{1}{4}A = \frac{1}{4}\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{4}\times 2 & \frac{1}{4}\times 3 & \frac{1}{4}\times(-1) \\ \frac{1}{4}\times 5 & \frac{1}{4}\times 6 & \frac{1}{4}\times 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} & \frac{-1}{4} \\ \frac{5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix}.$$

Notes

1. If k and l are any two scalars (i.e., numbers) and P is a matrix, then $k(lP) = (kl)P$.

   Examples:

   (i) $\frac{4}{5}(10P) = \left(\frac{4}{5}\times 10\right)P = 8P$

   (ii) $\frac{2}{3}(9P) = \frac{2}{3}\times 9P = 6P$

   (iii) $4\,(-2P) = 4(-2)P = -8P$

   (iv) $-2(-P) = -2(-1)P = 2P$

2. If m and n are any two scalars and A is a matrix, then $(m + n)\,A = mA + nA$.

Multiplication of Two Matrices

Two matrices A and B can be multiplied only if the number of columns in A (first matrix) is equal to the number of rows in B (second matrix). The product matrix of A and B (if exists) is written as AB.

Now consider a matrix A of order 2×3 and another matrix B of order 3×4. As the number of columns in A (i.e., 3) is equal to number of rows in B (i.e., 3). So AB exists and it is of the order 2×4. We can obtain the product matrix AB as follows:

$$A_{m\times q} \times B_{q\times n} = AB_{m\times n}.$$

Following Example will Clearly Illustrate the Method:

EXAMPLE 7.3

Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 7 \\ 3 & 5 \end{bmatrix}_{3\times 2}$ and $B = \begin{bmatrix} -5 & 6 & 4 \\ 9 & 11 & 8 \end{bmatrix}_{2\times 3}$

SOLUTION

$$AB = \begin{bmatrix} 2(-5)+(-1)(9) & 2(6)+(-1)(11) & 2(4)+(-1)(8) \\ 1(-5)+7(9) & (1)6+7(11) & (1)4+7(8) \\ 3(-5)+5(9) & (3)6+(5)11 & 3(4)+5(8) \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} -10+(-9) & 12+(-11) & 8+(-8) \\ -5+63 & 6+77 & 4+56 \\ -15+45 & 18+55 & 12+40 \end{bmatrix}_{3\times 3} = \begin{bmatrix} -19 & 1 & 0 \\ 58 & 83 & 60 \\ 30 & 73 & 52 \end{bmatrix}_{3\times 3}.$$

In general if $A = [a_{ip}]$ is a matrix of order $m \times q$ and $B = [b_{pj}]$ is a matrix of order $q \times n$, then the product matrix $AB = [x_{ij}]$ will be of order $m \times n$ and is given by $[x_{ij}] = \sum_{p=1}^{q} a_{ip}b_{pj}$.

Properties of Matrix Multiplication

1. In general, matrix multiplication is not commutative, i.e., $AB \neq BA$.

EXAMPLE 7.4

Let $A = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$

SOLUTION

$$AB = \begin{bmatrix} -3(2)+1(0) & -3(-1)+1(1) \\ (0)2+2(0) & 0(-1)+2(1) \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ 0 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-3)+(-1)0 & 2(1)+(-1)2 \\ 0(-3)+1(0) & (0)1+1(2) \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & 2 \end{bmatrix}.$$

So, we can observe that $AB \neq BA$.

2. Matrix multiplication is associative, i.e., $A(BC) = (AB)C$.
3. Matrix multiplication is distributive over addition, i.e.,
   (i) $A(B + C) = AB + AC$ (left distributive)
   (ii) $(B + C)A = BA + CA$ (right distributive).
4. For any two matrices A and B, if $AB = O$, then it is not necessarily imply that $A = O$ or $B = O$ or both A and B are O.

EXAMPLE 7.5

Let, $A = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -12 \\ -4 & 6 \end{bmatrix}$, then

SOLUTION

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} 8 & -12 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2(8)+4(-4) & 2(-12)+4(6) \\ 4(8)+8(-4) & 4(-12)+8(6) \end{bmatrix}$$

$$= \begin{bmatrix} 16-16 & -24+24 \\ 32-32 & -48+48 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, we can observe that, though $AB = O$, $A \neq O$ and $B \neq O$.

5. For any three matrices A, B and C, if $AB = AC$, then it is not necessarily imply that $B = C$ or $A = O$ (But in case of any three real number a, b and c if $ab = ac$ and $a \neq 0$, then it is necessary that $b = c$.)

EXAMPLE 7.6

Let $A = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$, $B = \begin{bmatrix} 10 & 5 \\ 15 & 10 \end{bmatrix}$ and $C = \begin{bmatrix} -10 & 35 \\ 25 & -5 \end{bmatrix}$. Prove that $AB = AC$ but $B \neq C$

SOLUTION

$$AB = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} 10 & 5 \\ 15 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} (5)10+10(15) & (5)5+(10)10 \\ (10)\,10+(20)15 & (10)5+(20)10 \end{bmatrix}$$

$$= \begin{bmatrix} 50+150 & 25+100 \\ 100+300 & 50+200 \end{bmatrix} = \begin{bmatrix} 200 & 125 \\ 400 & 250 \end{bmatrix}.$$

$$AC = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} -10 & 35 \\ 25 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 5(-10)+10(25) & 5(35)+10(-5) \\ 10(-10)+20(25) & 10(35)+20(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -50+250 & 175-50 \\ -100+500 & 350-100 \end{bmatrix}$$

$$= \begin{bmatrix} 200 & 125 \\ 400 & 250 \end{bmatrix}.$$

Here $AB = AC$ but $B \neq C$.
Hence Proved.

6. If A is a square matrix of order n and I is an identity matrix of order n, then $AI = IA = A$. That is, I is the multiplicative identity matrix.

EXAMPLE 7.7

Prove that $AI = IA = A$, for

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

SOLUTION

$$AI = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = A.$$

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = A.$$

$\therefore$ $AI = IA = A$. Here I is called multiplicative identity matrix.

7. If the matrix A is multiplied by a null matrix, then the resultant matrix is a null matrix, i.e., $AO = OA = O$.

8. If A and B are two matrices such that AB exists, then $(AB)^T = B^T A^T$.

Note If $A_1, A_2, A_3, \ldots, A_n$ are n matrices, then $(A_1A_2A_3\ldots A_n)^T = A_n{}^T A^T{}_{n-1} \ldots A_1{}^T$.

9. If A is any square matrix then $(A^T)^n = (A^n)^T$.

10. If A and B are any two square matrices, then

(i) $(A + B)^2 = (A + B)(A + B) = A(A + B) + B(A + B)$
$= A^2 + AB + BA + B^2$.

(ii) $(A - B)^2 = (A - B)(A - B) = A(A - B) - B(A - B)$
$= A^2 - AB - BA + B^2$.

(iii) $(A + B)(A - B) = A(A - B) + B(A - B)$
$= A^2 - AB + BA - B^2$.

11. For two matrices A and B, if AB exists, then BA may or may not exist.

Example: If $A_{3\times4}$ and $B_{4\times2}$ then $(AB)_{3\times2}$ exists. BA does not exist as number of columns of B is not equal to number of rows of A.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. What is the order of matrix $\begin{bmatrix} 7 & 1 & 2 \\ 6 & 2 & 2 \end{bmatrix}$?

2. For a matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & 3 \\ 5 & 8 & 6 \end{bmatrix}$, what is the second row and third column element?

3. Trace of a scalar matrix of order 4×4 whose one of the principal diagonal elements is 4 is _____.

4. If order of matrix A is 4×3 and AB is 4×5, then the order of matrix B is _____.

5. Two matrices $A = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} x & 3 \\ 5 & -3-y \end{bmatrix}$ are equal, then $x + y$ is _______.

6. If $A = \begin{bmatrix} 10 & 8 \\ -6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 5 & 0 \end{bmatrix}$, then find $A + B$.

7. If $A = \begin{bmatrix} 12 & 7 \\ 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 10 \\ -2 & -5 \end{bmatrix}$, then find $(A - B)$.

8. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then $A^T =$ _______.

9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then kA (where 'k' is a scalar) $=$ ___________.

10. If $A = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$, then $(A^T)^T =$ ___________.

11. The order of matrix $A + B^T$ is 4×3, then the order of matrix B is _______.

12. If $A = \begin{bmatrix} 3 & 24 \\ -4 & 8 \end{bmatrix}$, then check whether it is symmetric matrix or not.

13. $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ is a _____ matrix.

14. The product of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $[3 \quad 4]$ is _____.

15. The product of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is _____.

16. The product of $[2 \quad -4 \quad 4]$, and $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ is _____.

17. If all the diagonal elements in a diagonal matrix is 0, then it is a ________ matrix.

18. If $A = \begin{bmatrix} 2 & -1 & 0 \\ 9 & 2 & 4 \\ 6 & 3 & -9 \end{bmatrix}$, find $(k + l)A$.

19. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 5 \\ -6 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -6 \\ -3 & 1 \end{bmatrix}$, then find $(A + B) + C$.

20. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, then find $3A + 7B$.

21. If $5\begin{bmatrix} -3 & 1 \\ x & 2 \end{bmatrix} + \begin{bmatrix} y & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -15 & 9 \\ 6 & z \end{bmatrix}$, then find x, y, z.

22. If $A = \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 20 & 31 \\ 44 & 53 \end{bmatrix}$, then find the matrix X such that $A + X = B$.

23. If $A = \begin{bmatrix} 11 & -13 \\ -13 & 11 \end{bmatrix}$, then find A^T. What do you observe?

24. If $A = [a_{ij}]_{2\times3}$, defined as $a_{ij} = i^2 - j + 1$, then find matrix A.

25. If $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 9 & -11 \end{bmatrix}$, $B = \begin{bmatrix} 22 & 31 & 43 \\ 16 & 11 & 44 \end{bmatrix}$ and $C = \begin{bmatrix} 51 & 33 & 2 \\ 41 & 5 & -14 \end{bmatrix}$, then find $4C - 2B - A$.

26. Compute:
$$2\begin{bmatrix} 2 & 3 & 4 & 1 \\ -5 & 11 & 6 & 7 \end{bmatrix} + 5\begin{bmatrix} 11 & 13 & -3 & 4 \\ 5 & -3 & 4 & -7 \end{bmatrix} - 6\begin{bmatrix} 6 & 12 & 4 & -2 \\ -3 & 5 & 11 & 7 \end{bmatrix}.$$

27. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, then prove that $\frac{1}{2}(A + A^T)$ is a symmetric matrix.

PRACTICE QUESTIONS

28. Find a, p, q and b if

$$\begin{bmatrix} a-3 & 6 & b-q \\ 1 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 2p-q & -4 \\ \frac{q-p}{2} & 5 & a+b \end{bmatrix}.$$

29. If A, B and C are three matrices of order 3×4, 3×2 and 1×2 respectively, then find the order of matrix $A^T BC^T$.

30. If X and Y are two matrices such that $X + Y = \begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 6 & -5 \\ 3 & 2 \end{bmatrix}$, then find the matrices X and Y.

Short Answer Type Questions

31. If $A = \begin{bmatrix} 1 & 2 & 3 & 5 & 6 \\ 4 & -1 & 2 & -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 3 \\ 2 & 1 \\ -3 & 5 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$, then find AB and BA.

32. If $A - B^T = \begin{bmatrix} 2 & 3 & 4 & -1 \\ 0 & 3 & -1 & 5 \end{bmatrix}$ and $A^T + B = \begin{bmatrix} 5 & 3 \\ 1 & 0 \\ 2 & 5 \\ 3 & -1 \end{bmatrix}$, then find the matrices A and B.

33. If $A = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then find AB and BA. What do you observe?

34. If $A = \begin{bmatrix} 2 & 3 & 5 & 6 \\ -1 & 2 & -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 & 3 & 7 \\ 8 & -3 & -1 & 2 \end{bmatrix}$, then find $A^T + B^T$ and $(A + B)^T$. What do you notice?

35. If $A = \begin{bmatrix} -1 & 3 \\ 4 & 5 \end{bmatrix}$, then find $(A - I)(A - 2I)$.

36. If $A = \begin{bmatrix} 8 & -4 \\ 12 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 12 & -8 \\ 8 & 8 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 8 \\ 4 & 8 \end{bmatrix}$, then show that $AC = BC$.

37. If $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 5 \\ -5 & 6 \end{bmatrix}$, then verify whether $A(B - C) = AB - AC$.

38. If $A = \begin{bmatrix} 1 & 7 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then find $A(B + C)$ and $AB + AC$.

39. Given $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 9 \\ 8 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}$ find $(AB)C$ and $A(BC)$. What do you notice?

40. If $A = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$, then show that $A^2 = I$.

41. If $A = \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix}$, then show that $A^2 + 4A - 7I = O$.

42. If $A = \begin{bmatrix} -1 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$, then verify whether $(A + B)(A - B) = A^2 - B^2$.

43. If $P = \begin{bmatrix} ab & bc \\ ca & ab \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then show that $P^2 - 2abP = ba(c^2 - ab)I$.

44. If $\begin{bmatrix} 1 & -1 & 2 \\ 3 & a & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} b \\ 14 \end{bmatrix}$, then find $(a + b)$.

45. If $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $Q = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$, then show that $(PQ)^T = Q^T P^T$.

PRACTICE QUESTIONS

Essay Type Questions

46. If $P = \begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix}$ and $f(x) = x^2 - 2x + 2$, then find $f(P)$.

47. If $A\begin{bmatrix} -10 & 11 \\ 6 & -3 \end{bmatrix} = [-32 \;\; 46]$, then find A.

48. If $A = \begin{bmatrix} -5 & 3 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 14 & -13 \\ 1 & 3 \\ 14 & -3 \end{bmatrix}$, then find the matrix X such that $AX = B$.

49. If $P = \begin{bmatrix} 2005 & 2004 \\ 2004 & 2005 \end{bmatrix}$, then find X such that $PX = XP = P$.

50. If $x = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$ and $g(x) = x^2 + x - 2$, find $g(x)$.

CONCEPT APPLICATION

Level 1

1. If $\begin{pmatrix} -2 & -1 \\ 3a & b \\ 4 & -6+x \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 9 & -1 \\ 4 & 2 \end{pmatrix}$, then $\dfrac{x}{a+b} =$

(a) $\dfrac{1}{4}$ (b) $\dfrac{-1}{11}$

(c) -11 (d) 4

2. If $A = \begin{pmatrix} -5 & 7 \\ 3 & -8 \end{pmatrix}$ and $B = \begin{pmatrix} -12 & 24 \\ -36 & 52 \end{pmatrix}$, then find matrix X such that $A + X = B$.

(a) $\begin{pmatrix} -7 & 17 \\ 39 & -60 \end{pmatrix}$ (b) $\begin{pmatrix} -7 & 17 \\ -39 & 60 \end{pmatrix}$

(c) $\begin{pmatrix} -7 & 17 \\ 39 & 60 \end{pmatrix}$ (d) $\begin{pmatrix} -7 & -17 \\ 39 & 60 \end{pmatrix}$

3. If $A = (a \; -a \; b \; -b)$, then $\left(\dfrac{1}{ab}\right)A =$

(a) $\begin{pmatrix} \frac{1}{b} & \frac{1}{b} & \frac{1}{a} & \frac{-1}{a} \end{pmatrix}$

(b) $\begin{pmatrix} \frac{1}{a} & \frac{-1}{a} & \frac{1}{b} & \frac{-1}{b} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{1}{b} & \frac{-1}{b} & \frac{1}{a} & \frac{-1}{a} \end{pmatrix}$

(d) $(a^2b \quad -a^2b \quad b^2a \quad -b^2a)$

4. If the matrix $\begin{bmatrix} a & a+b \\ a+b+c & a+b+c+d \end{bmatrix}$ is symmetric, then which of the following holds good?

(a) $a = 0$ (b) $b = 0$

(c) $c = 0$ (d) $d = 0$

5. If $A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 2 \\ 5 & 9 \\ 3 & 4 \end{bmatrix}$, then $2A + 7B =$

(a) $\begin{bmatrix} 41 & 10 \\ 40 & 60 \\ 26 & 38 \end{bmatrix}$ (b) $\begin{bmatrix} -41 & 10 \\ 41 & 61 \\ 26 & 38 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & 5 \\ 30 & -60 \\ 24 & 26 \end{bmatrix}$ (d) $\begin{bmatrix} -41 & 10 \\ 41 & 61 \\ 25 & 38 \end{bmatrix}$

6. If $A = \begin{pmatrix} 1 & -3 & 4 \\ 2 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} -2 & -4 & 5 \\ 1 & -1 & 3 \end{pmatrix}$ and $5A - 3B + 2X = O$, then $X =$

(a) $\begin{pmatrix} -11 & 3 & -5 \\ -7 & -8 & 19 \end{pmatrix}$

(b) $\dfrac{1}{2}\begin{pmatrix} -11 & 3 & -5 \\ -7 & -8 & 19 \end{pmatrix}$

(c) $\dfrac{1}{2}\begin{pmatrix} 11 & -3 & 5 \\ 7 & 8 & -19 \end{pmatrix}$

(d) None of these

7. If the matrix $\begin{bmatrix} a+b & 0 & 0 \\ 0 & b+c & 0 \\ 0 & 0 & c+a \end{bmatrix}$, (where a, b, c are positive integers) is a scalar matrix, then the value of $(a+b+c)$ can be

(a) 6 (b) 8

(c) 5 (d) 7

8. If $A = \begin{bmatrix} 1 & 4 & -3 \\ 5 & 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -7 \\ 4 & 5 \end{bmatrix}$, then

(a) AB exists. (b) BA exists.

(c) $(A + B)$ exists. (d) $(A - B)$ exists.

9. If $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ is a

(a) scalar matrix.

(b) symmetric matrix.

(c) skew symmetric.

(d) diagonal matrix.

10. If $A = \begin{bmatrix} 2 & 4 \\ -3 & 7 \end{bmatrix}$ and $B = A^T$, then $A^T + B^T$ is

(a) $\begin{bmatrix} 4 & 1 \\ 1 & 14 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 8 \\ 7 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -6 \\ 8 & 14 \end{bmatrix}$ (d) None of these

11. If A and B are commute, then $(A + B)^2 =$

(a) $A^2 + B^2$ (b) $A^2 + 2AB + B^2$

(c) $A^2 - B^2$ (d) $A^2 - 2AB + B^2$

12. If $A = [2 \quad -3 \quad 1]$ and $B = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$, then find $2A^T + B$.

(a) $\begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix}$ (b) $\begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 \\ -8 \\ 0 \end{bmatrix}$

13. If $A = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ and $B = [3 \quad 4 \quad 5]$, then AB is

(a) $\begin{bmatrix} 3p & 4p & 5p \\ 3q & 4q & 5q \\ 3p & 4q & 5r \end{bmatrix}$

(b) $\begin{bmatrix} 3p & 3q & 3p \\ 4p & 4q & 4q \\ 5p & 5p & 5r \end{bmatrix}$

(c) $\begin{bmatrix} 3p & 4p & 5p \\ 3q & 4q & 5q \\ 3r & 4r & 5r \end{bmatrix}$

(d) None of these

14. If the orders of matrices A^T, B and C^T are 3×4, 2×3 and 1×2 respectively, then the order of the matrix $(AB^T)C$ is

(a) 3×2 (b) 2×3

(c) 4×2 (d) 4×1

15. A is a 2×2 matrix, such that $A = [a_{ij}]$, where $a_{ij} = 2i - j + 1$. The matrix A is

(a) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

16. If $A + B = \begin{bmatrix} 10 & -11 \\ 9 & 7 \end{bmatrix}$ and $A - B = \begin{bmatrix} -8 & 9 \\ 9 & -5 \end{bmatrix}$, then $B =$

(a) $\begin{bmatrix} 9 & -10 \\ 0 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & 10 \\ 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & -10 \\ 0 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 10 \\ 0 & 6 \end{bmatrix}$

17. If $A = \begin{pmatrix} -2 & -1 \\ -5 & -3 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 1 \\ 5 & -2 \end{pmatrix}$ and $(AB)^n = I$, then n is (a/an)

(a) odd number. (b) even number.

(c) $\forall n \in N$. (d) None of these

PRACTICE QUESTIONS

18. If $A=\begin{pmatrix}4 & 3\\-5 & 2\end{pmatrix}$, then $A^2-6A=$

(a) $23I$ (b) $24I$

(c) $-23I$ (d) $-24I$

19. If $A=\begin{pmatrix}1 & 5\\3 & 6\end{pmatrix}$ and $B=\begin{pmatrix}34\\39\end{pmatrix}$, then find the matrix X such that $AX=B$.

(a) $\begin{pmatrix}0\\7\end{pmatrix}$ (b) $\begin{pmatrix}7\\-1\end{pmatrix}$

(c) $\begin{pmatrix}-1\\7\end{pmatrix}$ (d) $\begin{pmatrix}9\\-1\end{pmatrix}$

20. If $\begin{pmatrix}2 & 3\\p & 1\end{pmatrix}\begin{pmatrix}5 & 2\\-4 & 6\end{pmatrix}=\begin{pmatrix}-2 & q\\16 & r\end{pmatrix}$, then $2p+r=$

(a) q (b) 21

(c) $2q$ (d) $q-r$

21. If $A=\begin{pmatrix}2 & -2\\-2 & 2\end{pmatrix}$, $B=\begin{pmatrix}5 & 3\\-7 & 2\end{pmatrix}$ and $C=\begin{pmatrix}4 & 2\\-8 & 1\end{pmatrix}$, then

(a) $AB=BC$ (b) $AB=AC$

(c) $BC=AC$ (d) None of these

22. If $A=\begin{bmatrix}-5 & -3 & 4\\3 & 2 & -4\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 5 & -2\\3 & 1 & 5\end{bmatrix}$, then find X such that $3A-2B+X=0$.

(a) $\begin{bmatrix}7 & 19 & 16\\3 & 4 & 22\end{bmatrix}$ (b) $\begin{bmatrix}-7 & -19 & 16\\3 & 4 & 22\end{bmatrix}$

(c) $\begin{bmatrix}7 & 19 & -16\\-3 & -4 & 22\end{bmatrix}$ (d) $\begin{bmatrix}7 & 19 & 16\\-3 & -4 & 22\end{bmatrix}$

23. If $A+B=\begin{bmatrix}7 & 6\\-3 & 2\end{bmatrix}$ and $A-B=\begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}$, then find A.

(a) $\begin{bmatrix}4 & 4\\0 & 4\end{bmatrix}$ (b) $\begin{bmatrix}8 & 8\\0 & 8\end{bmatrix}$

(c) $\begin{bmatrix}-4 & -4\\0 & -4\end{bmatrix}$ (d) $\begin{bmatrix}-8 & -8\\0 & -8\end{bmatrix}$

24. If $B-A^T=\begin{bmatrix}3 & 4 & -2\\5 & -3 & 7\end{bmatrix}$ and $B^T+A=\begin{bmatrix}2 & 0\\5 & -1\\3 & 4\end{bmatrix}$, then find Matrix A.

(a) $\begin{bmatrix}1/2 & -3/2\\1/2 & 1\\-5/2 & 3/2\end{bmatrix}$ (b) $\begin{bmatrix}1/2 & 3/2\\1/2 & 1\\-5/2 & 3/2\end{bmatrix}$

(c) $\begin{bmatrix}-1/2 & -5/2\\1/2 & 1\\5/2 & -3/2\end{bmatrix}$ (d) $\begin{bmatrix}1/2 & 3/2\\-1/2 & -1\\-5/2 & 3/2\end{bmatrix}$

25. If $2A-3B=\begin{pmatrix}-27 & 4 & 5\\7 & 6 & -15\end{pmatrix}$ and $5A-2B=\begin{pmatrix}-40 & -1 & 18\\12 & 15 & -21\end{pmatrix}$, then $B=$

(a) $\begin{pmatrix}55 & -22 & -11\\-11 & 0 & 33\end{pmatrix}$ (b) $\begin{pmatrix}5 & -2 & 1\\-1 & 0 & 3\end{pmatrix}$

(c) $\begin{pmatrix}-55 & 22 & 11\\11 & 0 & -33\end{pmatrix}$ (d) $\begin{pmatrix}-5 & -2 & -1\\1 & 0 & 3\end{pmatrix}$

26. If $A=\begin{bmatrix}-1 & 2\\3 & 4\end{bmatrix}$ and $B=A^T$, then $A^T+B^T=$

(a) $\begin{bmatrix}-3 & 4\\6 & 8\end{bmatrix}$ (b) $\begin{bmatrix}-2 & 0\\0 & 4\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-2 & 5\\5 & 8\end{bmatrix}$

27. $A_x=\begin{bmatrix}0 & 0 & x\\0 & 0 & 0\end{bmatrix}$, then find the maximum number of possibilities of a matrix B_x, in which x can be placed in a_{11}, a_{21}, or a_{31}, position such that $A_xB_x=O_{2\times1}$.

(a) 1 (b) 2

(c) 3 (d) Cannot be determined

28. If $A-2B=\begin{bmatrix}3 & 6\\7 & 6\end{bmatrix}$ and $A-3B=\begin{bmatrix}2 & 6\\7 & 5\end{bmatrix}$, then the matrix A is

(a) $\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix}$ (b) $\begin{bmatrix}5 & 8\\7 & 6\end{bmatrix}$

(c) $\begin{bmatrix}6 & 5\\7 & 8\end{bmatrix}$ (d) $\begin{bmatrix}5 & 8\\2 & 6\end{bmatrix}$

29. Which of the following matrices satisfies the equation $A^2 + A = O$?

(a) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(c) $A = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$ (d) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

30. If $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ -1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & -1 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find $2A + 3B - 7C$.

(a) $\begin{bmatrix} 2 & 4 & 23 \\ 8 & 0 & 8 \\ -2 & -5 & 16 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 4 & 23 \\ 8 & -2 & 8 \\ -2 & -5 & -16 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 & 23 \\ 8 & -2 & -8 \\ -2 & 5 & -16 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 4 & -23 \\ 8 & -2 & 8 \\ -2 & -5 & 16 \end{bmatrix}$

Level 2

31. If $A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 7 & -1 \end{bmatrix}$, find the matrix C such that $C = AB + A^2B$.

(a) $\begin{bmatrix} -96 & 21 \\ 36 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 96 & 21 \\ 36 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} -96 & -21 \\ 36 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} -96 & 21 \\ 36 & 7 \end{bmatrix}$

32. If $A = \begin{pmatrix} 3 & 1 \\ -4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 1 \\ 4 & -5 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$, then

(a) $AB = AC$ (b) $AC = BC$

(c) $BC = CB$ (d) $A(B + C) = AB + AC$

33. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$, then $AB^2 =$

(a) $\begin{bmatrix} -3 & -4 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -4 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & -4 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 4 \\ 1 & 0 \end{bmatrix}$

34. If $X = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, and $X^2 - 3X =$

(a) I (b) $3I$

(c) $4I$ (d) $-4I$

35. If $A = \begin{bmatrix} -3 & -2 \\ -1 & 3 \end{bmatrix}$, then $A^2 - 11I =$

(a) A (b) I

(c) A^2 (d) O

36. If $A = \begin{bmatrix} a-3 & -5 \\ c+1 & b-2 \end{bmatrix}$ is a skew-symmetric matrix, then $a + b - c =$ ________.

(a) 2 (b) 1

(c) −1 (d) 0

37. If $A = \begin{bmatrix} 0 & y \\ y & 0 \end{bmatrix}$ which of the following is true?

(a) $A^3 = \begin{bmatrix} 0 & y^3 \\ y^3 & 0 \end{bmatrix}$

(b) $A^4 = \begin{bmatrix} y^4 & 0 \\ 0 & y^4 \end{bmatrix}$

(c) Both (a) and (b)

(d) Neither (a) nor (b)

38. If $\begin{bmatrix} 4 & -5 \\ 3 & 6 \end{bmatrix} + 2X = \begin{bmatrix} 8 & -1 \\ -7 & 2 \end{bmatrix}$, then $X^T =$

(a) $\begin{bmatrix} 2 & 2 \\ -5 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -5 \\ 2 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -5 \\ -2 & 2 \end{bmatrix}$

39. If $A = \begin{bmatrix} p & 2 \\ -4 & -5 \end{bmatrix}$, $B = \begin{bmatrix} q & r-s \\ r & -5 \end{bmatrix}$ and $A = B$. Find $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$, if the trace of $A + B = -2$.

(a) $\begin{bmatrix} 2 & 6 \\ -4 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4 \\ -4 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 5 \\ -4 & -7 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

40. If $[3 \quad 7]\begin{bmatrix} 1 & -2 \\ -2 & x \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = [13]$, then the value of x is

(a) 0 (b) −1

(c) 1 (d) 2

41. If $A = \begin{bmatrix} 0 & x \\ 1 & 0 \end{bmatrix}$, such that $A^2 = 4I$, then x is

(a) 4 (b) 3

(c) 2 (d) 1

42. If $A = \frac{1}{\sqrt{3}}\begin{bmatrix} -8 & 3 \\ 3 & 8 \end{bmatrix}$, then $A^5 =$

(a) I (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) A

43. Which of the following is square root of $\begin{bmatrix} 16 & 4 \\ 3 & 13 \end{bmatrix}$?

(a) $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 16 & 4 \\ 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -4 \\ -3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 16 & -4 \\ 3 & -11 \end{bmatrix}$

44. If $5A - 2B = \begin{bmatrix} 11 & 2 & -5 \\ 4 & -3 & 6 \end{bmatrix}$ and $-2A + B = \begin{bmatrix} 4 & -2 & 4 \\ -5 & 3 & 1 \end{bmatrix}$, then $A =$

(a) $\begin{bmatrix} -19 & 2 & 3 \\ -6 & 3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 19 & -2 & 3 \\ -6 & 3 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 19 & 2 & 3 \\ 6 & 3 & 8 \end{bmatrix}$ (d) None of these

45. If $[4 \quad 5]\begin{bmatrix} 1 & 6 \\ x & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = [9]$, then the value of x is

(a) 9 (b) −9

(c) 10 (d) −10

46. If $A = \begin{bmatrix} 5 & -6 \\ 2 & 4 \end{bmatrix}$ and $B = A^T$, then $A^T - B^T$ is ______.

(a) $\begin{bmatrix} 0 & 6 \\ 6 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

47. If $A = \begin{bmatrix} 8 \\ -1 \\ -7 \end{bmatrix}$ and $B = [2 \quad 5 \quad -6]$, then find $3A + B^T$.

(a) $\begin{bmatrix} 26 \\ 2 \\ -27 \end{bmatrix}$ (b) $\begin{bmatrix} 36 \\ 4 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 18 \\ 2 \\ -9 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 26 \\ -27 \end{bmatrix}$

48. If $A = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$, then find $A^n (n \in N)$ ______.

(a) $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7^n & 0 \\ 0 & 7^n \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 7^n \\ 0 & 7^n \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 7^n \\ 7^n & 0 \end{bmatrix}$

49. If $A = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$, then A^{n+1} is ______.

(a) $\begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$ (b) $\begin{bmatrix} p^{n+1} & 0 \\ 0 & p^{n+1} \end{bmatrix}$

(c) $\begin{bmatrix} np & 0 \\ 0 & np \end{bmatrix}$ (d) None of these

50. If $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, then $\frac{A + A^T}{2}$ is ______.

(a) symmetric (b) skew symmetric

(c) diagonal (d) unit matrix

PRACTICE QUESTIONS

Level 3

51. If $A = \begin{bmatrix} p & -1 \\ q & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$ and $(A + B)^2 = A^2 + 2AB + B^2$, then $p - q =$ ________.

(a) 2 (b) −1

(c) 3 (d) 1

52. If $P = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & -2 \\ -3 & 0 \end{bmatrix}$ and $R = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$, then find $PQ + PR$.

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix}$

53. If $\begin{pmatrix} a & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ b & 8 \end{pmatrix} = \begin{pmatrix} 30 & 20 \\ 52 & c \end{pmatrix}$, then find $2a + b - c$.

(a) 16 (b) 15

(c) 12 (d) −20

54. If $P = [3 \quad 2 \quad 1]$, $Q = [1 \quad 2 \quad 3]$, $R = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $S = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, then $PR + QS =$

(a) [20] (b) [10]

(c) [15] (d) [12]

55. If $y = \begin{pmatrix} 4 & -2 \\ 3 & 0 \end{pmatrix}$ and $f(y) = y^2 - y - 6$, find $f(y)$.

(a) $\begin{pmatrix} 0 & 6 \\ 9 & 12 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 6 \\ 9 & -12 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & -6 \\ 9 & -12 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 6 \\ 9 & -12 \end{pmatrix}$

56. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$, then $A^{n+1} =$ ________.

(a) $\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 5 & 3 \end{bmatrix}$

(c) $2^n \begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$ (d) $2^{n+1} \begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

57. If $A(x) = \begin{bmatrix} e^x & e^x \\ e^{-x} & e^{-x} \end{bmatrix}$, then $A(x)\,A(y) =$

(a) $A(x + y)\,A(x - y)$

(b) $A(x + y) + A(x - y)$

(c) $A(x + y) - A(x - y)$

(d) $A(x + y)$

58. If $A = \begin{bmatrix} 6 & -7 & 1 \\ 3 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 & 2 \\ -4 & 3 & -5 \end{bmatrix}$ then find X such that $2A - B + X = O$.

(a) $\begin{bmatrix} 7 & 6 & 0 \\ 1 & 3 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 7 & 0 \\ 10 & 3 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 15 & 0 \\ -10 & 7 & -7 \end{bmatrix}$

59. If $\begin{bmatrix} 2 & 4 \\ p & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & q \\ -2 & r \end{bmatrix}$, then $pq =$ ______.

(a) $4(r - 1)$ (b) $5r$

(c) $4r + 2$ (d) r

60. If $A = \begin{bmatrix} -13 & 24 \\ -7 & 13 \end{bmatrix}$, then $A^8 =$ ________.

(a) A (b) I

(c) O (d) None of these

61. If $A + B = \begin{bmatrix} 8 & -9 \\ 3 & 5 \end{bmatrix}$ and $A - B = \begin{bmatrix} 6 & 5 \\ 1 & 7 \end{bmatrix}$, then $A =$ ______.

(a) $\begin{bmatrix} 7 & -2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} -7 & 2 \\ 2 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & -2 \\ -2 & -6 \end{bmatrix}$

62. If $A = \begin{bmatrix} 7 & 9 \\ 3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 13 \\ 18 \end{bmatrix}$, then find the matrix X such that $AX = B$.

(a) $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

63. If $A = \begin{bmatrix} a & b \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then $(b, a) =$ _______.

(a) $(1, -1)$ (b) $(-1, 1)$

(c) $(1, 1)$ (d) $(-1, 0)$

64. If $A = \begin{bmatrix} 6 & x \\ 7 & -6 \end{bmatrix}$ and $A^2 = I$, then $x =$ _______.

(a) 5 (b) $\frac{-5}{2}$

(c) -5 (d) 1

65. If $A - B = \begin{bmatrix} 18 & 1 & -9 \\ 11 & 12 & 2 \end{bmatrix}$ and $A + 3B = \begin{bmatrix} 26 & 1 & 3 \\ -13 & -16 & 10 \end{bmatrix}$, then find A.

(a) $\begin{bmatrix} 20 & 1 & 6 \\ 5 & 5 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 20 & 1 & 6 \\ -5 & -5 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 20 & 1 & -6 \\ 5 & 5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 20 & 1 & -6 \\ -5 & -5 & -4 \end{bmatrix}$

66. If $A = \begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -1 \\ -8 & 2 \end{bmatrix}$, then $AB + BA =$ _______.

(a) $\begin{bmatrix} -42 & 22 \\ 40 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} -42 & -22 \\ -40 & -10 \end{bmatrix}$

(c) $\begin{bmatrix} -42 & 22 \\ -40 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} 42 & 22 \\ 40 & 10 \end{bmatrix}$

67. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $f(x) = x^2 - 4x + 3$, then find $f(A)$.

(a) $\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$

68. If $[5 \quad 6]\begin{bmatrix} 1 & -3 \\ -1 & x \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = [-15]$, then the value of x is _______.

(a) 4 (b) 2

(c) 3 (d) 1

69. If $A = \begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -1 \\ 3 & 2 \end{bmatrix}$, then find the matrix C such that $C = AB + B^2$.

(a) $\begin{bmatrix} 37 & 25 \\ 16 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 27 & -17 \\ 48 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 5 \\ 17 & 16 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -15 \\ 6 & 18 \end{bmatrix}$

70. If $A - B^T = \begin{bmatrix} -2 & 2 & 1 \\ 3 & 6 & 5 \end{bmatrix}$ and $A^T + B = \begin{bmatrix} -4 & 5 \\ 8 & -2 \\ 9 & 1 \end{bmatrix}$, then matrix $A =$ _______.

(a) $\begin{bmatrix} 3 & -5 & -5 \\ 6 & 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 & 5 \\ 3 & 5 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & 5 & 7 \\ 3 & 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 5 & 5 \\ 4 & 2 & 3 \end{bmatrix}$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 2×3
2. 3
3. 16
4. 3×5
5. 0
6. $\begin{bmatrix} 12 & 5 \\ -1 & 4 \end{bmatrix}$
7. $\begin{bmatrix} 4 & -3 \\ 11 & 11 \end{bmatrix}$
8. $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$
9. $\begin{bmatrix} k & 2k \\ 3k & 4k \end{bmatrix}$
10. A
11. 3×4
12. It is not a symmetric matrix
13. skew-symmetric
14. $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$
15. $\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$
16. [10]
17. null
18. $\begin{bmatrix} 2k + 2l & -k - l & 0 \\ 9k + 9l & 2k + 2l & 4k + 4l \\ 6k + 6l & 3k + 3l & -9k - 9l \end{bmatrix}$
19. $\begin{bmatrix} 13 & 2 \\ -5 & 3 \end{bmatrix}$
20. $\begin{bmatrix} 38 & 48 \\ 58 & 68 \end{bmatrix}$
21. $x = \dfrac{3}{5}$, $y = 0$ and $z = 12$
22. $\begin{bmatrix} 18 & 34 \\ 39 & 46 \end{bmatrix}$
23. A is a symmetric matrix.
24. $A = \begin{bmatrix} 1 & 0 & -1 \\ 4 & 3 & 2 \end{bmatrix}$
25. $\begin{bmatrix} 155 & 67 & -79 \\ 130 & -11 & -133 \end{bmatrix}$
26. $\begin{bmatrix} 23 & -1 & -31 & 34 \\ 33 & -23 & -34 & -63 \end{bmatrix}$
28. $a = 3$, $b = 6$, $p = 8$ and $q = 10$
29. 4×1
30. $X = \begin{bmatrix} 5 & -4 \\ 4 & 2 \end{bmatrix}, Y = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

Short Answer Type Questions

31. $\begin{bmatrix} 26 & 37 \\ 1 & 20 \end{bmatrix}, \begin{bmatrix} 17 & 7 & 21 & 16 & 33 \\ 6 & 3 & 8 & 7 & 13 \\ 17 & -11 & 1 & -30 & -13 \\ 8 & 7 & 14 & 17 & 25 \\ 9 & 0 & 7 & -1 & 8 \end{bmatrix}$
32. $A = \begin{bmatrix} 7/2 & 2 & 3 & 1 \\ 3/2 & 3/2 & 2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3/2 & 3/2 \\ -1 & -3/2 \\ -1 & 3 \\ 2 & -3 \end{bmatrix}$

33. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

We can observe that $AB = BA = O$ and none of the two matrices A or B is null matrix.

ANSWER KEYS

34. $(A+B)^T = (A^T + B^T)$

35. $(A-I)(A-2I) = \begin{bmatrix} 18 & 3 \\ 4 & 24 \end{bmatrix}$

38. $\begin{bmatrix} 20 & 41 \\ 13 & 31 \end{bmatrix}, \begin{bmatrix} -10 & 37 \\ -21 & 40 \end{bmatrix} \begin{bmatrix} 20 & 41 \\ 13 & 31 \end{bmatrix}$

39. $\begin{bmatrix} 35 & -105 \\ -95 & -1 \end{bmatrix},$

$(AB)C = A(BC)$

44. 5

Essay Type Questions

46. $f(P)\begin{bmatrix} 5 & 0 \\ -8 & 5 \end{bmatrix}$

47. [5 3]

48. $\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

49. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

50. $\begin{bmatrix} 1 & -3 \\ 9 & -5 \end{bmatrix}$

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (a)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (b)

Level 2

31. (d)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (b)	**39.** (b)	**40.** (c)
41. (a)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (a)

Level 3

51. (b)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (d)	**59.** (a)	**60.** (b)
61. (a)	**62.** (c)	**63.** (b)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (b)	**70.** (d)

CONCEPT APPLICATION

Level 1

1. Find a, b and x by equating corresponding elements of the matrices.
2. Subtract A from B.
3. Apply scalar product of a matrix.
4. If A is symmetric matrix, then $A^T = A$.
5. First find the scalar product and then add the two matrices.
6. $2X = 3B - 5A$.
7. In a scalar matrix, principal diagonal elements are same.
8. Verify the addition and multiplication rules.
9. Find A^T and $-A$.
10. Substitute 'B' in $A^T + B^T$ and find the sum.
11. If A and B are commute, then $AB = BA$.
12. Find A^T and find the sum.
13. Multiply the matrices A and B.
14. If A is $m \times n$ matrix, B is $n \times p$ matrix, then the order of AB is $m \times p$.
15. (i) $A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

    (ii) $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

    (iii) Find the values of a_{11}, a_{12}, a_{21} and a_{22} by substituting the values of i and j in the given relation.
16. $2B = (A + B) - (A - B)$.
17. (i) Find the product of AB.

    (ii) Find AB, $(AB)^2$ and so on till you get I to know the value of n.
18. (i) Find the square of A, i.e., $(A \times A)$.

    (ii) $A^2 - 6A = A(A - 6I)$.
19. (i) Find the order of X using product rule.

    (ii) Order of X is 2×1.

    (iii) Verify from the options.
20. (i) Equate the corresponding elements on both sides.

    (ii) Find the product of the two matrices which are on LHS.

    (iii) Equate the corresponding elements and get the values p, q and r, then find $2p + r$.
21. (i) Find the products AB, BC, AC.

    (ii) Find the products AB, BC and AC and verify from options.
22. (i) $X = 2B - 3A$

    (ii) $3A - 2B + X = 0$

    $\Rightarrow X = 2B - 3A$.

    (iii) Multiply each element of B with 2 to get $2B$.

    (iv) Multiply each element of A with 3 to get $3B$.
23. (i) Add the given matrix equations.

    (ii) Add $(A - B)$ to $(A + B)$ to find matrix A.
24. (i) Transpose $B^T + A = \begin{bmatrix} 2 & 0 \\ 5 & -1 \\ 3 & 4 \end{bmatrix}$ and add it to the other equation.

    (ii) $(B - A^T)^T = B^T - A$.

    (iii) Subtract $(B^T - A)$ from $B^T + A$.
25. (i) Solve as linear equations.

    (ii) Multiply $2A - 3B$ by 5.

    (iii) Multiply $5A - 2B$ by 2 and then add to get matrix B.
26. Find A^T and B^T and add the two matrices.
27. Check for the possibilities of B_x such that $A_x B_x = O$.
28. (i) Multiply $A - 2B$ by 3.

    (ii) Multiply $A - 3B$ by 2, then subtract to get A.
29. (i) Take A as common.

    (ii) $A^2 + A = 0$

    $\Rightarrow \quad A(A + I) = 0$

    (iii) From the options $A \neq 0$, $A + I = 0$.
30. (i) Evaluate $2A$, $3B$, $7C$ and simplify.

    (ii) $2A$ means multiply each element of A with 2.

    (iii) $3B$ means multiply each element of B with 3.

    (iv) Similarly find $7C$, then find $2A + 3B - 7C$.

Level 2

31. (i) $C = A(B + AB)$.

(ii) $C = AB + A^2B = A(B + AB)$.

32. (i) Try from the choices.

(ii) Find the products AB, BC and AC and verify from options.

33. (i) $AB^2 = (AB)B$.

(ii) Find B^2, then find AB^2.

34. (i) Evaluate X^2, $3X$.

(ii) $X^2 - 3X = X(X - 3I)$.

35. (i) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(ii) Find $A \times A$.

(iii) Subtract $\begin{pmatrix} 11 & 0 \\ 0 & 11 \end{pmatrix}$ from $(A \times A)$.

40. (i) Multiply matrices of LHS and then equate it to [13].

(ii) Find the product of first two matrices and then multiply product matrix with third matrix.

(iii) Equate the above resultant matrix to [13].

41. (i) Find A^2, then equating the corresponding elements in $4I$ Matrix.

(ii) Evaluate $A \times A$.

(iii) Equate the above product to $4I$, i.e., $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ and find x.

42. (i) Evaluate A^2.

(ii) Find A^2 and A^3, and induce it to find A^5.

43. (i) Try from the option.

(ii) Find the square of each matrix in the options.

(iii) Compare it with given matrix to find the square root.

44. (i) $A = (5A - 2B) + 2(-2A + B)$.

(ii) Multiply $-2A + B$ with 2.

(iii) Add $(5A - 2B)$ and $(-4A + 2B)$ to obtain A.

45. (i) Evaluate the product in LHS and equate it to RHS.

(ii) Find the product of first two matrices and then multiply product matrix with third matrix of LHS.

(iii) Equate the above resultant matrix with [9] and find the value of x.

46. $A = \begin{bmatrix} 5 & -6 \\ 2 & 4 \end{bmatrix}$

$$B = A^T = \begin{bmatrix} 5 & 2 \\ -6 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 2 \\ -6 & 4 \end{bmatrix}; B^T = \begin{bmatrix} 5 & -6 \\ 2 & 4 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 5-5 & 2+6 \\ -6-2 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}.$$

47. Given that $A = \begin{bmatrix} 8 \\ -1 \\ -7 \end{bmatrix}$

$$B = [2 \quad 5 \quad -6]$$

$$B^T = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}, 3A = \begin{bmatrix} 24 \\ -3 \\ -21 \end{bmatrix}$$

$$3A + B^T = \begin{bmatrix} 24+2 \\ -3+5 \\ -6-22 \end{bmatrix} = \begin{bmatrix} 26 \\ 2 \\ -27 \end{bmatrix}.$$

48. Given $A = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$A - 7 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = 7I$$

$$A^n = 7^n I^n$$

$$A^n = 7^n I \ (\because I^n = I)$$

$$= 7^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7^n & 0 \\ 0 & 7^n \end{bmatrix}.$$

49. Given $A = \begin{bmatrix} P & 0 \\ 0 & P \end{bmatrix}$

$A = P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = PI$

$A^{n+1} = P^{n+1} \times I^{n+1} = P^{n+1}\, I \; (\because I^n = I)$

$= P^{n+1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} P^{n+1} & 0 \\ 0 & P^{n+1} \end{bmatrix}.$

50. $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}, A^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 5+5 & 6+7 \\ 7+6 & 8+8 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 10 & 13 \\ 13 & 16 \end{bmatrix}$

$\frac{1}{2}(A + A^T) = \begin{bmatrix} \frac{10}{2} & \frac{13}{2} \\ \frac{13}{2} & \frac{16}{2} \end{bmatrix} = \begin{bmatrix} 5 & \frac{13}{2} \\ \frac{13}{2} & 8 \end{bmatrix}$

$\frac{1}{2}[(A + A^T)]^T = \frac{1}{2}[A + A^T]$

$\therefore$ It is symmetric.

Level 3

56. $A = \begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

$\begin{bmatrix} 25-15 & -15+9 \\ 25-15 & -15+9 \end{bmatrix}$

$\begin{bmatrix} 10 & -6 \\ 10 & -6 \end{bmatrix} = 2\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

$A^3 = A^2 \cdot A$

$= \begin{bmatrix} 10 & -6 \\ 10 & -6 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

$= \begin{bmatrix} 50-30 & -30+18 \\ 50-30 & -30+18 \end{bmatrix} = \begin{bmatrix} 20 & -12 \\ 20 & -12 \end{bmatrix}$

$= 4\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix} = 2^2\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}$

$A^{n+1} = 2^n\begin{bmatrix} 5 & -3 \\ 5 & -3 \end{bmatrix}.$

57. $A(x) = \begin{bmatrix} e^x & e^x \\ e^{-x} & e^{-x} \end{bmatrix}$

$A(y) = \begin{bmatrix} e^y & e^y \\ e^{-y} & e^{-y} \end{bmatrix}$

$A(x)\, A(y) = \begin{bmatrix} e^{x+y} + e^{x-y} & e^x e^y + e^x e^{-y} \\ e^{-x}e^y + e^{-x}e^{-y} & e^{-x}e^y + e^{-x}e^{-y} \end{bmatrix}$

$= \begin{bmatrix} e^{x+y} + e^{x-y} & e^{x+y} + e^{x-y} \\ e^{-x+y} + e^{-x-y} & e^{-x+y} + e^{-x-y} \end{bmatrix}$

$= \begin{bmatrix} e^{x+y} + e^{x-y} & e^{x+y} + e^{x-y} \\ e^{-(x-y)} + e^{-(x+y)} & e^{-(x-y)} + e^{-(x+y)} \end{bmatrix}$

$= A\,(x + y) + A\,(x - y).$

58. $A = \begin{bmatrix} 6 & -7 & 1 \\ 3 & -2 & 1 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 12 & -14 & 2 \\ 6 & -4 & 2 \end{bmatrix}$

Given that $2A - B + X = O$

$2A - B + X - 2A + B = I + B - 2A$

(Adding B and subtracting $2A$)

$X = O + B - 2A$

$= O^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 1 & 2 \\ -4 & 3 & -5 \end{bmatrix}$

$X = \begin{bmatrix} 0+5-12 & 0+1+14 & 0+2-2 \\ 0-4-6 & 0+3+4 & 0-5-2 \end{bmatrix}$

$= \begin{bmatrix} -7 & 15 & 0 \\ -10 & 7 & -7 \end{bmatrix}.$

59. Given $\begin{bmatrix} 2 & 4 \\ p & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & q \\ -2 & r \end{bmatrix}$

$$\begin{bmatrix} -2+12 & 4+4 \\ -p+3 & 2p+1 \end{bmatrix} = \begin{bmatrix} 10 & q \\ -2 & r \end{bmatrix}$$

$$\begin{bmatrix} 10 & 8 \\ -p+3 & 2p+1 \end{bmatrix} = \begin{bmatrix} 10 & q \\ -2 & r \end{bmatrix}$$

$-p + 3 = -2$, $q = 8$, $r = 2p + 1$

$-p = -5$, $r = 2(5) + 1$

$p = 5$, $r = 11$.

Therefore $pq = 5(8) = 40 = 4(r - 1)$.

60. $A = \begin{bmatrix} -13 & 24 \\ -8 & 13 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -13 & 24 \\ -7 & 13 \end{bmatrix}\begin{bmatrix} -13 & 24 \\ -7 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 169-168 & -312+312 \\ 91-91 & -168+169 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^2 = I$

$(A^2)^4 = I^4$

$A^8 = I$.

61. $A + B = \begin{bmatrix} 8 & -9 \\ 3 & 5 \end{bmatrix}$ (1)

$A - B = \begin{bmatrix} 6 & 5 \\ 1 & 7 \end{bmatrix}$ (2)

Adding Eqs. (1) and (2) we get

$$A + B + A - B = \begin{bmatrix} 8 & -9 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 1 & 7 \end{bmatrix}$$

$$2A = \begin{bmatrix} 14 & -4 \\ 4 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{14}{2} & \frac{-4}{2} \\ \frac{4}{2} & \frac{12}{2} \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 2 & 6 \end{bmatrix}.$$

62. Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$AX = B \Rightarrow \begin{bmatrix} 7 & 9 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} 7x+9y \\ 3x+8y \end{bmatrix} = \begin{bmatrix} 13 \\ 18 \end{bmatrix}$$

$7x + 9y = 13$ (1)

$3x + 8y = 18$ (2)

Solving Eqs. (1) and (2), we get

$x = -2$ and $y = 3$

$\therefore \; x = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

63. $A + B = \begin{bmatrix} a & b \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$

$$= \begin{bmatrix} a+1 & b+1 \\ 6 & -2 \end{bmatrix}.$$

$$(A+B)^2 = \begin{bmatrix} a+1 & b+1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} a+1 & b+1 \\ 6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (a+1)^2 + 6(b+1) & (a+1)(b+1) - 2(b+1) \\ 6(a+1) - 12 & 6(b+1) + 4 \end{bmatrix}.$$

$$A^2 = \begin{bmatrix} a & b \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+2b & ab-b \\ 2a-2 & 2b+1 \end{bmatrix}.$$

$$B^2 = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1+4 & 1-1 \\ 4-4 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}.$$

$$A^2 + B^2 = \begin{bmatrix} 5+a^2+2b & ab-b \\ 2a-2 & 6+2b \end{bmatrix}$$

Given $(A + B)^2 = A^2 + B^2$

$\therefore \; 6(a + 1) - 12 = 2a - 2$

$6a + 6 - 12 = 2a - 2$

$6a - 6 = 2a - 2$

$4a = 4$

$a = 1$.

$6(b + 1) + 4 = 6 + 2b$

$6b + 6 + 4 = 6 + 2b$

$4b = -4$

$b = -1.$

$(b, a) = (-1, 1).$

64. $A = \begin{bmatrix} 6 & x \\ 7 & -6 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 6 & x \\ 7 & -6 \end{bmatrix}\begin{bmatrix} 6 & x \\ 7 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 36+7x & 6x-6x \\ 42-42 & 7x+36 \end{bmatrix}$$

$$= \begin{bmatrix} 36+7x & 0 \\ 0 & 36+7x \end{bmatrix}.$$

Given, $A^2 = I \Rightarrow \begin{bmatrix} 36+7x & 0 \\ 0 & 36+7x \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore\ 36 + 7x = 1$

$7x = -35$

$x = -5.$

65. $A - B = \begin{bmatrix} 18 & 1 & -9 \\ 11 & 12 & 2 \end{bmatrix}$ (1)

$A + 3B = \begin{bmatrix} 26 & 1 & 3 \\ -13 & -16 & 10 \end{bmatrix}$ (2)

$3\times(1) \Rightarrow 3A - 3B = \begin{bmatrix} 54 & 3 & -27 \\ 33 & 36 & 6 \end{bmatrix}$ (3)

Add Eqs. (2) and (3), we get

$A + 3B + 3A - 3B$

$$= \begin{bmatrix} 26 & 1 & 3 \\ -13 & -16 & 10 \end{bmatrix} + \begin{bmatrix} 54 & 3 & -27 \\ 33 & 36 & 6 \end{bmatrix}$$

$$4A = \begin{bmatrix} 80 & 4 & -24 \\ 20 & 20 & 16 \end{bmatrix}$$

$$A = \begin{bmatrix} 20 & 1 & -6 \\ 5 & 5 & 4 \end{bmatrix}.$$

66. Given $A = \begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -1 \\ -8 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} -6 & -1 \\ -8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times-6+4\times-8 & 1\times-1+4\times2 \\ -2\times-6+5\times-8 & -2\times-1+5\times2 \end{bmatrix}$$

$$= \begin{bmatrix} -6-32 & -1+8 \\ +12-40 & 2+10 \end{bmatrix} = \begin{bmatrix} -38 & 7 \\ -28 & 12 \end{bmatrix}.$$

$$BA = \begin{bmatrix} -6 & -1 \\ -8 & 2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -6+2 & -24-5 \\ -8-4 & -32+10 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -29 \\ -12 & -22 \end{bmatrix}.$$

$$AB + BA = \begin{bmatrix} -38-4 & 7-29 \\ -28-12 & 12-22 \end{bmatrix}$$

$$= \begin{bmatrix} -42 & -22 \\ -40 & -10 \end{bmatrix}.$$

67. Given $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$f(x) = x^2 - 4x + 3$

$f(A) = A^2 - 4A + 3I$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4+0 & 2+3 \\ 0+0 & 0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix}.$$

$$4A = \begin{bmatrix} 8 & 4 \\ 0 & 12 \end{bmatrix}$$

$$3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A^2 - 4A + 3I = \begin{bmatrix} 4-8+3 & 5-4+0 \\ 0-0+0 & 9-12+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}.$$

68. $[5 \quad 6]\begin{bmatrix} 1 & -3 \\ -1 & x \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = [-15]$

$$[5-6-15+6x]\begin{bmatrix} 3 \\ 4 \end{bmatrix} = [-15]$$

$$[-1-15+16x]\begin{bmatrix}3\\4\end{bmatrix}=[-15]$$

$[-3 + 4\ (6x - 15)] = [-15]$

$[-3 + 24x - 60] = [-15]$

$[24x - 63] = [-15]$

$24x - 63 = -15$

$24x = 63 - 15$

$24x = 48$

$x = 2.$

69. $AB=\begin{bmatrix}1 & -4\\3 & 2\end{bmatrix}=\begin{bmatrix}6 & -1\\3 & 2\end{bmatrix}$

$$=\begin{bmatrix}6-12 & -1-8\\18+6 & -3+4\end{bmatrix}$$

$$=\begin{bmatrix}-6 & -9\\24 & 1\end{bmatrix}.$$

$$B^2=\begin{bmatrix}6 & -1\\3 & 2\end{bmatrix}=\begin{bmatrix}6 & -1\\3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}36-3 & -6-2\\18+6 & -3+4\end{bmatrix}$$

$$=\begin{bmatrix}33 & -8\\24 & 1\end{bmatrix}.$$

$C = AB + B^2$

$$=\begin{bmatrix}-6+33 & -9-8\\24+24 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}27 & -17\\48 & 2\end{bmatrix}.$$

70. $A-B^T=\begin{bmatrix}-2 & 2 & 1\\3 & 6 & 5\end{bmatrix}$

$$(A-B^T)^T=\begin{bmatrix}-2 & 3\\2 & 6\\1 & 5\end{bmatrix}$$

$$A^T-B=\begin{bmatrix}-2 & 3\\2 & 6\\1 & 5\end{bmatrix} \quad (1)$$

$$\text{and } A^T+B=\begin{bmatrix}-4 & 5\\8 & -2\\9 & 1\end{bmatrix} \quad (2)$$

Add Eqs. (1) and (2),

$$A^T-B+A^T+B=\begin{bmatrix}-2 & 3\\2 & 6\\1 & 5\end{bmatrix}+\begin{bmatrix}-4 & 5\\8 & -2\\9 & 1\end{bmatrix}$$

$$2A^T=\begin{bmatrix}-6 & 8\\10 & 4\\10 & 6\end{bmatrix}$$

$$A^T=\begin{bmatrix}-3 & 4\\5 & 2\\5 & 3\end{bmatrix}$$

$$A=\begin{bmatrix}-3 & 5 & 5\\4 & 2 & 3\end{bmatrix}.$$

Chapter 8

Significant Figures

REMEMBER

Before beginning this chapter, you should be able to:

- Know rational and irrational numbers
- Work on decimal system
- Understand how to apply basic operations on numbers

KEY IDEAS

After completing this chapter, you should be able to:

- Round off to a certain number of decimal places
- Concept of significant figures
- Find addition and subtraction values of significant figures
- Calculate multiplication and division of significant figures
- Understand absolute error and relative error

INTRODUCTION

Numerical work is of two kinds—Computation with exact numbers and computation with approximate numbers. The numbers 1, 2, 3, 1.21, 1.41 are examples of exact numbers. Numbers like π, $\sqrt{2}$, $\sqrt{3}$, e are also exact numbers—when they are written in this manner. But when 1.41 is used instead of $\sqrt{2}$, it is an approximate number. Similarly 3.14 and 2.72 are approximate values of π and e respectively. We write $\sqrt{2} \approx 1.41$, $\pi \approx 3.14$ and $e \approx 2.72$. In the decimal notation we use the digits (or figures) 0, 1,…, 9 to represent numbers, i.e., both exact and approximate.

The digits that are used to express a number are called significant digits or significant figures. In an exact number, any initial zeroes on the left of the decimal point and terminal zeroes on the right of the decimal point are not significant.

Before we see which digits of an approximate number are significant, we shall see how we round off numbers to a certain number of decimal places. The table given below shows how to round off numbers to two or three decimal places:

Number	Number Rounded off to 2 Decimal Places	Number Rounded off to 3 Decimal Places
7.0034	7.00	7.003
7.1004	7.10	7.100
6.3249	6.32	6.325
5.0155	5.02	5.016
5.0145	5.01	5.015
4.0049	4.00	4.005
2.1266	2.13	2.127

We use the following rules to round off numbers to 2 decimal places:

1. If the digit in the third decimal place is 5 or more than 5, we increase the digit in the second decimal place by 1. Thus in the seventh row in the table above, we round off 2.1266 as 2.13 and write $2.1266 \approx 2.13$.
2. If the digit in the third decimal place is less than 5, we leave the digit in the second place as it is. Thus in the first row in the table above, we round off 7.0034 as 7.00 and write $7.0034 \approx 7.00$.

Similarly, we can round off numbers to m decimal places by considering the digit in the $(m + 1)$th place. We can see from the above examples, (first, second and sixth rows in the table above) that the terminal zeroes in approximate numbers on the right of the decimal point do signify something.

Thus in approximate numbers, all non-zero digits and all zeroes which lie between significant digits as well as terminal zeroes (on the right of the decimal point) are significant.

To decide whether terminal zeroes on the left of the decimal point or initial zeroes to the right of the decimal point are significant or not, we need further information. We try to understand this with the following example:

Suppose several values of a particular quantity are given and we are required to consider all these values together, we ensure that all the values have the same number of digits after the decimal place (say n). If the largest number has m digits before the decimal place we say that there are $m + n$ significant digits in these values. Even if we express these values in smaller or bigger units the number of significant digits does not change. We consider the following examples to understand this:

Object	Length (in mm)	Length (in m)	Length (in km)	Length (in mm)
A	$1\,0\,\underline{0}\,\underline{0}$	1.0	$0.\,\underline{0}01$	$0.\,\underline{0}\,\underline{0}\,\underline{0}\,\underline{0}\,0\,1$
B	$2\,0\,2\,\underline{0}\,\underline{0}$	20.2	$0.\,\underline{0}202$	$0.\,\underline{0}\,\underline{0}\,\underline{0}\,\underline{0}\,2\,0\,2$
C	$3\,5\,7\,8\,\underline{0}\,\underline{0}$	357.8	0.3578	$0.\,\underline{0}\,\underline{0}\,\underline{0}\,3\,5\,7\,8$

In the table above, none of the zeroes that have been underlined are significant. They appear only because the values that have been measured in metres are being reported in smaller (mm) or in bigger (Mm) units. An Mm is a megametre or 10^6 m.

When numbers are expressed in the exponential (or scientific) notation, i.e., as $N(10^K)$, the significant figures are figures which appear in N.

Example: 1.02 (10^{12}) has 3 significant figures.

$2.000(10^{15})$ has 4 significant figures.

Examples:

1. The distance between two places is 1600 km, correct to the nearest hundred km.
   Then, the unit of measurement is one hundred km.
   $\therefore$ 1600 km = (16) hundred km.
   $\therefore$ 1, 6 are the significant figures in this case.

2. The length of a line segment is 32 cm, correct to the nearest cm.
   Then, the unit of measurement is cm
   $\therefore$ 3, 2 are the significant figures.

3. The length of a line segment is 6.0 cm, correct to the nearest mm.
   Then, the unit of measurement is mm.
   $\therefore$ 6.0 cm = 60 mm
   $\therefore$ 6, 0 are significant figures.

Thus, to decide whether terminal zeroes before the decimal point or initial zeroes after the decimal point are significant, we have to see whether they are coming from the measured value or from using a smaller or bigger unit (respectively) than the unit used in the measurement. The zeroes which are there in the measured value are significant. These which come from the change in unit are not.

When there is no reference to the measurement, we assume that the given value is the measured value. Specifically, we assume that terminal zeroes in whole numbers and initial zeroes after the decimal point are not significant.

Example: 7200 has 2 significant figures, they are 7 and 2 and 0.001 has 1 significant figure, i.e., 1.

Serial Number	Comment	Examples	Number of Significant Figures
1.	The digits 1, 2, 3, …, 9 are all significant	47532 14.23	5 4
2.	Zeroes occurring between non-zero digits are significant.	10.4062 50.01	6 4
3.	The ending zeroes of an approximated decimal numbers are significant.	327.200 10.0	6 3

(Continued)

(*Continued*)

Serial Number	Comment	Examples	Number of Significant Figures
4.	In the notation $N(10^k)$, all the digits which appear in N are significant.	13.05×110^6 110.00×10^{-5}	4 5
5.	The zeroes that indicate the location of the decimal point alone are not significant.	0.00013 0.01230	2 (only 1, 3) 4 (only 1, 2, 3, 0)
6.	The ending zeroes of a whole number are not always significant, unless it is specified.	1400 ml 1400 ml (correct to nearest integers)	2 (only 1, 4) 4 (1, 4, 0, 0)

Addition and Subtraction

Rule: When adding or subtracting, the result can only show as many decimal places as the measurement showing the least number of decimal places.

EXAMPLE 8.1

Add 632.73 and 24.082.

SOLUTION

$$\begin{array}{r} 632.73 \\ +24.082 \\ \hline 656.812 \approx 656.81. \end{array}$$

EXAMPLE 8.2

Subtract 6.235 cm from 8.4 cm.

SOLUTION

$$\begin{array}{r} 8.4 \\ -6.235 \\ \hline 2.165 \approx 2.2. \end{array}$$

EXAMPLE 8.3

Evaluate (8.253 + 6.7289 − 2.334) and correct to 3 significant figures.

SOLUTION

$8.253 + 6.7289 - 2.334 = 12.6479$

$= 12.6$ (Correct to three significant figures).

Multiplication and Division

Rule: When multiplying or dividing, the result can have only AS MANY significant figures as the LEAST of the measurements used in the operation.

EXAMPLE 8.4

3.42×3.2

SOLUTION

$3.42 \times 3.2 = 10.944 \approx 10.9.$

EXAMPLE 8.5

$8.635 \div 0.25$

SOLUTION

$8.635 \div 0.25 = 34.54.$

ABSOLUTE ERROR AND RELATIVE ERROR

The absolute difference between the exact value and the approximated value of a number is called absolute error.

The absolute error divided by the exact value of a given number is called a relative error and it is represented as percentage.

EXAMPLE 8.6

A factory manufactured 2678 bolts and this is approximated to the nearest thousands. Find

(a) the absolute error.

(b) the relative error.

SOLUTION

When 2678 is approximated to the nearest thousand its value is 3,000.

(a) Absolute error = 3000 − 2678 = 322.

(b) Relative error $= \dfrac{322}{2678}(100\%) = 12.023\%$ or 12%.

EXAMPLE 8.7

Number of significant digits of the HCF of 0.5, 0.75 and 1.25 is _______.

(a) 1 **(b)** 2 **(d)** 3 **(d)** 4

SOLUTION

Given numbers are 0.5, 0.75 and 1.25.

That is, $\dfrac{50}{100}, \dfrac{75}{100}, \dfrac{125}{100}$

$$\text{HCF} = \frac{\text{HCF }(50, 75, 125)}{\text{LCM }(100, 100, 100)} = \frac{25}{100} = 0.25$$

Number of significant digits = 2.

EXAMPLE 8.8

There are certain number of students in a class. This is approximated to nearest hundreds. Approximate value is 600 and the absolute error is 25. If exact value is less than the approximated value, then find the relative error. (Correct to two decimal places).

(a) 4.36 **(b)** 3.37 **(c)** 4.35 **(d)** 4.34

SOLUTION

Approximate value = 600

Absolute error = 25

Exact value = 600 ± 25 = 575 or 625

$\therefore$ Relative error $= \dfrac{25}{575} \times 100$

~ 4.3478

~ 4.35.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Number of significant figures in 5.00 is _____.
2. Number of significant figures in 7.06 is _____.
3. The number of significant figures in 0.00203040 is ______.
4. Taking mm as the unit, the number of significant figures in 5.0 cm is ______.
5. Taking 10^3 as the unit, the number of significant figures in 1000 is ______.
6. Value of 0.072 correct to 1 significant figure is ______.
7. 378.4629 kg approximated to the nearest kg is ______.
8. 1638 is approximated to the nearest hundred. Absolute error = ______.
9. The difference between exact value and approximated value is _______.
10. The ratio of absolute error to the exact value is ______.

Short Answer Type Questions

11. 81.75 when expressed, correct to the nearest integer would be ____.
12. The unit of measurement is cm. If the measure is 5.00 metres, then find the number of significant figures in the measurement.
13. The unit of measurement is litre. If the measure is 4.000 kilolitres, then find the number of significant figures in the measurement.
14. Write the value of each of the following, correct to 4 significant figures.

    (i) 343.92 (ii) 0.0010829 (iii) 76.0065
15. The unit of measurement is mm. If the measure is 4.2 cm, then find the number of significant figures in the measurement.
16. 5268.75 when expressed correct to the nearest thousand would be ____.
17. Evaluate:

    (i) 6.02 + 3.7602 − 0.9327, correct to four significant figures.

    (ii) 0.529 − 42.78 + 70.062, correct to three significant figures.
18. Find the value of $\sqrt{5}$, correct to two decimal places. Use this value to evaluate $\frac{3}{\sqrt{5}}$, correct to two significant figures.
19. Express the fraction $\frac{7.566}{0.00600}$ in the form of $m \times 10^3$. Also find the expression correct to three significant figures.
20. By rationalizing the denominator, evaluate $\frac{1}{3+\sqrt{2}}$ correct to two significant figures. Given that $\sqrt{2} = 1.414$.

CONCEPT APPLICATION

Level 1

1. The value of 56.0023 corrected to four significant figures is

    (a) 56.00 (b) 56.01
    (c) 56.02 (d) 56.03
2. The number of significant figures in 0.00250 is

    (a) 2 (b) 3
    (c) 6 (d) 5
3. The value of 2.00885 corrected to four significant figures is ________.

    (a) 2.009

    (b) 2.008

    (c) 2.010

    (d) 2.018

4. The length of object measured to the nearest centimetre is 120 cm. If the length is expressed in mm, then the number of significant figures is
(a) 4 (b) 2
(c) 3 (d) 5

5. Find the sum of the significant digits of the LCM of 0.12, 0.18 and 0.24.
(a) 7 (b) 8
(c) 6 (d) 9

6. Find the sum $0.23 + 0.234 + 0.2345$ corrected to three significant figures.
(a) 0.697 (b) 0.677
(c) 0.699 (d) 0.688

7. The value obtained when $\frac{22}{7}$ rounded off to five significant figures is ______.
(a) 3.1427 (b) 3.1428
(c) 3.1426 (d) 3.1429

8. If $234.a42b6 \approx 234.a43$, then what can be the possible value of b?
(a) $5 < b < 10$
(b) $5 \leq b < 10$
(c) $1 < b < 9$
(d) None of these

9. The unit of measurement is one hundredth of 1 cm. If the measure is 0.09 cm, the number of significant figures is ______.
(a) 1 (b) 2
(c) 3 (d) 4

10. If there are three significant digits in $0.abc$, then what are the possible values of a, b and c?
(a) $a < 10$, $b \neq 0$ and $c \neq 0$
(b) $a = 0$, $b = 0$ and $c = 9$
(c) $a \neq 0$
(d) None of these

Level 2

11. The number of significant digits of 8.65000×10^{50} is ______.
(a) 6 (b) 3
(c) 50 (d) 55

12. If the number of significant digits of $abc \times 10^{-10}$ is 1, then which of the following holds good?
(a) $a = 0$, $b \neq 0$, $c = 0$
(b) $a = 0$, $b = 0$ and $0 < c < 10$
(c) Cannot be determined
(d) None of these

13. The number of zeros before the first significant digit of $\frac{1}{3125}$ is ______.
(a) 1 (b) 2
(c) 3 (d) 4

14. The value of $\sqrt{115}$ corrected to four significant figures is ______.
(a) 10.72 (b) 10.73
(c) 10.74 (d) 10.75

15. There are 25425 people in a town. This is approximated to the nearest thousands. Calculate the relative error (approximately).
(a) 0.1% (b) 0.3%
(c) 1.67% (d) 2.26%

16. After simplification, the first significant digit of the fraction $\frac{0.12345}{125}$ is ______.
(a) 6 (b) 7
(c) 8 (d) 9

17. A trader weighs 8.99 quintals of iron rods with the help of kilogram weights. Find the smallest possible limiting relative error. (approximately)
(a) 1.1%
(b) 0.11%
(c) 12.5%
(d) None of these

18. The value of $\frac{12}{\sqrt{5}-1}$ corrected to three significant digits is ______. $(\sqrt{5} = 2.2361)$

PRACTICE QUESTIONS

(a) 9.71 (b) 9.75

(c) 9.81 (d) 9.85

19. The value of $\frac{1}{5-\sqrt{3}}$ correct to two significant figures [given that $\sqrt{3} = 1.732$] is ______.

(a) 0.31 (b) 0.30

(c) 0.32 (4) 0.33

20. A pole is measured with a scale marked in centimetres. If the length of the pole is 16.52 m, then find the smallest possible limiting absolute error.

(a) 48 cm

(b) 52 cm

(c) 2 cm

(d) 1 cm

21. Evaluate 3.768 – 1.876 corrected to two significant figures.

(a) 1.89 (b) 1.9

(c) 2.89 (d) 1.98

22. When a number is approximated to the nearest hundreds, the approximated value is 600 and absolute error is 24. Then the exact number is ______.

(a) 624

(b) 548

(c) 576

(d) Either (a) or (c)

23. The distance between two places is 2000 km, correct to nearest thousand km. Then number of significant figures is ______.

(a) 4 (b) 3

(c) 2 (d) 1

24. The height of a person is 6.0 ft, correct to nearest foot (in integer) then number of significant figures is ______.

(a) 2 (b) 1

(c) 3 (d) Cannot be determined

25. Number of significant figures in the square root of 42.25 is ______.

(a) 4 (b) 3

(c) 2 (d) 1

26. Evaluate (0.304)(0.12) corrected to four significant figures.

(a) 0.03648

(b) 0.3648

(c) 0.0365

(d) None of these

27. Which of the following numbers has two significant figures?

(a) 24 (b) 240

(c) 2400 (d) All of these

28. The length of a pole is 42.3 m. Find the number of significant figures, when its length is expressed in kilometres.

(a) 2 (b) 3

(c) 1 (d) 4

29. Number of significant figures in 3.0 is ______.

(a) 1 (b) 3

(c) 4 (d) 2

30. Evaluate $\frac{3}{7}$ correct to three decimal places.

(a) 0.428

(b) 0.429

(c) 0.431

(d) 0.421

31. Evaluate $\frac{15}{\sqrt{2}-1}$ correct to four significant digits (Take $\sqrt{2} = 1.414$).

(a) 32.63

(b) 32.36

(c) 36.23

(d) 36.32

32. Evaluate $\sqrt{80}$ correct to three significant digits.

(a) 8.944 (b) 8.94

(c) 8.95 (d) 8.945

PRACTICE QUESTIONS

33. After simplification, numbers of zeroes before first significant digit of $\left(\frac{0.03125}{25}\right)$ is ______.

(a) 4 (b) 3

(c) 2 (d) 1

34. The distance between two places is measured with a scale marked in metres. If the distance is 2.013 m, then find the number of significant digits in it.

(a) 1 (b) 2

(c) 3 (d) 4

35. There are certain number of students in a class. This is approximated to nearest hundreds. Approximate value is 600 and the absolute error is 25. If exact value is more than the approximate value, then find the relative error.

(a) 4% (b) 4.5%

(c) 5% (d) 5.5%

36. Evaluate $\frac{1}{3+\sqrt{7}}$ correct to one significant digit (Take $\sqrt{7} = 2.646$).

(a) 0.2 (b) 0.1

(c) 0.3 (d) 0.02

37. Evaluate 2.304×23.05 correct to four significant figures.

(a) 53.1072 (b) 53.11

(c) 53.1172 (d) 53.12

38. If number of significant digits of $a \times 3a$ is 3, then which of the following is true? (where a is whole number)

(a) $a = 0$

(b) $0 \leq a \leq 9$

(c) $0 < a < 9$

(d) $0 < a \leq 9$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 3
2. 3
3. 6
4. 2
5. 1
6. 0.07
7. 378 kg
8. 38
9. absolute error
10. relative error

Short Answer Type Questions

11. 82
12. 3
13. 4
14. (i) 343.9
    (ii) 0.001083
    (iii) 76.01
15. 2
16. 5000
17. (i) 8.848 (ii) 27.8
18. 1.3
19. 1.26×10^3
20. 0.23

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (c)

Level 2

11. (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (d)
21. (b)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (b)
31. (c)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (d)		

HINTS AND EXPLANATION

CONCEPT APPLICATION

Level 1

1. The number *ab.cdef* correct to four significant digits is *ab.cd* if $0 \le e \le 4$.
4. There is no change in the number of significant figures in a measurement, even if it is changed from one system to another system.
5. (i) Apply the concept of LCM of decimals and then find the required sum.
   (ii) Write the significant digits of three numbers.
   (iii) Find the LCM.
   (iv) Add the sum of the digits of the number obtained by the LCM.
6. (i) Add the given decimals and correct the sum to three significant figures.
   (ii) Round off to 3 significant figures after adding the numbers.
7. The number *a.bcdef* correct to five significant digits is *a.bcde* if $0 \le f \le 4$.
8. Check from the options.
10. Check from the options.

Level 2

11. (i) Consider 10^{50} as one unit and find the number of significant digits in the given number.
    (ii) Assume 10^{50} as 1 digit.
    (iii) The significant figures in $N(10^k)$ is the number of significant figures in N.
12. (i) Assume 10^{-10} as 1 digit.
    (ii) The significant figures is $N(10^k)$ is the number of significant figures in N.
13. Convert the fraction into decimal and proceed.
14. Find $\sqrt{115}$ by division method.
15. (i) Relative error
    $$= \frac{\text{Absolute error}}{\text{Original population}} \times 100.$$
    (ii) Absolute error = Exact value − Nearest thousand.
    (iii) Relative error
    $$= \frac{\text{Absolute error}}{\text{Exact value}} \times 100.$$
17. (i) Smallest possible limiting relative error
    $$= \frac{\text{Smallest possible limiting error}}{\text{Actual weight}} \times 100.$$
    (ii) Convert the weight into kilograms.
    (iii) Find the absolute error.
    (iv) Relative error
    $$= \frac{\text{Absolute error}}{\text{Exact value}} \times 100.$$
18. (i) Rationalize the denominator.
    (ii) Put $\sqrt{5} = 2.2361$ and simplify, correct to three significant figures.
19. (i) Rationalize the denominator.
    (ii) Put $\sqrt{3} = 1.732$ and simplify to two significant figures.
20. (i) Convert the given length into centimetres.
    (ii) Find a number nearer to thousand of the given length.
    (iii) Find the difference between the two numbers.
21. $$\begin{array}{r} 3.768 \\ (-)\ 1.876 \\ \hline 1.892 \sim 1.9. \end{array}$$
    (Corrected to two significant figures).
22. Let the exact number be x and its approximated value = 600
    Absolute error = 24.
    $\therefore x = 600 \pm 24 = 624$ or 576.

23. The distance between two places = 2000 km = (2) thousand km.

$\therefore$ Number of significant figures = 1.

24. The height of a person is 6.0 ft.

When it is corrected to nearest ft, the height is 6 ft.

$\therefore$ Number of significant digit is 1.

25. $\sqrt{42.25} = 6.5.$

Number of significant digits = 2.

26. $0.304 \times 0.12 = 0.03648$

There are four significant figures in the result.

$\therefore$ The required result is 0.03648.

27. We have, ending zeroes of exact numbers are not significant.

$\therefore$ In each of the given options significant figures are 2

$\therefore$ Choice (d) follows.

28. The length of a pole is 42.3 m = 0.0423 km, number of significant figures is 3.

29. Significant figures of 3.0 are 3, 0.

Number of significant figures is 2.

30. $\frac{3}{7} \sim 0.42857$

~ 0.429 ($\because$ corrected to three significant figures.)

31. $\frac{15}{\sqrt{2}-1} = \frac{15(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{15(\sqrt{2}+1)}{2-1}$

$= 15\left(1.414+1\right) = 36.23.$

32. $\sqrt{80} \sim 8.94427$

~ 8.94

($\because$ Corrected to three significant digits.)

33. $\frac{0.03125}{25} = 0.00125$

Number of zeroes = 2.

34. In 2.013, there are 4 significant digits.

35. Approximate value = 600

Absolute error = 25

Exact value = 600 ± 25 = 575 or 625

$\therefore$ Relative error $= \frac{25}{625} \times 100 = 4\%.$

36. $\frac{1}{3+\sqrt{7}} = \frac{3-\sqrt{7}}{(3+\sqrt{7})(3-\sqrt{7})}$

$= \frac{3-\sqrt{7}}{9-7} = \frac{3-2.646}{2} = \frac{0.354}{2} = 0.177$

$\sim 0.2.$

($\because$ Corrected to one significant digit.)

37. $2.304 \times 23.05 = 53.1072$

~ 53.11 ($\because$ corrected to four significant figures).

38. Number of significant digits of $a \times 3a$ is 3.

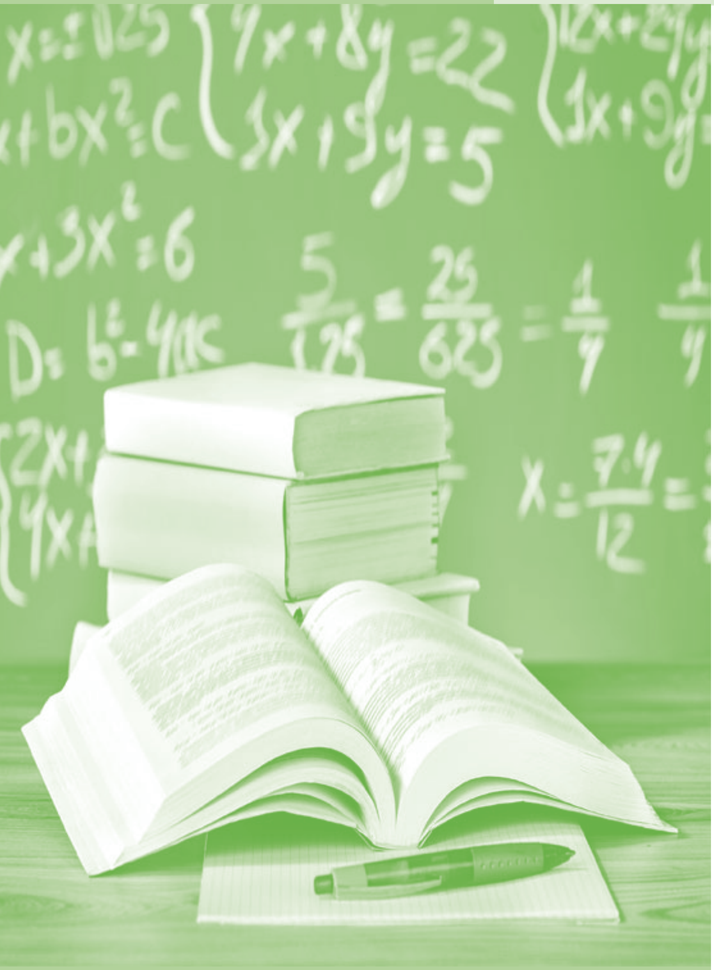
$\because$ a cannot be 0

$\because$ $0 < a \leq 9$ and $a \in N$.

HINTS AND EXPLANATION

Chapter 9

Statistics



REMEMBER

Before beginning this chapter, you should be able to:

- Know basic terms like data and information
- Understand the presentation of data in daily life

KEY IDEAS

After completing this chapter, you will be able to:

- Study about data, types of data, tabulation of data and statistical graphs
- Understand the measures of central tendencies for grouped and ungrouped data such as mean, median and mode
- Study range, quartiles, and estimation of median and quartiles from ogives
- Find mean deviation

INTRODUCTION

The word 'statistics' is derived from the Latin word 'status' which means political state. Political states had to collect information about their citizens to facilitate Governance and plan for the development. Then, in course of time, statistics came to mean a branch of mathematics which deals with the collection, classification and analysis of numerical data.

In this chapter, we shall learn about the classification of data, *viz*., grouped and ungrouped, measures of central tendencies and their properties.

DATA

The word 'data' means information in the form of numerical figures or a set of given facts.

For example, the percentage of marks scored by 10 pupils of a class in a test are:

36, 80, 65, 75, 94, 48, 12, 64, 88 and 98.

The set of these figures is the data related to the marks obtained by 10 pupils in a class test.

Types of Data

Statistics is basically the study of numerical data. It includes methods of collection, classification, presentation, analysis of data and inferences from data. Data as such can be qualitative or quantitative in nature. If one speaks of honesty, beauty, colour, etc., the data is qualitative while height, weight, distance, marks, etc., are quantitative. Data can also be classified as raw data and grouped data.

Raw Data

Data obtained from direct observation is called raw data. The marks obtained by 10 students in a monthly test is an example of raw data or ungrouped data.

In fact, very little can be inferred from this data. So, to make this data clearer and more meaningful, we group it into ordered intervals.

Grouped Data

To present the data in a more meaningful way, we condense the data into convenient number of classes or groups, generally not exceeding 10 and not less than 5. This helps us in perceiving, at a glance, certain salient features of data.

Some Basic Definitions

Before getting into the details of tabular representation of data, let us review some basic definitions:

Observation

Each numerical figure in a data is called an observation.

Frequency

The number of times a particular observation occurs is called its frequency.

Tabulation or Presentation of Data

A systematical arrangement of the data in a tabular form is called tabulation or presentation of the data. This grouping results in a table called the frequency table which indicates the number of

scores within each group. Many conclusions about the characteristics of the data, the behaviour of variables etc., can be drawn from this table.

The quantitative data that is to be analysed statistically, can be divided into three categories:

1. Individual series
2. Discrete series
3. Continuous series

Individual Series

Any raw data that is collected, form is an individual series.

Examples:

1. Runs scored by 6 batsmen in a test match:

   13, 89, 56, 24, 48 and 17

2. Number of students in different classes in a school:

   54, 46, 52, 38, 45, 50, 29, 60, 35, 56

Discrete Series

A discrete series is formulated from the raw data by taking the frequency of the observations into consideration.

Example: Given below is the data showing the number of children in 10 families of a locality:

1, 1, 2, 2, 3, 1, 2, 3, 2, 2

Arranging the data in the ascending order, we get 1, 1, 1, 2, 2, 2, 2, 2, 3, 3.

To count, we can use tally marks. We record tally marks in bunches of five, the fifth one crossing the other four diagonally, i.e., $\cancel{\vert\vert\vert\vert}$.

Thus, we may prepare a frequency table as below:

Number of Children	Tally Marks	Number of Families (Frequency)
1	$\vert\vert\vert$	3
2	$\cancel{\vert\vert\vert\vert}$	5
3	$\vert\vert$	2

Continuous Series

When the data contains large number of observations, we put them into different groups, called class intervals such as, 1–10, 11–20, 21–30, etc.

Here, 1–10 means data whose values lie between 1 and 10, including both 1 and 10.

This form is known as an **inclusive form**. Also, 1 is called the **lower limit** and 10 is called the **upper limit**.

EXAMPLE 9.1

Given below are the ages of 40 people in a colony:

33	8	7	28	30	25	6	50	24	44
56	32	27	21	17	62	58	16	14	19
24	31	27	5	12	46	15	42	67	34
4	21	10	40	20	50	48	63	9	21

Taking class intervals 1–10, 11–20, 21–30, 31–40, 41–50, 51–60 and 61–70, we construct a frequency distribution table for the above data.

SOLUTION

First, we write the ages in the ascending order as

4	5	6	7	8	9	10	12	14	15
16	17	19	20	21	21	21	24	24	25
27	27	28	30	31	32	33	34	40	42
44	46	48	50	50	56	58	62	63	67

Now, we can prepare the frequency distribution table as below:

Class Interval	Tally Marks	Frequency
1–10	~~||||~~ ||	7
11–20	~~||||~~ ||	7
21–30	~~||||~~ ~~||||~~	10
31–40	~~||||~~	5
41–50	~~||||~~ |	6
51–60	||	2
61–70	|||	3

Class Interval

A group into which the raw data is condensed is called a class interval.

Each class is bounded by two figures, which are called the class limits. The figure on the LHS is called the lower limit and the figure on the RHS is called the upper limit of the class. Thus 0–10 is a class with lower limit being 0 and the upper limit being 10.

Class Boundaries

In an exclusive form, the lower and upper limits are known as class boundaries or true lower limit and true upper limit of the class respectively.

Thus, the boundaries of 15–25 which is in exclusive form are 15 and 25.

The boundaries in an inclusive form are obtained by subtracting 0.5 to the lower limit and adding 0.5 to the upper limit, if the difference between upper limit of a class and lower limit or the succeeding class is 1.

Thus, the boundaries of 15–25 which is in inclusive form are 14.5–25.5.

Class Size

The difference between the true upper limit and the true lower limit is called the class size. Hence, in the above example, the class size = 25 − 15 = 10.

Class Mark or Mid-value

Class mark = $\frac{1}{2}$(upper limit + lower limit).

Thus, the class mark of 15–25 is $\frac{1}{2}(25+15)=20$.

STATISTICAL GRAPHS

The information provided by a numerical frequency distribution is easily understood when represented by diagrams or graphs. The diagrams act as visual aids and leave a lasting impression on the mind. This enables the investigator to make quick conclusions about the distribution.

There are different types of graphs or diagrams to represent statistical data. Some of them are:

1. Bar chart or bar graph (for unclassified frequency distribution)
2. Histogram (for classified frequency distribution)
3. Frequency polygon (for classified frequency distribution)
4. Frequency curve (for classified frequency distribution)
5. Cumulative frequency curves (for classified frequency distribution)
   (i) Less than cumulative frequency curve
   (ii) Greater than cumulative frequency curve

Bar Graph

The important features of bar graphs are:

1. Bar graphs are used to represent unclassified frequency distributions.
2. Frequency of a value of a variable is represented by a bar (rectangle), whose length (i.e., height) is equal (or proportional) to the frequency.
3. The breadth of the bar is arbitrary and the breadth of all the bars are equal. The bars may or may not touch each other.

EXAMPLE 9.2

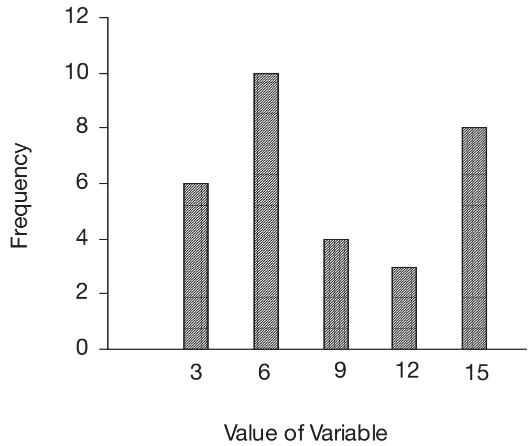
Represent the following frequency distribution by bar graph:

Value of Variable	3	6	9	12	15
Frequency	6	10	4	3	8

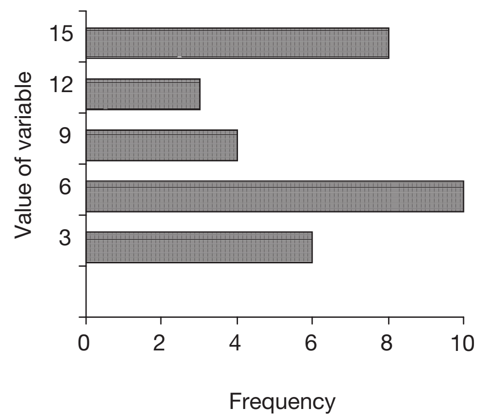
SOLUTION

Either of the following bar graphs [Fig. 9.1(a) or Fig. 9.2(b)] may be used to represent the above frequency distribution. The first graph takes value of the variable along the *X*-axis and the frequency along the *Y*-axis, whereas the second one takes the frequency along the *X*-axis and the value of the variable on the *Y*-axis.

All the rectangles (bars) should be of same width and uniform spaces should be left between any two consecutive bars.



(a)



(b)

Figure 9.1

Histograms

Classified or grouped data is represented graphically by histograms. A histogram consists of rectangles each of which has its breadth proportional to the size of concerned class interval and its height proportional to the corresponding frequency. In a histogram, two consecutive rectangles have a common side.

Hence, in a histogram, we do the following:

1. We represent class boundaries along the *X*-axis.
2. Along the *Y*-axis, we represent class frequencies.
3. We construct rectangles with bases along the *X*-axis and heights along the *Y*-axis.

EXAMPLE 9.3

Construct a histogram for the frequency distribution below:

Class Interval	10–20	20–30	30–40	40–50	50–60
Frequency	6	3	8	4	7

SOLUTION

Here, the class intervals are continuous. The following histogram is drawn according to the method described in Fig 9.2.

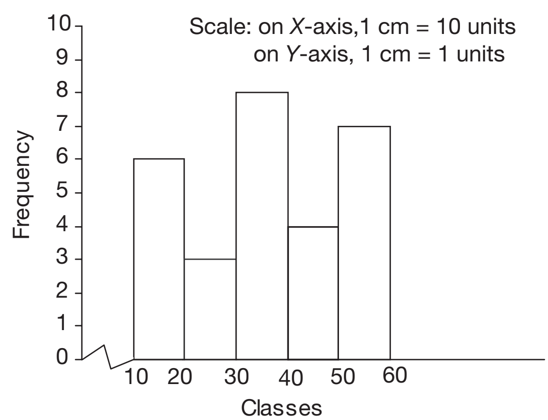


Figure 9.2

Remarks

The following points may be noted:

1. A link mark (∿) made on the horizontal axis, between the vertical axis and first vertical rectangle, if there is a gap between 0 and the lower boundary of first class interval.
2. We may shade all rectangles. A heading for the histogram may also be given.

Important Observations

1. If the class intervals are discontinuous, the distribution has to be changed into continuous intervals and then the histogram has to be drawn.
2. Bar graphs are used for unclassified frequency distributions, whereas histograms are used for classified frequency distribution. The breadths of rectangles in a bar graph are arbitrary, while those in histogram are determined by class size.

Frequency Polygon

Frequency polygons are used to represent classified or grouped data graphically. It is a polygon whose vertices are the mid-points of the top sides of the rectangles, forming the histogram of the frequency distribution.

To draw a frequency polygon for a given frequency distribution, the mid values of the class intervals are taken on *X*-axis and the corresponding frequencies on *Y*-axis and the points are plotted on a graph sheet. These points are joined by straight line segments which form the frequency polygon.

EXAMPLE 9.4

Construct a frequency polygon for the following data:

Class Interval	5–10	11–16	17–22	23–28	29–34	Total
Frequency	7	13	8	12	10	50

SOLUTION

Here, the class intervals are discontinuous. Hence, first we convert the class intervals to continuous class intervals and then find mid-points of each class interval. We do this by adding 0.5 to each upper limit and subtracting 0.5 from each lower limit.

Class Interval	Exclusive	Mid-value of Class	Frequency
5–10	4.5–10.5	7.5	7
11–16	10.5–16.5	13.5	13
17–22	16.5–22.5	19.5	8
23–28	22.5–28.5	25.5	12
29–34	28.5–34.5	31.5	10

Now, taking the mid-values of class intervals on the *X*-axis and the corresponding frequencies on the *Y*-axis, we draw a frequency polygon as shown in the Fig. 9.3.

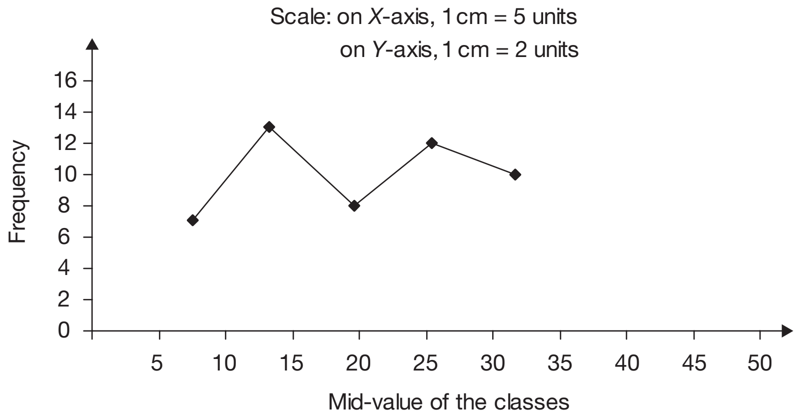


Figure 9.3

Frequency Curve

Frequency curves are used to represent classified or grouped data graphically. As the class-interval in a frequency distribution decreases, the points of the frequency polygon become closer and closer and then the frequency polygon tends to become a frequency curve. So, when the number of scores in the data is sufficiently large and the class intervals become smaller (ultimately tending to zero), the limiting form of frequency polygon becomes frequency curve.

EXAMPLE 9.5

Draw a frequency curve for the data given below:

Class Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	3	2	4	6	9	7	10

SOLUTION

Following table shows the mid-values of classes and corresponding frequencies for the given data:

Class Interval	Mid-value	Frequency
0–10	5	3
10–20	15	2
20–30	25	4
30–40	35	6
40–50	45	9
50–60	55	7
60–70	65	10

Now, taking the mid-values of the classes along the *X*-axis and the corresponding frequencies along the *Y*-axis, we mark the points obtained from the above table in a graph sheet and join them with a smooth curve, which gives the frequency curve as shown in the Fig. 9.4.

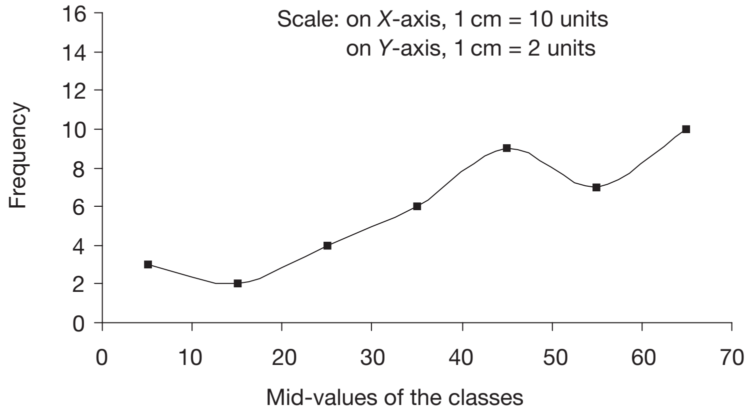


Figure 9.4

Cumulative Frequency Curves

The curves drawn for cumulative frequencies, less than or more than the true limits of the classes of a frequency distribution are called cumulative frequency curves. The curve drawn for the 'less than cumulative frequency distribution' is called the 'less than cumulative frequency curve' and the curve drawn for the 'greater than cumulative frequency distribution' is called the 'greater than cumulative frequency curve'.

From these curves, we can find the total frequency above or below a particular value of the variable.

EXAMPLE 9.6

For the given distribution, draw the less than and greater than cumulative frequency curves.

Class Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	5	3	7	2	14	9	15	10

SOLUTION

Less than cumulative frequency distribution:

Upper Class Boundary	Frequency	Less than Cumulative Frequency
10	5	5
20	3	8
30	7	15
40	2	17
50	14	31
60	9	40
70	15	55
80	10	65

Greater than cumulative frequency distribution:

Lower Class Boundary	Frequency	Greater than Cumulative Frequency
0	5	65
10	3	60
20	7	57
30	2	50
40	14	48
50	9	34
60	15	25
70	10	10

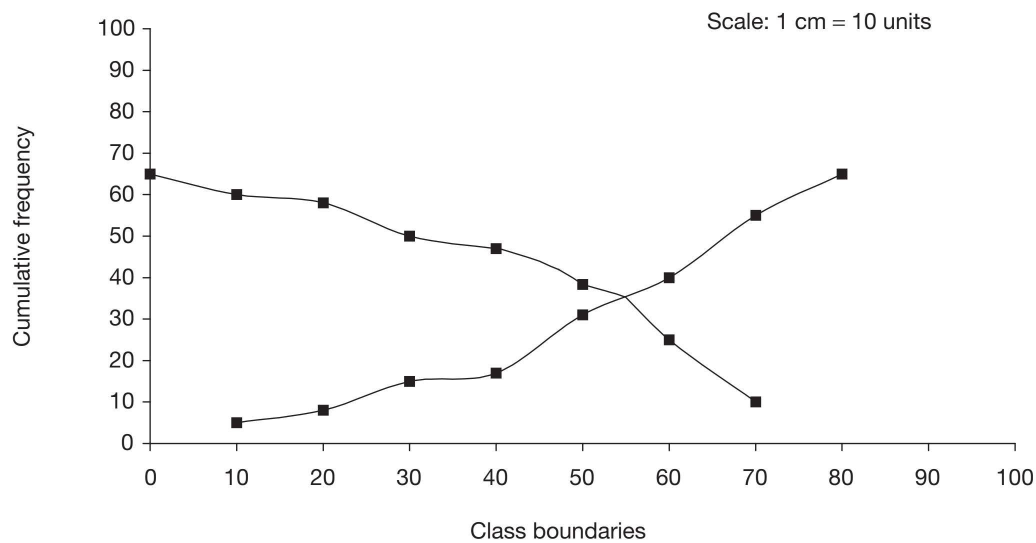


Figure 9.5

MEASURES OF CENTRAL TENDENCIES FOR UNGROUPED DATA

Till now, we have seen that the data collected in statistical enquiry or investigation is in the form of raw data. If the data is very large, the user cannot get much information from such data. For this reason, the data is grouped together to obtain some conclusions.

The measure of central tendency is a value which represents the total data, i.e., it is the value in a data around which the values of all the other observations tend to concentrate.

The most commonly used measures of central tendencies are:

1. Arithmetic mean
2. Median
3. Mode

These measures give an idea about how the data is clustered or concentrated.

Arithmetic Mean or Mean (AM)

The arithmetic mean (or simply the mean) is the most commonly used measure of central tendency.

Arithmetic Mean for Raw Data

Definition The arithmetic mean of a statistical data is defined as the quotient obtained when the sum of all the observations or entries is divided by the total number of items.

If $x_1, x_2, \ldots, x_n$ are the n items, then

$$\text{AM} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{or briefly} \quad \frac{\sum x}{n}.$$

AM is usually denoted by $\overline{x}$.

EXAMPLE 9.7

Find the mean of the first seven natural even numbers.

SOLUTION

Given data is 2, 4, 6, 8, 10, 12, 14.

$$\therefore \text{Arithmetic mean (AM)} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$= \frac{2+4+6+8+10+12+14}{7} = \frac{56}{7} = 8.$$

Mean of Discrete Series

Let $x_1, x_2, x_3 \ldots, x_n$ be n observations with respective frequencies $f_1, f_2, \ldots, f_n$.

This can be considered as a special case of raw data where the observation x_1 occurs f_1 times, x_2 occurs f_2 times, and so on.

$$\therefore \text{ The mean of the above data} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n}.$$

It can also be represented by $\overline{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$.

Weighted Arithmetic Mean

When the variables $x_1, x_2, \ldots, x_n$ do not have same importance, and the weights $w_1, w_2, \ldots, w_n$ are given to each of the variables, the weighted arithmetic is given by $\overline{x}_w = \dfrac{\sum x_i w_i}{\sum w_i}$.

EXAMPLE 9.8

The population of 50 villages in a state is given below:

Population	Number of Villages
6000	8
7000	10
9000	12
10000	5
11000	7
13000	6
15000	2
Total	50

Find the mean population of the villages of the state.

SOLUTION

The mean $\bar{x}$ is given by

$$\bar{x} = \frac{(6000 \times 8) + (7000 \times 10) + (9000 \times 12) + (10000 \times 5) + (11000 \times 7) + (13000 \times 6) + (15000 \times 2)}{8 + 10 + 12 + 5 + 7 + 6 + 2}$$

$$= \frac{461000}{50}$$

$$\bar{x} = 9220.$$

That is, the mean population of the villages is 9220.

Some Important Results about AM

1. The algebraic sum of deviations taken about the mean is zero, i.e., $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$.
2. The value of the mean depends on all the observations.
3. The AM of two numbers a and b is $\frac{a+b}{2}$.
4. Combined mean: If $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic means of two series with n_1 and n_2 observations respectively, then the combined mean is:

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

   The above result can be extended to any number of groups of data.
5. If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then the mean of $x_1 + a, x_2 + a, x_3 + a, \ldots, x_n + a$ is $\bar{x} + a$, for all values of a.
6. If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then the mean of $ax_1, ax_2, \ldots, ax_n$ is $a\bar{x}$ and that of $\frac{x_1}{a}, \frac{x_2}{a}, \ldots, \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.
7. The mean of the first n natural numbers is $\left(\frac{x+1}{2}\right)$.

8. The mean of the squares of the first n natural numbers $= \dfrac{(n+1)(2n+1)}{6}$.

9. The mean of the cubes of the first n natural numbers $= \dfrac{n(n+1)^2}{4}$.

Median

Another measure of the central tendency of a given data is the median.

Definition

If the values x_i in the raw data are arranged either in the increasing or decreasing order of their magnitude, then the middle-most value in this arrangement is called the median.

Thus, for the raw (ungrouped) data, the median is computed as follows:

1. The values of the observations are arranged in the order of magnitude.
2. The middle-most value is taken as the median. Hence, depending on the number of observations (odd or even), we determine median as follows.
   - **(i)** When the number of observations (n) is odd, then the median is the value of $\left(\dfrac{n+1}{2}\right)$th observation.
   - **(ii)** If the number of observations (n) is even, then the median is the mean of $\left(\dfrac{n}{2}\right)$th and $\left(\dfrac{n}{2}+1\right)$th observations.

EXAMPLE 9.9

Find the median of the following data:

1, 12, 5, 3, 7, 13, 9, 23, 17, 11 and 6

SOLUTION

Arranging the given numbers in the ascending order, we have 1, 3, 5, 6, 7, 9, 11, 12, 13, 17, 23.

Here, the middle term is 9

$\therefore$ Median = 9.

EXAMPLE 9.10

Find the median of the data 11, 5, 3, 13, 16, 9, 18, 10.

SOLUTION

Arranging the given data in the ascending order, we have 3, 5, 9, 10, 11, 13, 16, 18.

As the given number of values is even, we have two middle values, they are $\left(\dfrac{n}{2}\right)$th and $\left(\dfrac{n}{2}+1\right)$th observations. Here, they are 10, 11.

$\therefore$ Median of the data = Average of 10 and 11

$= \dfrac{10+11}{2} = 10.5.$

Some Important Facts about Median

1. The median does not take all the items into consideration.
2. The sum of absolute deviations taken about the median is the least.
3. The median can be calculated graphically while the mean cannot be.
4. The median is not effected by extreme values.
5. The sum of deviations taken about median is less than the sum of absolute deviations taken from any other observation in the data.

Mode

The third measure of central tendency of a data is the mode.

Definition

The most frequently found value in the data is called the mode. This is the measure which can be identified in the simplest way.

EXAMPLE 9.11

Find the mode of 1, 0, 2, 1, 0, 2, 3, 1, 3, 1, 0, 4, 2, 4, 1 and 2.

SOLUTION

Among the observations given, the most frequently found observation is 1. It occurs 5 times.

$\therefore$ Mode = 1.

Some Important Facts about Mode

1. For a given data, the mode may or may not exist. In a series of observations, if no item occurs more than once, then the mode is said to be ill-defined.
2. If the mode exists for a given data, it may or may not be unique.
3. Data having unique mode is uni-modal while data having two modes is bi-modal.

Properties of Mode

1. It can be calculated graphically.
2. It is not effected by extreme values.
3. It can be used for open-ended distribution and qualitative data.

Empirical Relationship among Mean, Median and Mode

For a moderately symmetric data, the above three measures of central tendency can be related by the formula,

$$\text{Mode} = 3\text{ Median} - 2\text{ Mean}.$$

EXAMPLE 9.12

Find the mode when median is 8 and mean is 10 of a data.

SOLUTION

$$\begin{aligned}\text{Mode} &= 3\text{ Median} - 2\text{ Mean}\\ &= (3\times 8) - (2\times 10) = 24 - 20 = 4.\end{aligned}$$

Observations

1. For a symmetric distribution,

$$\text{Mean} = \text{Median} = \text{Mode}.$$

2. Given any two of the mean, median and mode the third can be calculated.
3. This formula is to be applied in the absence of sufficient data.

MEASURE OF CENTRAL TENDENCIES FOR GROUPED DATA

We studied the measure of central tendencies of ungrouped or raw data. Now we study the measures of central tendencies (mean, median and mode) for grouped data.

Mean of Grouped Data

If the frequency distribution of 'n' observations of a variable x has k classes, x_i is the mid-value and f_i is the frequency of ith class, then the mean $\overline{x}$ of grouped data is defined as

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_kx_k}{f_1 + f_2 + \cdots + f_k} = \frac{\sum_{i=1}^{k} f_ix_i}{\sum_{i=1}^{k} f_i}$$

(or) simply,

$$\overline{x} = \frac{\sum f_ix_i}{N} \quad \text{where } N = \sum_{i=1}^{k} f_i.$$

In grouped data, it is assumed that the frequency of each class is concentrated at its mid-value.

EXAMPLE 9.13

Calculate the arithmetic mean (AM) of the following data:

Salaries of Employees (in thousand rupees)	0–10	10–20	20–30	30–40	40–50
Number of Employees	4	13	8	9	6

SOLUTION

Let us write the tabular form as given below:

Salaries of Employees	Number of Employees (f_i)	Mid-points of Classes (x_i)	f_ix_i
0–10	4	5	20
10–20	13	15	195
20–30	8	25	200
30–40	9	35	315
40–50	6	45	270

$$\therefore \text{ Mean } = \overline{x} = \frac{\sum f_ix_i}{N} = \frac{1000}{40} = 25.$$

Short-cut Method for Finding the Mean of Grouped Data (Deviation Method)

Sometimes, when the frequencies are large in number, the calculation of mean using above formula is cumbersome. This can be simplified if the class interval of each class of grouped data is the same. Under the assumption of equal class intervals, we get the following formula for the mean of grouped data:

$$\bar{x} = A + \frac{1}{N}\left(\sum_{i=1}^{k} f_i u_i\right) \times c.$$

Where, A = assumed mean value from among mid-values

c = length of class interval

k = number of classes of the frequency distribution

$$N = \text{Sum of frequencies} = \sum_{i=1}^{k} f_i$$

$$u_i = \frac{x_i - A}{C},\ i = 1, 2, 3, \ldots, k$$

x_i = mid-value of the ith class

u_i is called the deviation or difference of the mid-value of the ith class from the assumed value, divided by the class interval.

Using this method the previous example can be worked out as follows:

Short-cut Method for the Above Example

Salaries of Employees	Number of Employees (f_i)	Mid-values (x_i)	Deviation $u_i = \frac{x_i - A}{C}$	$f_i u_i$
0–10	4	5	−3	−12
10–20	13	15	−2	−26
20–30	8	25	−1	−8
30–40	9	35(A)	0	0
40–50	6	45	1	6
	$N = 40$			$\Sigma f_i u_i = -40$

Here, $A = 35,\ N = 40,\ C = 10,\ \Sigma f_i u_i = -40$

$$\therefore \text{AM} = A + \frac{1}{N}(\Sigma f_i u_i) \times C$$

$$= 35 + \frac{1}{40}(-40) \times 10 = 25.$$

Median of Grouped Data

Before finding out how to obtain the median of grouped data, we first review what a median class is.

If n is the number of observations, then from the cumulative frequency distribution, the class in which $\left(\frac{n}{2}\right)$ th observation lies is called the median class.

Formula for calculating median:

$$\text{Median } (M) = L + \frac{\frac{n}{2} - F}{f}(c)$$

Where, L = Lower boundary of the median class, i.e., class in which $\left(\frac{n}{2}\right)$th observation lies
N = Sum of frequencies
F = cumulative frequency of the class just preceding the median class
f = frequency of the median class
C = length of class interval

EXAMPLE 9.14

Given below is the data showing heights of 50 students in a class. Find its median.

Height (in cm)	162	164	166	167	168	170	173	175	177	180
Number of Students	6	4	5	12	8	3	7	2	2	1

SOLUTION

To find median, we prepare less than cumulative frequency table as given below:

Height (in cm)	Number of Students	Cumulative Frequency (f)
162	6	6
164	4	10
166	5	15
167	12	27
168	8	35
170	3	38
173	7	45
175	2	47
177	2	49
180	1	50

Here $N = 50$, which is even.

$\therefore$ Median = $\frac{N}{2}$ value $\frac{50}{2}$ or 25th observation.

From the column of cumulative frequency, the value of 25th observation is 167.

$\therefore$ Median = 167 cm.

Note In the above example, we do not have any class interval. As there is no class interval, we cannot use the formula.

EXAMPLE 9.15

Find the median of the following data:

Class Interval	0–10	10–20	20–30	30–40	40–50
f	6	8	5	4	7

SOLUTION

To find the median, we prepare the following table:

Class Interval	Frequency	Cumulative Frequency
0–10	6	6
10–20	8	14(F)
20–30	5(f)	19
30–40	4	23
40–50	7	30
Total	$N = 30$	

Here $N = 30 \Rightarrow \frac{N}{2} = 15.$

This value appears in the class 20–30.

L = Lower boundary of the median class 20–30 = 20

$F = 14, f = 5,\ C = 10$ (class length)

$$\text{Median } = L + \frac{\left(\frac{N}{2} - F\right)}{f} \times C$$

$$\therefore \text{ Median } = 20 + \frac{(15 - 14)}{5} \times 10 = 22.$$

Mode of Grouped Data

The formula for determining the mode of grouped data is $L_1 + \dfrac{\Delta_1 C}{\Delta_1 + \Delta_2}$.

Where, L_1 = lower boundary of the modal class (class with highest frequency)

$\Delta_1 = f - f_1$ and $\Delta_2 = f - f_2$ where f is the frequency of modal class

f_1 = frequency of previous class of the modal class

f_2 = frequency of next class of the modal class

Rewriting the formula,

$$\text{Mode} = L_1 + \frac{(f - f_1)C}{(f - f_1) + (f - f_2)}$$

$$\text{Mode} = L_1 + \frac{(f - f_1)C}{2f - (f_1 + f_2)}.$$

EXAMPLE 9.16

The following information gives the marks scored by of students of a class in an examination. Find the mode of the data.

Marks	1–20	21–40	41–60	61–80	81–100
Number of Students	3	18	23	37	19

SOLUTION

Here, the given classes are not continuous. Hence, we first rewrite it as shown below:

Marks	Adjusted Marks	Number of Students
1–20	0.5–20.5	3
21–40	20.5–40.5	18
41–60	40.5–60.5	$23(f_1)$
61–80	60.5–80.5	$37(f)$
81–100	80.5–100.5	$19(f_2)$

From the above table it can be observed that the maximum frequency occurs in the class interval 61–80.

$\therefore f = 37,\ f_1 = 23,\ f_2 = 19,\ L_1 = 60.5,\ C = 20.$

$$\therefore \text{Mode} = L_1 + \frac{(f - f_1)C}{(f - f_1) + (f - f_2)}$$

$$= 60.5 + \frac{(37 - 23)20}{14 + 18} = 60.5 + 87.5 = 69.25.$$

RANGE

The difference between the maximum and the minimum values of the given observations is called the range of the data.

Given $x_1, x_2, \ldots, x_n$ (n individual observations)

$$\text{Range} = (\text{Maximum value}) - (\text{Minimum Value}).$$

EXAMPLE 9.17

Find the range of 1, 3, 8, 6, 2, 11, 10, 15 and 13.

SOLUTION

Arranging the given data in the ascending order,

We have 1, 2, 3, 6, 8, 10, 11, 13, 15.

$\therefore$ Range = (Maximum value) − (Minimum value) = 15 – 1 = 14.

Note The range of the class interval is the difference of the actual limits of the class.

QUARTILES

In a given data, the observations that divide the given set of observations into four equal parts are called quartiles.

First Quartile or Lower Quartile

When the given observations are arranged in ascending order, the observation which lies midway between the lower extreme and the median is called the first quartile or the lower quartile and is denoted as Q_1.

Third Quartile or Upper Quartile

Of the data when the given observations are arranged in ascending order, the observation that lies in midway between the median and the upper extreme observation is called the third quartile or the upper quartile and is denoted by Q_3.

We can find Q_1 and Q_3 for an ungrouped data containing n observations as follows.

Rearrange the given n observations or items in the ascending order then, lower or first quartile, Q_1 is $\left(\frac{n}{4}\right)$th item or observation if n is even and $\left(\frac{n+1}{4}\right)$th item or observation, when n is odd.

EXAMPLE 9.18

Find Q_1 for the data 23, 7, 11, 9, 15, 12, 20 and 18.

SOLUTION

Arranging the given observations in ascending order, we have 7, 9, 11, 12, 15, 18, 20, 23.

Here $n = 8$ (n is even)

$\therefore$ first quartile,

$$Q_1 = \left(\frac{n}{4}\right)\text{th item} = \left(\frac{8}{4}\right)\text{th item}$$

$$= \text{second observation of the data, i.e., } 9$$

$$\therefore Q_1 = 9.$$

EXAMPLE 9.19

Find Q_1 for the observations 13, 8, 11, 15, 19, 4 and 10.

SOLUTION

Arranging the observations in ascending order, we have 4, 8, 10, 11, 13, 15, 19.

Here $n = 7$ (odd)

$$\therefore Q_1 = \left(\frac{n+1}{4}\right)\text{th item, i.e., } \left(\frac{7+1}{4}\right)\text{th item} = \text{2nd observation.}$$

$$\therefore Q_1 = 8.$$

EXAMPLE 9.20

The ages of 10 employees in an organization are 26, 23, 27, 33, 39, 43, 41, 36, 42, 25. Find Q_1.

SOLUTION

The given observations when arranged in ascending order, we get 23, 25, 26, 27, 33, 36, 39, 41, 42, 43.

Here $n = 10$ (even)

$$\therefore Q_1 = \left(\frac{n}{4}\right)\text{th observation}$$

$$= \left(2\frac{1}{2}\right)\text{th observation of the data}$$

$$\therefore Q_1 = \text{2nd observation} + \frac{1}{2}(\text{3rd} - \text{2nd})\text{ observation}$$

$$= 25 + \frac{1}{2}(26 - 25) = 25.5$$

$$\therefore Q_1 = 25.5.$$

Third Quartile

$$Q_3 = \left(\frac{3n}{4}\right)\text{th item, when } n \text{ is even.}$$

$$= 3\left(\frac{n+1}{4}\right)\text{th item, when } n \text{ is odd.}$$

EXAMPLE 9.21

Find Q_3 for the data 8, 13, 18, 9, 20, 11.

SOLUTION

Arranging the data in ascending order, we have 8, 9, 11, 13, 18, 20.

Here $n = 6$ (even)

$$\therefore Q_3 = 3\left(\frac{n}{4}\right)\text{th observation} = 4\frac{1}{2}\text{th observation}$$

$$\Rightarrow Q_3 = \text{4th observation} + \frac{1}{2}(\text{5th observation} - \text{4th observation})$$

$$= 13 + \frac{1}{2}(18 - 13) \qquad \therefore Q_3 = 15.5.$$

Semi-inter Quartile Range or Quartile Deviation (QD)

Quartile deviation, $\mathrm{QD} = \dfrac{Q_3 - Q_1}{2}$.

EXAMPLE 9.22

Find semi-inter quartile range of the following data:

X	3	6	7	9	10	11	13
f	2	9	13	17	10	14	15

SOLUTION

X	Frequency (f)	Cumulative Frequency
3	2	2
6	9	11
7	13	24
9	17	41
10	10	51
11	14	65
13	15	80
	$N = 80$	

Here, $N = 80$

$\therefore Q_1 = \left(\frac{N}{4}\right)$th observation = 20th observation

$\therefore Q_1 = 7$ (as 20th items lies in the class having 24 as cumulative frequency)

$\therefore Q_3 = 3\left(\frac{N}{4}\right)$th observation = 60th observation

$\therefore Q_3 \equiv 11$ (as 60th item lies in the class having 65 as cumulative frequency)

Semi-inter quartile range, $(\text{QD}) = \frac{Q_3 - Q_1}{2} = \frac{11-7}{2} = 2.$

Note For an individual data, the second quartile Q_2 coincides with median.

$\Rightarrow Q_2 =$ Median of the data.

Estimation of Median and Quartiles from Ogive

1. Prepare the cumulative frequency table with the given data.
2. Draw an ogive graph.
3. Let, the total number of observations = sum of all frequencies = N.
4. Mark the points A, B and C on Y-axis, corresponding to $\frac{N}{4}, \frac{N}{2}$ and $\frac{3N}{4}$ respectively.
5. Mark three points (P, Q and R) on ogive corresponding to $\frac{3N}{4}, \frac{N}{2}$ and $\frac{N}{4}$ respectively.
6. Draw vertical lines from the points R, Q and P to meet X-axis Q_1, M and Q_3 respectively.
7. Then, the abscissas of Q_1, M and Q_3 gives lower quartile, median and upper quartile respectively.

EXAMPLE 9.23

The following table shows the distribution of the percentage of marks of a group of students:

Percentage of Marks	30–40	40–50	50–60	60–70	70–80	80–90
Number of Students	4	11	3	7	5	2

SOLUTION

Class Interval	Number of Students (f)	Cumulative Frequency
30–40	4	4
40–50	11	15
50–60	3	18
60–70	7	25
70–80	5	30
80–90	2	32
	$N = 32$	

$N = 32$, $\frac{N}{2} = 16$ and $\frac{3N}{4} = 24$.

From the graph,

Lower quartile $(Q_1) = 44$

Upper quartile $(Q_3) = 69$

Median $(M) = 53$

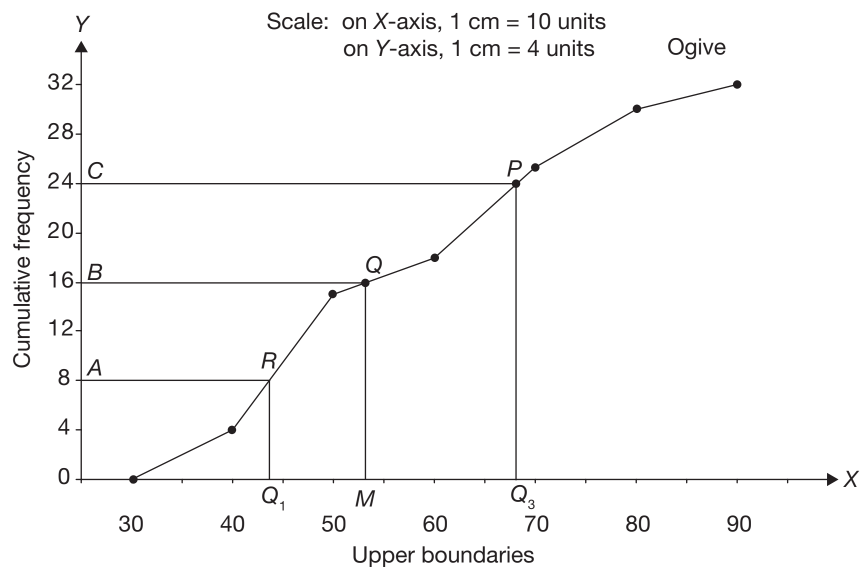


Figure 9.6

Estimation of Mode from Histogram

1. Draw a histogram to represent the given data.
2. From the upper corners of the highest rectangle, draw the line segments to meet the opposite corners of adjacent rectangles, diagonally as shown in the given example. Mark the intersecting point as P.
3. Draw PM perpendicular to X-axis, to meet X-axis at M.
4. Abscissa of M gives the mode of the data.

EXAMPLE 9.24

Estimate the mode of the following data from the histogram.

Class Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	8	10	15	16	20	13	14	12

From the graph, Mode (M) = 44.

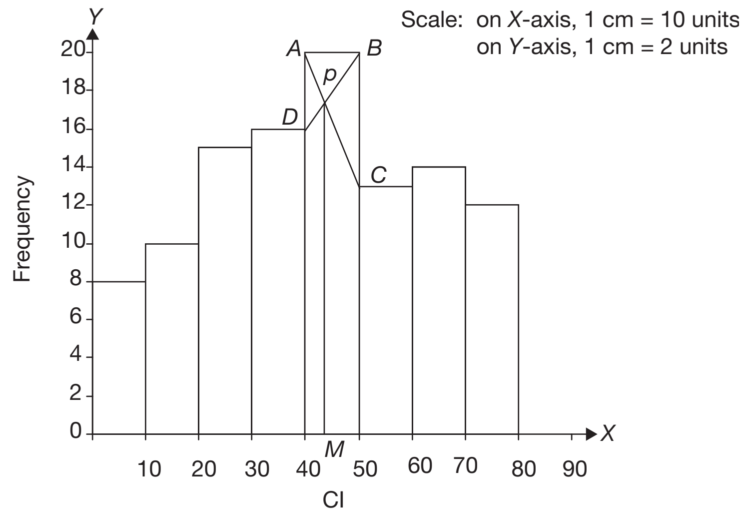


Figure 9.7

MEAN DEVIATION (MD)

Mean deviation is defined as the average or mean of the deviations taken about mean or median or mode. Hence, it is also called an average deviation.

Mean deviation gives the best results when deviations are taken about median. We should take only the positive values of deviations.

Mean Deviation (MD) for Ungrouped or Raw Data

$$\text{MD} = \frac{\sum |x_i - \overline{x}|}{n}$$

Where, x_i is each observation

$\overline{x}$ is arithmetic mean, or median or mode as specified in the problem

n is the number of observations.

EXAMPLE 9.25

Find the mean deviation of the following from median 13, 18, 15, 10, 17, 19 and 21.

SOLUTION

Writing the given observations in ascending order, we get 10, 13, 15, 17, 18, 19, 21.

Given number of observations are 7.

$\therefore$ $n = 7$ (odd)

$\therefore$ Median, $\overline{x} = 17$

$\therefore$ Mean Deviation

$$= \frac{|10-17|+|13-17|+|15-17|+|17-17|+|18-17|+|19-17|+|21-17|}{7}$$

$$= \frac{7+4+2+0+1+2+4}{7} = \frac{20}{7} = 2.857.$$

Mean Deviation for Discrete Data

$$\text{Mean deviation (MD)} = \frac{\sum |x_i - \overline{x}|}{N} \quad \text{or} \quad \frac{\sum fD}{N}.$$

Where, x_i is each of the given observations

f_i is their corresponding frequencies.

$N = \Sigma f_i$ (sum of the frequencies).

$D = |x_i - \overline{x}|$.

EXAMPLE 9.26

Find the mean deviation about mean for the following data:

x	1	3	7	9	13	14
f	2	5	8	6	3	1

SOLUTION

x_i	f_i	$f_i x_i$	$D = \lvert x_i - \overline{x}\rvert$	$f_i D$
1	2	2	6.2	12.4
3	5	15	4.2	21.0
7	8	56	0.2	1.6
9	6	54	1.8	10.8
13	3	39	5.8	17.4
14	1	14	6.8	6.8
	$N = \Sigma f = 25$	$\Sigma fx = 180$		$\Sigma fD = 70$

$$\overline{x} = \text{AM} = \frac{\sum fx}{\sum f} \Rightarrow \overline{x} = \frac{180}{25} = 7.2$$

$$\therefore \text{MD} = \frac{\sum fD}{N} = \frac{70}{25} = 2.8.$$

Mean Deviation for Grouped Data

$$\text{MD} = \frac{\sum f_i |x_i - \overline{x}|}{N} \quad \text{or} \quad \frac{\sum fD}{N}$$

Where, x_i is mid-value of each class
f_i is the corresponding frequency
$\bar{x}$ is mean or median or mode
N is sum of frequencies
$D = |x_i - \bar{x}|$.

Note The formula for discrete data and grouped data is same except for one change, i.e., x_i is to be replaced by mid values of the classes for grouped data.

EXAMPLE 9.27

Find mean deviation for the following data about median:

Class Interval	5–9	10–14	15–19	20–24	25–29
f	5	9	12	8	6

SOLUTION

Class Interval	f	x	Cumulative Frequency	$D = \|x - \bar{x}\|$	fD
5–9	5	7	5	10	50
10–14	9	12	14F	5	45
15–19	12f	17	26	0	0
20–24	8	22	34	5	40
25–29	6	27	40	10	60
					$\Sigma fD = 195$

$$\frac{N}{2} = \frac{40}{2} = 20, L = 14.5, F = 14, f = 12, C = 5$$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times C$$

$$= 14.5 + \frac{20 - 14}{12} \times 5$$

$$= 14.5 + 2.5 = 17$$

$$\therefore \text{MD} = \frac{\sum f|x - \bar{x}|}{N} \text{ or } \frac{\sum fD}{N}$$

$$= \frac{195}{40} = 4.875.$$

Some Important Results Based on MD

1. MD depends on all observations.
2. By default, MD is to be computed about mean.
3. MD about median is the least.
4. MD of two numbers a and b is $\frac{|a - b|}{2}$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If 1–10, 11–20, 21–30, 31–40, …, are the classes of a frequency distribution, then the upper boundary of the class 31–40 is _______.

2. If 1–5, 6–10, 11–15, …, are classes of a frequency distribution, then the mid-value of the class 11–15 is _______.

3. In a class, seven students got 90 marks each, the frequency of the observation 90 is _______.

4. If the lower boundary of a class is 35 and length of the class is 5, then the upper boundary is _______.

5. Range of the scores 27, 35, 47, 36, 25 and x is 23, where $x < 25$, then x is ______.

6. In a histogram, width of the rectangle represents ______ of the class and length of the rectangle represents ______ of the class.

7. If the median of the scores 1, 2, x, 4 and 5 (where $1 < 2 < x < 4 < 5$) is 3, then the mean of the scores is ______.

8. In a less than cumulative frequency distribution, frequency and cumulative frequency of a class are 10 and 20 respectively, then the cumulative frequency of the previous class is ______.

9. Mode of the scores 2, 3, 2, 4, 3, 2, 4, 6 is ______.

10. If median of the scores $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \frac{x}{5}$ and $\frac{x}{6}$ (where $x > 0$) is 6 then $\frac{x}{6}$ is ______.

11. The coefficient of range of the scores 1, 2, 3, 4 and 5 is _______.

12. If sum of the 20 deviations from the mean is 100, then the mean deviation is _______.

13. If lower and upper quartiles of the data are x and y respectively, then quartile deviation is _______.

14. If mode and median of the certain scores are 2 and 2, then mean is _______.

15. If the mean of the scores x_1, x_2, x_3, x_4, x_5 and x_6 is x, then mean of $5x_1$, $5x_2$, $5x_3$, $5x_4$, $5x_5$ and $5x_6$ is _______.

16. If the mean of a, b and c is b, then $a + c$ in terms of b is _______.

17. If lower and upper quartiles of the data are 23 and 25 respectively, then coefficient of quartile deviation is _______.

18. Mode of certain scores is x. If each score is decreased by 3, then the mode of the new series is _______.

19. If the mean of 2, 4, 6, 8, x and y is 5, then $x + y$ is _______.

20. Find the mean of the following:

$$10\frac{1}{4}, 9, 4\frac{3}{4}, 8, 2\frac{2}{3}, 12 \text{ and } 2\frac{1}{3}$$

21. Find the median of the following data:

2.0025, 2.0205, −2.06, −2.206 and −2.006

22. Find the median of the following data:

23, 92, 43, 34, 54, 48, 82, 14, 62 and 46

23. If the strengths of 10 classes in a school are given as 28, 42, 25, 30, 45, 22, 25, 34, 26 and 36, then find the median strength.

24. The mean of 8 observations was found to be 20. Later it was detected that one of the observations was misread as 62. What is the correct observation, if the correct mean is 15.5?

25. Find the coefficient of range of the scores 25, 30, 22, 34, 50, 56 and 67.

26. If the median of 33, 28, 20, 25, 34 and x is 29, then find the maximum possible value of x.

27. If the mode of the scores 3, 4, 3, 5, 4, 6, 6 and x is 4, then find the value of x.

28. If the heights of five students in a class are 132 cm, 158 cm, 150 cm, 145 cm and 155 cm, then find their mean height.

29. Find the median of the following data:

$$3\frac{3}{7}, 3\frac{5}{8}, 3\frac{1}{2}, 3\frac{1}{4}, 3\frac{7}{9} \text{ and } 3\frac{7}{11}$$

30. Following are the runs scored by 11 members of a cricket team in a test innings. Calculate the quartile deviation of the data.

20, 22, 30, 32, 39, 41, 42, 60, 62, 65 and 80

PRACTICE QUESTIONS

Short Answer Type Questions

31. If the mean weight of 10 students is 25 kg and the mean weight of another 10 students is 35 kg, then the mean weight of 20 students is ______.

32. Given below are the number of students in 30 class rooms in a school. Construct a frequency distribution table for this data with a class interval of 4.

25 30	24 18	20 24	32 35	22 20
22 32	40 28	30 25	26 29	34 15
38 28	19 16	15 20	24 30	26 18

33. The average marks of 50 students of a class is 76. If the average marks of all boys is 70 and that of all girls is 80 in that class, then find the number of boys in the class.

34. The mean of 30 observations is 25. If two observations 30 and 60 are misread as 20 and 40, then find the correct mean.

35. The mean expenditure of a person from Monday to Wednesday is ₹250, and the mean expenditure from Wednesday to Friday is ₹400. If he spend ₹300 on Wednesday, find the mean expenditure of the person from Monday to Friday.

36. The sum of deviations of n observations about 25 is 25 and sum of deviations of the same n observations about 35 is –25. Find the mean of the observations.

37. If the sum of mode and mean of certain observations is 129 and the median of the observations is 63, then find mode and mean.

38. The height (in cms) of 20 children of class 9 is given, find the mean.

Height (in cm)	Number of Children
120	2
121	4
122	3
123	2
124	5
125	4

39. Find the median of the following distribution:

x	f	x	f
10	2	30	4
20	3	40	5
50	6	80	3
60	5	90	3
70	4	100	1

40. Find the quartile deviation of the following data:

x	f	x	f
2	4	13	2
3	6	17	4
5	8	19	6
7	9	23	6
11	10		

41. Calculate the mean deviation for the following data about median.

15, 20, 16, 13, 10, 11, 18

42. Draw a histogram for the following data:

Class Interval	f
0–5	2
5–10	10
10–15	8
15–20	6
20–25	4

43. Draw the frequency polygon for the following data by drawing a histogram:

Class Interval	f
0–5	10
5–10	20
10–15	15
15–20	30
20–25	32
25–30	44

44. Draw the less than cumulative frequency curve for the following data:

Class Interval	f
10–14	10
15–19	8
20–24	15
25–29	20
30–34	25

45. If the mean of the following data is 26, then find the missing frequency x.

Class Interval	f
0–10	4
10–20	X
20–30	9
30–40	5
40–50	6

Essay Type Questions

46. Find the AM of the following data by short-cut method.

Class Interval	f
1–5	5
6–10	10
11–15	15
16–20	10
21–25	5

47. Find the quartile deviation for the following grouped data.

Class Interval	f
0–4	1
5–9	3
10–14	2
15–19	4
20–24	5
25–29	3
30–34	2
35–39	4
40–44	3
45–49	3

48. Find the mode of the following distribution given below.

Class Interval	f
0–19	12
20–39	20
40–59	23
60–79	22
80–99	13

49. Calculate the mean deviation about mean for the following data.

Class Interval	f
10–14	5
15–19	6
20–24	3
25–29	10
30–34	4
35–39	8
40–44	15
45–49	10

50. Draw the frequency curve for the following data.

Class Interval	f
2–5	12
5–8	7
8–11	8
11–14	6
14–17	10
17–20	9

CONCEPT APPLICATION

Level 1

1. The mean of first n natural numbers is $\frac{5n}{9}$. Find n.

(a) 5 (b) 4

(c) 9 (d) None of these

2. Mean of a certain number of observations is m. If each observation is divided by $x (x \neq 0)$ and increased by y, then the mean of new observations is

PRACTICE QUESTIONS

(a) $mx + y$ (b) $\frac{mx+y}{x}$

(c) $\frac{m+xy}{x}$ (d) $m + xy$

3. If the difference of mode and median of a data is 24, then the difference of median and mean is

(a) 12 (b) 24

(c) 8 (d) 36

4. The mode of the observations $2x + 3$, $3x - 2$, $4x + 3$, $x - 1$, $3x - 1$, $5x + 2$ (x is a positive integer) can be

(a) 3 (b) 5

(c) 7 (d) 9

5. The median of 21 observations is 18. If two observations 15 and 24 are included to the observations, then the median of the new series is

(a) 15 (b) 18

(c) 24 (d) 16

6. If the quartile deviation of a set of observations is 10 and the third quartile is 35, then the first quartile is

(a) 24 (b) 30

(c) 17 (d) 15

7. The upper class limit of inclusive type class interval 10–20 is ______.

(a) 10.5 (b) 20

(c) 20.5 (d) 17.5

8. The semi-inter quartile range of the observations 9, 12, 14, 6, 23, 36, 20, 7, 42 and 32 is

(a) 12.75 (b) 12.5

(c) 9.75 (d) 9.5

9. Find the mode of the following discrete series.

x	f	x	f
1	5	6	8
3	7	12	6
5	3	15	5

(a) 3 (b) 12

(c) 8 (d) 6

10. The mean deviation of $a^3 + b^3$ and $a^3 - b^3$ (where a and $b > 0$) is ______.

(a) a^3 (b) b^3

(c) $2a^3$ (d) $2b^3$

11. If the arithmetic mean of the observations x_1, x_2, x_3, ..., x_n is 1, then the arithmetic mean of $\frac{x_1}{k}, \frac{x_2}{k}, \frac{x_3}{k}, \ldots, \frac{x_n}{k}$ $(k > 0)$ is

(a) greater than 1. (b) less than 1.

(c) equal to 1. (d) None of these

12. Range of 14, 12, 17, 18, 16 and x is 20. Find x $(x > 0)$.

(a) 2 (b) 28

(c) 32 (d) Cannot be determined

13. The mean of a set of observation is a. If each observation is multiplied by b and each product is decreased by c, then the mean of new set of observations is ______.

(a) $\frac{a}{b} + c$ (b) $ab - c$

(c) $\frac{a}{b} - c$ (d) $ab + c$

14. The mean deviation of first 8 composite numbers is ______.

(a) 2.9375 (b) 4.83

(c) 5.315 (d) 3.5625

15. Find the mode of the following discrete series.

x	f	x	f
1	5	5	12
2	4	6	3
3	6	7	9
4	8	8	10

(a) 4 (b) 8

(c) 5 (d) 7

16. The the highest score of certain data exceeds its lowest score by 16 and coefficient of range is $\frac{1}{3}$. Find the sum of the highest score and the lowest score.

(a) 36 (b) 48

(c) 24 (d) 18

17. Find the mean deviation (approximately) about the mode for the following ungrouped data: 20, 25, 30, 18, 15, 40.

PRACTICE QUESTIONS

(a) 6.71 (b) 4.52

(c) 7.61 (d) 5.33

18. The mean of first n odd natural numbers is $\frac{n^2}{81}$. Find n.

(a) 9 (b) 81

(c) 27 (d) None of these

19. The arithmetic mean of 12 observations is 15. If two observations 20 and 25 are removed then the arithmetic mean of remaining observations is

(a) 14.5 (b) 13.5

(c) 12.5 (d) 13

20. The arithmetic mean and mode of a data is 24 and 12 respectively, then the median of the data is _______.

(a) 20 (b) 18

(c) 20 (d) 22

21. The inter-quartile range of the observations 3, 5, 9, 11, 13, 18, 23, 25, 32 and 39 is

(a) 24 (b) 17

(c) 31 (d) 8

22. Find the mean deviation from the mode for the following ungrouped data: 2.5, 6.5, 7.3, 12.3, 16.2.

(a) 4.34 (b) 5.57

(c) 2.33 (d) 6.72

23. The mean of the following distribution is 5, then find the value of b.

x	f
3	2
5	a
7	5
4	b

(a) 10 (b) 6

(c) 8 (d) None of these

24. The mean deviation of $\frac{a+b}{2}$ and $\frac{a-b}{2}$ (where a and $b > 0$) is _______.

(a) $\frac{b}{2}$ (b) $\frac{a}{2}$

(c) a (d) b

25. If the mean of $x + 2$, $2x + 3$, $3x + 4$ and $4x + 5$ is $x + 2$, then find the value of x.

(a) 0 (b) 1

(c) −1 (d) 2

26. The range of 15, 14, x, 25, 30, 35 is 23. Find the least possible value of x.

(a) 14 (b) 12

(c) 13 (d) 11

27. Find the median of the following data.

Class Interval	f
0–10	12
10–20	13
20–30	25
30–40	20
40–50	10

(a) 25 (b) 23

(c) 24 (d) 26

28. In the following table, pass percentage of three schools from the year 2001 to the year 2006 are given. Which school students' performance is more consistent?

	2001	2002	2003	2004	2005	2006
School X	80	89	79	83	84	65
School Y	92	94	76	75	80	63
School Z	93	97	67	63	70	85

(a) X (b) Y

(c) Z (d) X and Y

29. The median of the following discrete series is

x	f
3	5
6	2
5	4
8	6
12	7
7	6

(a) 7 (b) 8

(c) 9 (d) 6

30. Which of the following does not change for the observations 23, 50, 27, $2x$, 48, 59, 72, 89, $5x$, 100, 120, when x lies between 15 and 20?

(a) Arithmetic mean (b) Range
(c) Median (d) Quartile deviation

Level 2

31. If the ratio of mean and median of a certain data is 2 : 3, then find the ratio of its mode and mean.

(a) 4 : 3 (b) 7 : 6
(c) 7 : 8 (d) 5 : 2

32. If mean of the following distribution is 13, then the value of p is

x	5	10	12	17	16	20
f	9	3	p	8	7	5

(a) 6 (b) 7
(c) 10 (d) 4

33. If the ratio of mode and median of a certain data is 6 : 5, then find the ratio of its mean and median.

(a) 8 : 9 (b) 9 : 10
(c) 9 : 7 (d) 8 : 11

34. If the arithmetic mean of the following distribution is 8.2, then find the value of p.

x	1	3	5	9	11	13
f	3	2	7	p	4	8

(a) 5 (b) 6
(c) 9 (d) None of these

35. The median of the series 8, 12, 15, 7, x, 19 and 22 lies in the interval.

(a) [12, 15] (b) [7, 15]
(c) [15, 17] (d) [9, 12]

36. The mode of the following distribution is

Class Interval	f
1–5	4
6–10	7
11–15	10
16–20	8
21–25	6

(a) 14.5 (b) 16.5
(c) 10.5 (d) 13.5

37. The mean of the following data is

Class Interval	f
10–15	5
15–20	7
20–25	3
25–30	4
30–35	8

(a) 22
(b) 23.05
(c) 24.05
(d) 27.05

38. The median of the following frequency distribution is

Class Interval	f
0–10	5
10–20	8
20–30	7
30–40	10
40–50	20

(a) 35
(b) 30
(c) 40
(d) 45

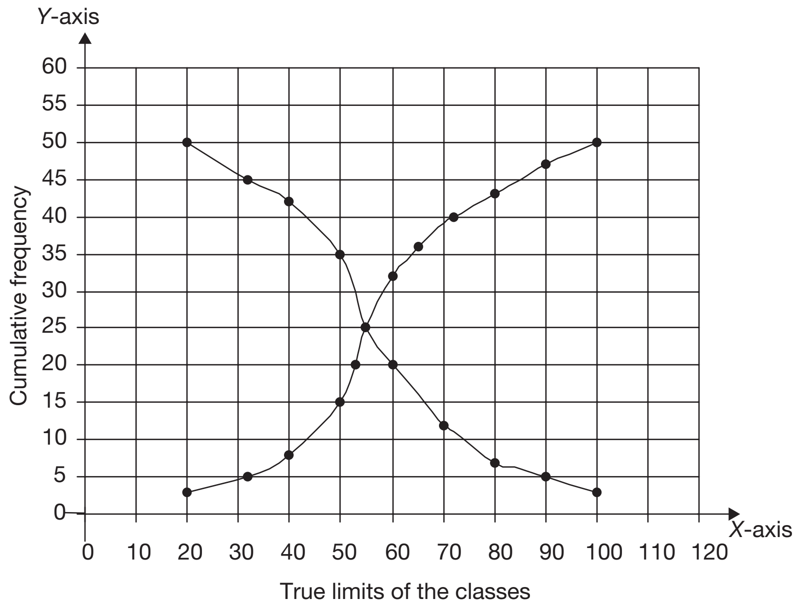
39. Find the quartile deviation of the following discrete series.

x	7	5	4	8	12	10
f	2	4	6	10	9	7

(a) 6.5
(b) 4.5
(c) 3.5
(d) 2.5

PRACTICE QUESTIONS

Direction for questions 40 and 41: These questions are based on the following data (figure).



The given figure represents the percentage of marks on X-axis and the number of students on Y-axis.

40. Find the number of students who scored less than or equal to 50% of marks.

(a) 35 (b) 15

(c) 20 (d) 30

41. Find the number of students who scored greater than or equal to 90% of marks.

(a) 47 (b) 45

(c) 5 (d) 10

42. In a class of 15 students, on an average, each student got 12 books. If exactly two students received same number of books, and the average of books received by remaining students be an integer, then which of the following could be the number of books received by each of the two students who received same number of books?

(a) 11 (b) 15

(c) 20 (d) 25

43. Find the quartile deviation of the following discrete series.

x	3	5	6	8	10	12
f	7	2	3	4	5	6

(a) 4 (b) 3

(c) 3.5 (d) 4.5

44.

Weight (in kg)	Number of Students
20	8
22	4
24	3
25	7
30	5

Find the mean deviation (approximately) about the median for the above data.

(a) 2.5 (b) 1.5

(c) 3 (d) 0.5

45. Find the mean deviation (approximately) about the mean for the following.

Class Interval	f
0–5	3
5–0	4
10–15	8
15–20	10
20–25	5

(a) 5 (b) 4

(c) 6 (d) 3

46. If the average mark of 15 students is 60 and the average mark of another 10 students is 70, then find the average mark of 25 students.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) Average marks of 25 students $= \frac{1600}{25} = 64$

(B) The total marks of 15 students = 15 × 60 = 900

The total marks of 10 students = 10 × 70 = 700

(C) The total marks of 25 students = 900 + 700 = 1600

(a) BCA (b) BAC

(c) CBA (d) CAB

47. In a class of 25 boys and 20 girls, the mean weight of the boys is 40 kg and the mean weight of the girls is 35 kg. Find the mean weight of the class.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) The total weight of 25 boys = 25 × 40 = 1000 kg

The total weight of 20 girls = 20 × 35 = 700 kg

(B) The mean weight of the class $= \frac{1700}{45} = 37\frac{7}{9}$ kg

(C) The total weight of 45 students = 1000 kg + 700 kg = 1700 kg

(a) ABC (b) ACB

(c) BCA (d) CBA

48. If $p < q < 2p$; the median and mean of p, q and $2p$ are 36 and 31 respectively, then find the mean of p and q.

(a) 21.5 (b) 23

(c) 27.5 (d) 24

49. If $x < y < 2x$; the median and the mean of x, y and $2x$ are 27 and 33 respectively, then find the mean of x and y.

(a) 23.5 (b) 24

(c) 23 (d) 25.5

50. The mean of a set of 12 observations is 10 and another set of 8 observations is 12. The mean of combined set is _______.

(a) 11 (b) 10.8

(c) 11.2 (d) 0.6

Level 3

51. A class of 40 students is divided into four groups named as A, B, C and D. Group-wise percentage of marks scored by them are given below in the table.

A	B	C	D
20	42	10	21
30	51	25	69
40	45	85	70
25	58	73	86
22	53	98	53
45	64	43	68
65	72	64	99

By using the coefficient of range find which of the group has shown good performance.

(a) A (b) B

(c) C (4) D

52. Life (in hour) of 10 bulbs from each of four different companies A, B, C and D are given below in the table.

A	B	C	D
120	700	950	530
1600	502	330	650
280	1430	1200	720
420	625	500	550
800	780	445	748
770	335	1260	570
270	224	375	635
455	1124	1130	804
150	473	185	500

By using the coefficient of range find which company has shown the best consistency in its quality?

(a) A (b) B

(c) C (d) D

PRACTICE QUESTIONS

53. If the mode of the observations 5, 4, 4, 3, 5, x, 3, 4, 3, 5, 4, 3 and 5 is 3, then find the median of the observations.

(a) 3 (b) 4
(c) 5 (d) 3.5

54. In a colony, the average age of the boys is 14 years and the average age of the girls is 17 years. If the average age of the children in the colony is 15 years, find the ratio of number of boys to that of girls.

(a) 1 : 2 (b) 2 : 1
(c) 2 : 3 (d) 3 : 2

55. Find the median of the following data.

Class Interval	f
0–4	3
4–8	6
8–12	6
12–16	6
16–20	8

(a) 13 (b) 12
(c) 11 (d) 10

56. In a class of 20 students, 10 boys brought 11 books each and 6 girls brought 13 books each. Remaining students brought atleast one book each and no two students brought the same number of books. If the average number of books brought in the class is a positive integer, then what could be the total number of books brought by the remaining students?

(a) 12 (b) 16
(c) 14 (d) 8

57. The mean of a set of 20 observations is 8 and another set of 30 observations is 10. The mean of combined set is _______.

(a) 9.2 (b) 10.8
(c) 11.2 (d) 9.8

58. Find the approximate value of mean deviation about the mode of the following data.

Class Interval	f
0–10	4
10–20	6
20–30	3
30–40	9
40–50	5

(a) 11.5 (b) 12.5
(c) 13.5 (d) 14.5

59. The mean of the following distribution is 4. Find the value of q.

x	2	3	4	5	7
f	4	4	2	3	q

(a) 2 (b) 3
(c) 0 (d) 4

60. If the ratio of mean and median of a certain data is 5 : 7, then find the ratio of its mode and mean.

(a) 2 : 5 (b) 11 : 5
(c) 6 : 5 (d) 2 : 3

61. Find the mode of the following discrete series.

x	1	2	3	4	5	6	7	8	9
f	3	8	15	1	9	12	14	5	7

(a) 7 (b) 5
(c) 2 (d) 3

62. Find the median of the following data.

x	12	15	18	21	24
f	4	7	2	3	4

(a) 12 (b) 16
(c) 18 (d) 15

63. Find the mean deviation about the median for the following data.

x	1	2	3	4	5	6
f	3	7	5	8	2	5

(a) 1 (b) $0.\overline{7}$
(c) 3 (d) $1.\overline{3}$

64. Find the mode for the following data.

Class Interval	f
0–9	2
10–19	4
20–29	7
30–39	5
40–49	3

(a) 30 (b) 25.5
(c) 32 (d) 33

65. Find the quartile deviation of the following discrete series.

x	8	10	13	16	19	22
f	4	7	8	3	5	4

(a) 3.5 (b) 6
(c) 5 (d) 4.5

66. Find the arithmetic mean of the observations $x + 5, x + 6, x + 10, x + 11, x + 14, x + 20$ (where x is any real number).

(a) $x + 11$ (b) $x + 5$
(c) $x + 13$ (d) $x + 7$

67. Find the mode of the following discrete series.

x	1	2	3	4	5	6	7	8	9
f	3	8	15	1	9	12	17	5	7

(a) 7 (b) 5
(c) 2 (d) 3

68. Find the mean of the following continuous distribution.

Class Interval	f
0–10	8
10–20	4
20–30	6
30–40	3
40–50	4

(a) 20.8 (b) 21.4
(c) 21.8 (d) 22.2

69. Which of the following is not changed for the observations 31, 48, 50, 60, 25, 8, $3x$, 26, 32? (where x lies between 10 and 15).

(a) Arithmetic mean
(b) Range
(c) Median
(d) Quartile deviation

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 40.5
2. 13
3. 7
4. 40
5. 24
6. size (length), frequency
7. 3
8. 10
9. 2
10. $\frac{x}{6} = 4$
11. $\frac{2}{3}$
12. 5
13. $\frac{y - x}{2}$
14. 2
15. $5x$
16. $a + c = 2b$
17. $\frac{1}{24}$
18. $(x - 3)$
19. 10
20. 7
21. -2.006
22. 47
23. 29
24. 26
25. $\frac{45}{89}$
26. 30
27. 4
28. 148 cm
29. $3\frac{1}{2}$
30. 16

Shot Answer Type Questions

31. 30 kg
33. 20
34. 26
35. ₹330
36. 30
37. 69
38. 122.8 cm
39. 50
40. 6
41. 15, 2.9
45. 6

Essay Type Questions

46. 13
47. 10.625
48. 54.5
49. 9.9 (approx)

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (b)
11. (d)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (b)

Level 2

31. (d)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (b)
41. (c)	**42.** (d)	**43.** (c)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (b)

Level 3

51. (b)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (b)	**59.** (b)	**60.** (b)
61. (d)	**62.** (d)	**63.** (d)	**64.** (b)	**65.** (d)	**66.** (a)	**67.** (a)	**68.** (b)	**69.** (b)	

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Sum of the first n natural numbers is $\dfrac{n(n+1)}{2}$.
2. If each observations divided by x, then the new mean is $\left(\dfrac{m}{x}\right)$.
3. Use empirical formula.
4. Substitute $x = 1$.
5. Observe median after including new observations.
6. Recall the formula for quartile deviation.
7. For inclusive type class, given values are the limits.
8. Semi-quartile range is the quartile deviation.
9. The 'x' value of the highest frequency class is the mode.
10. Mean deviation of x and y is $\dfrac{|x-y|}{2}$.
11. Mean depends on the value of k.
12. Range = Maximum value – Minimum value.
13. Use the properties of mean.
14. Mean deviation $= \dfrac{\Sigma|x_i - A|}{n}$, where A = Arithmetic mean.
15. The 'x' value of the highest frequency class is the mode.
16. Use the formula for coefficient of range.
17. Find the mode by using mean and median then find mean deviation.
18. Mean $= \dfrac{\text{Sum of the observations}}{\text{Total number of observations}}$.
19. AM $= \dfrac{\text{Sum of all observation}}{\text{Total number of observations}}$.
20. Mode = 3 median – 2 mean.
21. Inter-quartile range $= \dfrac{Q_3 - Q_1}{2}$

    $Q_3 = \dfrac{3(n+1)}{4}$th observation

    $Q_1 = \dfrac{(n+1)}{4}$th observation.
22. Mean deviation $= \dfrac{\Sigma|x_i - A|}{n}$

    Where A = Arithmetic mean.
23. (i) $\dfrac{\sum fx}{N} = 5$.

    (ii) Find the value of $\sum fx$ and $\sum f$.

    (iii) Mean $= \dfrac{\sum fx}{\sum f}$.
24. Mean deviation of x and y is $\dfrac{|x-y|}{2}$.
25. (i) Find the sum of observations.

    (ii) Use, $x + 2 = \dfrac{\text{Sum of observations}}{\text{Number of observations}}$.
26. (i) Consider x as the least value.

    (ii) Use, Range = Highest score – Lowest score and find x.
27. (i) Write the cumulative frequencies of each class.

    (ii) Trace the median class, i.e., the class corresponding the commutative frequency equal to $\left(\dfrac{N}{2}\right)$.

    (iii) Evaluate median, by using median $= L + \left(\dfrac{\frac{N}{2} - F}{f}\right) \times C$.
28. (i) Find the average (mean) pass percentage of all the schools.

    (ii) The school with the maximum pass percentage is more consistent.
29. (i) Find $\dfrac{N}{2}$ and proceed.

    (ii) Find the cumulative frequency.

    (iii) Identify the value of x corresponding to the $\left(\dfrac{N}{2}\right)$th observation.

    (iv) Use the formula to find median.

HINTS AND EXPLANATION

30. (i) Recall the properties of central tendencies.
(ii) For different values of x, AM, Median, and QD will be changed.
(iii) The maximum and minimum values are not changed in the given interval of x.

Level 2

31. (i) Substitute, median $= \frac{3}{2}$ mean in empirical relation.
(ii) Mode = 3 Median – 2 Mean.
(iii) Express median in terms of mean according to the given data.
(iv) Now write the ratio of the mode and mean using the relation in (i).

32. (i) $\frac{\sum fx}{N} = 13$.
(ii) Find $\sum fx$ and $\sum f$.
(iii) Mean $= \frac{\sum fx}{\sum f}$, where $\sum f = N$.

33. (i) Mode = 3 Median – 2 Mean.
(ii) Divide the above equation by median.
(iii) Substitute, $\frac{\text{Mode}}{\text{Median}} = \frac{6}{5}$ and obtain the required ratio.

34. (i) $\frac{\sum fx}{N} = 8.2$.
(ii) Find $\sum fx$ and $\sum f$.
(iii) Mean $= \frac{\sum fx}{\sum f}$.

35. (i) Recall the properties of central tendencies.
(ii) Arrange the given observations (except x) in ascending order.
(iii) Find the median for different values of x.
(iv) Write median interval for all values of x.

36. (i) Mode $= L + \frac{\Delta_1 C}{\Delta_1 + \Delta_2}$.
(ii) Identify f_1, f_2 and L.
(iii) Mode $= L + \frac{D_1}{D_1 + D_2} \times C$ (Where $D_1 = f - f_1$ and $D_2 = f - f_2$).

37. (i) Use deviation method.
(ii) Find the mid values of the class intervals.
(iii) Find $\sum fx$ and $\sum f$.
(iv) Mean $= \frac{\sum fx}{\sum f}$.

38. (i) Median $= l + \frac{\frac{N}{2} - m}{f} \times C$.
(ii) Find the cumulative frequency.
(iii) Find $\frac{N}{2}$, m, f and c.
(iv) Find the median by the formula,

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times C.$$

39. (i) QD $= \frac{Q_3 - Q_1}{2}$.
(ii) Find the cumulative frequency.
(iii) Identify the x value corresponding to $\left(\frac{N}{2}\right)$th observation and $\left(\frac{3N}{2}\right)$th observation.

40. (i) Inferences from the graph.
(ii) Use less than cumulative frequency curve.
(iii) Identify the cumulative frequency corresponding to 50.

41. (i) Inferences from the graph.
(ii) Use greater than cumulative frequency curve and identify the frequency corresponding to 90.

42. (i) Use mean concept.
(ii) Let the two students received x number of books each.
(iii) According to the data $2x + 13p = 180$.
(iv) From the options choose x such that p must be an integer.

43. (i) $QD = \frac{Q_3 - Q_1}{2}$.

(ii) Write cumulative frequency.

(iii) The corresponding values of x of $\frac{N}{4}$th and $\frac{3N}{4}$th observation is Q_1 and Q_3 respectively.

44. (i) Find median and then mean deviation.

(ii) Calculate the median of the given discrete data.

(iii) Evaluate mean deviation by using $\frac{\sum |x_i - A| f}{\sum f}$.

45. (i) Find mean and then mean deviation.

(ii) Find the mid-values of the series.

(iii) Find the mean.

(iv) Find the mean deviation using the formula,

$$MD = \sqrt{\sum \frac{(x_i - A) f}{\sum f}}$$

46. BCA is the required sequential order.

47. ACB is the required sequential order.

48. As p, q and $2p$ are in descending order, their median is 36.

$\therefore \quad q = 36$

$\frac{p + q + 2p}{3} = 31$

$\Rightarrow 3p + 36 = 93 \Rightarrow p = 19.$

$\therefore \quad$ Mean of 19 and 36 $= \frac{19 + 36}{2} = 27.5.$

49. As x, y and $2x$ are in ascending order, median is y.

$y = 27.$

Mean $= \frac{x + y + 2x}{3} = 33$

$\Rightarrow \frac{3x + 27}{3} = 33 \Rightarrow x = 24.$

$\therefore$ Mean of x and $y = \frac{x + y}{2} = \frac{24 + 27}{2} = 25.5.$

50. The mean of set of 12 observations is 10.

Sum of the observations $= 12 \times 10 = 120$

The mean of another set of 8 observations is 12

Sum of the observations $= 12 \times 8 = 96$

Total number of observations $= 12 + 8 = 20$

Total sum of observations $= 216$

Combined mean $= \frac{216}{20} = 10.8.$

Level 3

51. (i) Find the coefficient of range of A, B, C and D and compare.

(ii) Coefficient of range =

$$\frac{\text{Maximum value} - \text{Minimum Value}}{\text{Maximum value} + \text{Minimum Value}}.$$

(iii) Find the coefficient of range of A, B, C and D.

(iv) The least coefficient of range implies good performance.

52. (i) Find the coefficient of range of A, B, C and D and compare.

(ii) Coefficient of range =

$$\frac{\text{Maximum value} - \text{Minimum Value}}{\text{Maximum value} + \text{Minimum Value}}.$$

(iii) Find the coefficient of range of A, B, C and D.

(iv) The least coefficient of range implies good performance.

53. Given, Mode = 3

$\therefore x = 3$

Now, arrange the given data in ascending order,

3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5

$\therefore$ Median = 4

Hence, option (b) is correct.

54. Given,

Avg. age of boys = 14 years

Avg. age of girls = 17 years

Avg. age of children in society = 15 years

Let no. of boys be x, and

No. of girls be y

$\Rightarrow$ Sum of boys = 14 × no. of boys

Sum of girls = 17 × no. of girls

Sum of children = 15 × (no. of boys + no. of girls)

$\Rightarrow$ 14 × no. of boys + 17 × no. of girls = 15 × ($x + y$)

$\Rightarrow 14x + 17y = 15x + 15y$

$\Rightarrow x - 2y = 0$

$\Rightarrow x = 2y$

$\therefore x : y = 2 : 1$

Hence, option (b) is correct.

55.

Class Interval	f	Cumulative Frequency
0–4	3	3
4–8	5	8 m
8–12	6	14
12–16	3	17
16–20	8	25

Here, $N = 25$

$\frac{N}{2} = \frac{25}{2} = 12.5$.

$\therefore$ The median class = 8 – 12

$\therefore l = 8, f = 6, m = 8, c = 4.$

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c = 8 + \frac{12.5 - 8}{6} \times 4$$

$$= 8 + \frac{4.5}{3} \times 2 = 8 + 1.5 \times 2 = 8 + 3 = 11.$$

56. 10 boys brought 11 books each. Therefore, total number of books brought by 10 boys = 110. Six girls brought 13 books each. Therefore, total number of books brought by 6 girls = 78.

Remaining number of students are 4 and the total number of books brought by 4 students = x.

Given that $\frac{x + 78 + 110}{20}$ is a positive integer.

If $x = 12$ then the above value is an integer.

57. The mean of set of 20 observations is 8.

Sum of the observations = 20 × 8 = 160

The mean of another set of 30 observations is 10

Sum of the observations = 30 × 10 = 300

Total number of observations = 20 + 30 = 50

Total sum of observations = 460

Combined mean = $= \frac{460}{50} = 9.2.$

58.

Class Interval	f	x	$D = \lvert x - \bar{x} \rvert$	fD
0–10	4	5	31	124
10–20	6	15	21	126
20–30	3	25	11	33
30–40	9	35	1	9
40–50	5	45	9	45

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c.$$

Here, maximum frequency is 9.

$\therefore$ Model class is 30–40

$\therefore L = 10$

$\Delta_1 = f - f_1$ and $\Delta_2 = f - f_2$

$\Delta_1 = 9 - 3,\ \Delta_2 = 9 - 5 = 4$ and $C = 10$

$$\text{Mode} = 30 + \frac{6}{6+4} \times 10 = 30 + \frac{6}{10} \times 10$$

$= 30 + 6 = 36.$

$\sum fD = 337,\ N = 27$

$$\therefore \text{MD} = \frac{\sum f \cdot d}{N} = \frac{337}{27} \sim 12.5.$$

59.

x	f	fx
2	4	8
3	4	12
4	2	8
5	3	15
7	q	$7q$

Here $\sum fx = 43 + 7q$

$\sum f = 13 + q$

We know that, AM $= \frac{\Sigma fx}{\Sigma f}$

$$4 = \frac{43+7q}{13+q}$$

$52 + 4q = 43 + 7q \Rightarrow 3q = 9 \Rightarrow q = 3.$

60. Given that, $\frac{\text{Mean}}{\text{Median}} = \frac{5}{7}$

We know that, Mode = 3 Median – 2 Mean.

$$\text{Mode} = \text{Mean}\left(\frac{3\,\text{Median}}{\text{Mean}} - 2\right)$$

$$\frac{\text{Mode}}{\text{Mean}} = \left(3\cdot\frac{7}{5} - 2\right) = \frac{21-10}{5} = \frac{11}{5}.$$

61. Mode = 3

$\therefore$ Since the frequency of 3 is maximum.

62.

x	f	Cumulative Frequency
12	4	4
15	7	11
18	2	13
21	3	16
24	4	20

$N = 20$

The value of x corresponding to $\frac{N}{2}$th value class in the cumulative frequency is the median. Here $\frac{N}{2} = 10$ and the corresponding x value is median, i.e., 15.

63.

x_i	f_i	Cumulative Frequency	$D = x_i - A$	$f_i\|D\|$
1	3	3	–2	6
2	7	10	–1	7
3	5	15	0	0
4	8	23	1	8
5	2	25	2	4
6	5	30	3	15
				40

Here $N = 30$

$\frac{N}{2} = 15$, which falls in the class with $x = 3$.

$\therefore$ Median $= 3, \Sigma f\,|D| = 40$

$$\text{Mean deviation} = \frac{\Sigma f\,|D|}{N} = \frac{40}{30} = 1.33.$$

64. First we convert the given class intervals into continuous class intervals by subtracting 0.5 and adding 0.5 to lower and upper limits of each class respectively.

Class Interval	f
0–9	2
10–19	$4f_1$
20–29	$7f$
30–39	$5f_2$
40–49	3

Here, $f = 7$, $f_1 = 4$, $f_2 = 5$ and $L = 19.5$ (lower boundary of the highest frequency class).

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$$

$\Delta_1 = f - f_1 = 3$

$\Delta_2 = f - f_2 = 2$

$$\bar{x} = 19.5 + \frac{3}{5} \times 10$$

$19.5 + 6 = 25.5.$

65.

x	f	Less than Cumulative Frequency
8	4	4
10	7	11
13	8	19
16	3	22
19	5	27
22	4	31

$N = 31$

Q_1 = size of $\frac{(n+1)}{4}$th item = 8th item = 10.

Q_3 = size of $\frac{3(n+1)}{4}$th item = 24th item = 19.

$$\text{QD} = \frac{Q_3 - Q_1}{2} = \frac{19-10}{2} = 4.5.$$

66. $\text{AM} = \dfrac{\text{Sum of all observations}}{\text{total number of observations}}$

$$= \frac{x+5+x+6+x+10+x+11+x+14+x+20}{6}$$

$$= \frac{6x+66}{6} = \frac{6(x+11)}{6} = x+11.$$

67. Mode = 7

$\therefore$ Since the frequency of 7 is maximum.

68.

Class Interval	*D*	*x*	*fx*
0–10	8	5	40
10–20	4	15	60
20–30	6	25	150
30–40	3	35	105
40–50	4	45	180

Here $\Sigma fx = 535$

$\Sigma f = 25$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{535}{25} = 21.4.$$

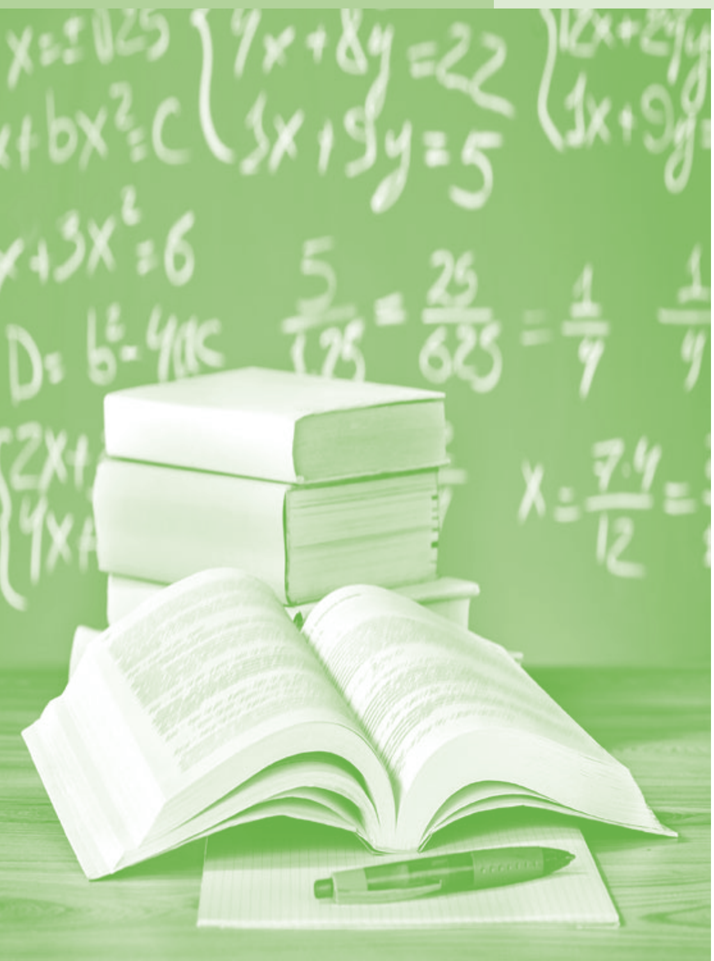
69. If we take x value between 10 and 15. There is no change in maximum value and the minimum value.

$\therefore$ Range will not change.

HINTS AND EXPLANATION

Chapter 10

Probability



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the basic concepts of probability
- Compute basic probability in daily life

KEY IDEAS

After completing this chapter, you should be able to:

- Learn the concepts related to probability
- Find probability of happening of any event
- Apply the concept of probability in daily life and solve problems

INTRODUCTION

In our daily life we frequently use some statements like the followings:

1. I will probably get a scholarship for my performance.
2. I doubt that I will catch the train.
3. The chances of the prices of all commodities going up are very high.
4. There is a fifty-fifty chance of Sania winning the match.
5. I will most probably return from my official tour in three days.
6. The chances of Sachin making a century in his next match are very few.

In the above statements the words probably, chances, doubt and most probably involve an uncertainty.

In statement (1), I will probably get a scholarship means he may get a scholarship or he may not get a scholarship. Here we are predicting a scholarship based on his performance. Similar predictions are also made in statements (2) to (6). In this chapter, we are going to learn how to measure the uncertainty (i.e., probability) numerically by using probability in many cases. Actually probability started with gambling but it has been used extensively in all fields like biological sciences, medical sciences, weather forecasting, commerce, etc.

Example: Take any ₹1 coin and toss it 5 times, 10 times, 15 times, 20 times, etc., and record your observations in three columns as shown below:

Number of Times Coin Tossed	Number of Times Head Shows up	Number of Times Tail Shows up
5	3	2
10	6	4
15	6	9
20	8	12
:	:	:
:	:	:

Now find the values in each of the following case:

1. $\dfrac{\text{Number of times head shows up}}{\text{Total number of times the coin is tossed}}$

2. $\dfrac{\text{Number of times a tail shows up}}{\text{Total number of times the coin is tossed}}$

Observations

1. As the number of times a head/a tail shows up is less than the total number of times the coin is tossed, the values of the above two fractions are proper fractions.
2. The values of the above two fractions are positive but less than one.
3. We observe that, when the number of times the coin tossed becomes larger and larger, then the value of the above two fractions approaches $\frac{1}{2}$ or 0.5.

Example: A six-faced (well-balanced) cube with its six faces marked with numbers 1 to 6 is called a dice.

Roll a dice 30 times and record the faces showing up with numbers 1 to 6 in each case as shown below:

Number of Times a Dice is Rolled	Number of Times Each Face with the Following Values Shows up					
	1	**2**	**3**	**4**	**5**	**6**
30	8	5	4	6	5	2

Find the values of the following:

1. $\dfrac{\text{Number of times 1 turned up}}{\text{Total number of times the dice is rolled}}$

2. $\dfrac{\text{Number of times 2 turned up}}{\text{Total number of times the dice is rolled}}$

3. $\dfrac{\text{Number of times 3 turned up}}{\text{Total number of times the dice is rolled}}$

4. $\dfrac{\text{Number of times 4 turned up}}{\text{Total number of times the dice is rolled}}$

5. $\dfrac{\text{Number of times 5 turned up}}{\text{Total number of times the dice is rolled}}$

6. $\dfrac{\text{Number of times 6 turned up}}{\text{Total number of times the dice is rolled}}$

By repeating the above 20, 40 or 50 times, we observe the following:

1. The value of each fraction is a proper fraction and the value of it lies between 0 and 1.

2. The value of each fraction approaches $\dfrac{1}{6}$.

In the above examples each toss of a coin or each roll of a dice is called a trial.

When a coin is tossed, the possible outcome is a head or a tail.

Getting a head or tail in a particular throw is called an event with outcome head or tail.

When a dice is rolled, the possible outcome is 1, 2, 3, 4, 5 or 6. Getting a particular numbered face is an event. Getting an odd number is also an event and this event consists of three outcomes 1, 3 and 5. Getting a prime number is also an event and consists of three outcomes 2, 3 and 5. Getting a multiple of 3 is an event which consists of two outcomes 3 and 6.

An event can be defined as the collection of some outcomes of a trial, or we can say an event is a subset of a set of all the outcomes of a trial.

The probability of happening of any event (E) of a trial is denoted by $P(E)$ and is defined as

$$P(E) = \frac{\text{Number of favourable cases for the event to happen}}{\text{Total number of trials}}.$$

EXAMPLE 10.1

A coin is tossed 1200 times whereby head occurred in 745 cases and in the remaining tail occurred. Find the probability of each event.

SOLUTION

Given that the coin is tossed 1200 times.

Total number of trials = 1200

Let the event of getting a head be H, and getting a tail be T.

The number of times the event H occurred = 745

$$\text{Probability of event } H, \text{ i.e., } P(H) = \frac{\text{Number of times head occured}}{\text{Total number of trials}} = \frac{745}{1200} = \frac{149}{240}$$

Number of times tail, i.e., event T occurred = 1200 − 745 = 455.

$$\therefore \text{Probability of event } T, \text{ i.e., } P(T) = \frac{\text{number of times tail occured}}{\text{total number of trials}}$$

$$= \frac{455}{1200} = \frac{91}{240}$$

Notes

1. The number of possible outcomes in each of the above trial is two (H and T).
2. $P(H) + P(T) = \frac{149}{240} + \frac{91}{240} = \frac{240}{240} = 1.$

EXAMPLE 10.2

A dice is rolled 100 times with the frequencies of the outcomes 1, 2, 3, 4, 5 and 6, which are given in the following table. Find the probability of getting each outcome.

Outcome	1	2	3	4	5	6
Frequency	10	18	15	25	20	12

SOLUTION

Given that the total number of trials = 100

Let each event be denoted by E_i, where i = 1, 2, 3, 4, 5 and 6.

$$\text{Then, probability of getting } 1 = P(E_1) = \frac{\text{Frequency of 1}}{\text{Total numbers of trials}} = \frac{10}{100}.$$

$$\text{Similarly, } P(E_2) = \frac{18}{100}, P(E_3) = \frac{15}{100}, P(E_4) = \frac{25}{100}, P(E_5) = \frac{20}{100} \text{ and } P(E_6) = \frac{12}{100}.$$

Notes

1. The possible outcomes in each trial are 1, 2, 3, 4, 5 and 6.
2. Sum of the probabilities, $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1.$

EXAMPLE 10.3

A bulb-manufacturing company kept a record of the number of hours it can glow before a bulb needed to be replaced. The following table shows the results of 500 bulbs:

Life of the Bulb (in hours)	Less than 1500	1500 to 2500	2500 to 5000	More than 5000
Frequency	10	150	250	90

If you buy a bulb of this company, then find the probability of the following events:

(a) The bulb needs to be replaced before it glows for less than 1500 hours.

(b) The bulb needs to be replaced when it glows between 1500 hours and 5000 hours.

(c) The bulb lasts for more than 2500 hours.

SOLUTION

Total number of bulbs tested = 500.

(a) Number of bulbs replaced for less than 1500 hours = 10

$$P(\text{bulb to be replaced before it glows for} < 1500 \text{ hours}) = \frac{10}{500} = 0.02.$$

(b) Number of bulbs replaced between 1500 hours and 5000 hours = 150 + 250 = 400

$$P(\text{bulb to glow between 1500 hours and 5000 hours}) = \frac{400}{500} = 0.8.$$

(c) Number of bulbs that last for more than 2500 hours = 250 + 90 = 340

$$P(\text{bulb lasts for more than 2500 hours}) = \frac{340}{500} = 0.68.$$

EXAMPLE 10.4

The percentage of monthly targets achieved in producing a certain type of bolts in a company for different months is given in the following table:

Month	Jan	Feb	Mar	April	May	June
Target Achieved (in per cent)	55	87	73	82	74	80

Find the probability that the company achieved 80% of the monthly target in producing the bolts.

SOLUTION

The total number of months for which target is fixed = 6.

The number of months the company achieved 80% of the target fixed = 3.

$$\therefore P(\text{achieving 80\% of the target in producing bolts}) = \frac{3}{6} = \frac{1}{2} = 0.5.$$

EXAMPLE 10.5

A pack has 90 cards. Each card was marked with a different number among 110 to 199. A card was selected at random. Find the probability that the number on it is not a perfect square.

(a) $\frac{37}{45}$ **(b)** $\frac{13}{15}$ **(c)** $\frac{41}{45}$ **(d)** $\frac{43}{45}$

SOLUTION

Among 110 to 199, the perfect squares are 121, 144, 169 and 196. Let the number on the card be x.

Probability (x being a perfect square) $= \frac{4}{90} = \frac{2}{45}$.

$\therefore$ Required probability $= 1 - \frac{2}{45} = \frac{43}{45}$.

EXAMPLE 10.6

A number was chosen at random from the first 300 three digits natural numbers. Find the probability of it ending with a zero.

(a) $\frac{1}{15}$ **(b)** $\frac{1}{25}$ **(c)** $\frac{1}{10}$ **(d)** $\frac{1}{20}$

SOLUTION

Let the number be x, $100 \le x \le 399$.

A number ending with a 0 is divisible by 10.

Least value of x, divisible by 10 = 100 = 10(10)

Greatest value of x, divisible by 10 = 390 = 10(39)

Number of values of x divisible by 10 = number of numbers from 10 to 39 = 30

$\therefore$ Required probability $= \frac{30}{300} = \frac{1}{10}$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. When two coins are tossed simultaneously, list out the possible outcomes.
2. When a dice is rolled, what are all the possible outcomes?
3. Two coins are tossed. Find the number of outcomes of getting one head.
4. What is the sum of all the probabilities of trials of an experiment?
5. When a dice is rolled, then what is the number of possible outcomes of obtaining an even number?
6. Probability of occurring of an event always lies between ______.
7. Let n be the number of trials that an event E occurred and m be the total number of trials, then find the probability of the event E.
8. A dice is rolled. Find the number of outcomes of getting a composite number.
9. When a dice is rolled the probability that the number on the face showing up is greater than 6 is ______.
10. A coin is tossed 20 times and head occurred 12 times. How many times did tail occur?

Short Answer Type Questions

11. A coin is tossed 500 times. Head occurs 343 times and tail occurs 157 times. Find the probability of each event.
12. A day is selected in a week, find the probability that the day is Monday.
13. In a monthly test, 10 students were awarded marks in a Mathematics examination as follows:

    23, 25, 18, 15, 20, 17, 10, 24, 15, 19

    If a student is selected at random, what is the probability that he gets more than 18 marks?
14. In a cricket match, Dhoni hits a six 4 times from 24 balls he plays. Find the probability of hitting a six.
15. Three coins are tossed 100 times, and three heads occurred 25 times, two heads occurred 38 times, one head occurred 14 times and head did not occur 23 times. Find the probability of getting more than one head.

Essay Type Questions

16. A dice is rolled 250 times, and the outcomes 1, 2, 3, 4, 5 and 6 occurred as given in the following table:

Outcome	1	2	3	4	5	6
Frequency	56	78	46	27	19	24

    Find the probability of getting an odd number.
17. A Class IX English book contains 200 pages. A page is selected at random. What is the probability that the number on the page is divisible by 10?
18. In a colony there are 75 families and each family has two children. The number of male children of the families is as follows:

Male Child	2	1	0
Number of Families	27	38	10

    If a family is selected at random, what is the probability that the family has only one male child?
19. The percentage of marks obtained by a student in a monthly test is as follows:

Test	I	II	III	IV
Marks	78%	63%	82%	65%

    What is the probability that the student gets more than 75% marks in a test?
20. A box contains 50 tickets. Each ticket is numbered from 1 to 50. One ticket is selected at random, find the probability that the number on the ticket is not a perfect square.

CONCEPT APPLICATION

Level 1

1. If a month is selected at random in a year, then find the probability that the month is either March or September.

   (a) $\frac{1}{12}$ (b) $\frac{1}{6}$

   (c) $\frac{3}{4}$ (d) None of these

2. A coin is tossed 1000 times. Head occurred 625 times. Find the probability of getting a tail.

   (a) $\frac{5}{8}$ (b) $\frac{7}{8}$

   (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

3. A dice is rolled 600 times and the occurrence of the outcomes 1, 2, 3, 4, 5 and 6 are given below in the table:

Outcome	1	2	3	4	5	6
Frequency	200	30	120	100	50	100

   Find the probability of getting a prime number.

   (a) $\frac{1}{3}$ (b) $\frac{2}{3}$

   (c) $\frac{49}{60}$ (d) $\frac{39}{125}$

4. A bag contains 50 coins and each coin is marked from 51 to 100. One coin is picked at random. What is the probability that the number on the coin is not a prime number?

   (a) $\frac{1}{5}$ (b) $\frac{3}{5}$

   (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

5. From the letters of the word 'MOBILE', if a letter is selected, what is the probability that the letter is a vowel?

   (a) $\frac{1}{3}$ (b) $\frac{4}{7}$

   (c) $\frac{3}{7}$ (d) $\frac{1}{2}$

6. The percentage of attendance of different classes in a year, in a school is given below:

Class	X	IX	VIII	VII	VI	V
Attendance	30	62	85	92	76	55

   What is the probability that the class attendance is more than 75%?

   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$

   (c) $\frac{5}{6}$ (d) $\frac{1}{2}$

7. In a book, the frequency of units digit of a number on the pages is given below:

Units Digits	Frequency
0	50
1	40
2	10
3	25
4	15
5	80
6	90
7	110
8	120
9	60

   Find the probability of getting 8 in the units place on the pages.

   (a) $\frac{1}{5}$ (b) $\frac{1}{10}$

   (c) $\frac{1}{4}$ (d) $\frac{1}{60}$

8. 10 bags of rice, each bag marked 10 kg, actually contained the following weights of rice (in kgs). 10.03, 10.09, 9.97, 9.98, 10.01, 9.94, 10.05, 9.99, 9.95, 10.02. Find the probability that the bag chosen at random contains more than 10 kg.

   (a) $\frac{1}{2}$ (b) $\frac{3}{5}$

   (c) $\frac{5}{8}$ (d) $\frac{2}{5}$

9. If a three digit number is chosen at random, what is the probability that the chosen number is a multiple of 2?

PRACTICE QUESTIONS

(a) $\frac{499}{900}$ (b) $\frac{5}{9}$

(c) $\frac{1}{2}$ (d) $\frac{500}{899}$

10. If a two digits number is chosen at random, what is the probability that the number chosen is a multiple of 3?

(a) $\frac{3}{10}$ (b) $\frac{29}{100}$

(c) $\frac{1}{3}$ (d) $\frac{7}{25}$

Level 2

11. A mathematics book contains 250 pages. A page is selected at random. What is the probability that the number on the page selected is a perfect square?

(a) $\frac{3}{25}$ (b) $\frac{7}{50}$

(c) $\frac{3}{50}$ (d) $\frac{7}{125}$

12. The runs scored by Sachin Tendulkar in different years is given below:

Year	Score
1996–97	1000
1997–98	3000
1999–2000	1000
2000–01	5000
2001–02	3000
2002–03	8000
2003–04	4000

What is the probability that in a year Sachin scored more than 3000 runs?

(a) $\frac{3}{7}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{5}{8}$

13. To know the opinion of people about the political leaders, a survey on 1000 members was conducted. The data recorded is shown in the following table:

Option	Number of People
Like	200
Dislike	500
No opinion	300

Find the probability that a person chosen at random is with no opinion on political leaders.

(a) $\frac{1}{2}$ (b) $\frac{3}{10}$

(c) $\frac{1}{5}$ (d) None of these

14. From 101 to 500, if a number is chosen at random, what is the probability that the number ends with 0?

(a) $\frac{41}{399}$ (b) $\frac{40}{399}$

(c) $\frac{1}{10}$ (d) $\frac{41}{400}$

15. A bag contains 12 pencils, 3 sharpeners and 7 pens. What is the probability of drawing a pencil from the bag?

(a) $\frac{6}{11}$ (b) $\frac{3}{22}$

(c) $\frac{7}{22}$ (d) $\frac{15}{22}$

16. Find the probability of getting a sum 10, when two dice are rolled.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) When the two dices are rolled, the number of possible outcomes $= 6 \times 6 = 36$.

(B) Favourable outcomes are (4, 6), (5, 5) and (6, 4).

(C) The required probability $= \frac{3}{36} = \frac{1}{12}$.

(D) When a dice is rolled, the possible outcomes are 1, 2, 3, 4, 5 and 6.

(a) BADC (b) DBAC

(c) BDAC (d) DABC

17. Find the probability of getting a difference of 4, when two dice are rolled.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) When two dices are rolled, the number of possible outcomes = $6 \times 6 = 36$.

(B) When a dice is rolled, the possible outcomes are 1, 2, 3, 4, 5 and 6.

(C) The required probability $= \frac{4}{36} = \frac{1}{9}$.

(D) Favourable outcomes are (1, 5), (5, 1), (2, 6) and (6, 2).

(a) ABCD (b) ABDC

(c) BADC (d) ADBC

18. In a football match, Ronaldo scores 4 goals from 10 penalty kicks. Find the probability of converting a penalty kick into a goal by Ronaldo.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{5}$

19. From the month of August, whose first day is Tuesday, a day is selected at random. Find the probability that the day selected is not a Tuesday.

(a) $\frac{5}{6}$ (b) $\frac{26}{31}$

(c) $\frac{6}{31}$ (d) $\frac{27}{31}$

20. In a cricket match, Warne took three wickets in every 27 balls that he bowled. Find the probability of a batsman not getting out by Warne's bowling.

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$

(c) $\frac{8}{9}$ (d) $\frac{5}{9}$

21. A day is selected at random from April, whose first day is Monday. Find the probability that the day selected is a Monday.

(a) $\frac{1}{7}$ (b) $\frac{1}{5}$

(c) $\frac{1}{6}$ (d) $\frac{2}{5}$

22. A month is selected at random in a year. Find the probability that it is either January or June.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{2}$

23. A biased dice was rolled 800 times. The frequencies of the various outcomes are given in the table below.

Outcome	1	2	3	4	5	6
Frequency	150	200	100	75	125	150

When the dice is rolled, the probability of getting a number which is a perfect square is ______ (approximately).

(a) $\frac{9}{32}$ (b) $\frac{11}{32}$

(d) $\frac{13}{32}$ (d) $\frac{15}{32}$

24. A two digits number is chosen at random. Find the probability that it is a multiple of 7.

(a) $\frac{11}{90}$ (b) $\frac{13}{90}$

(c) $\frac{7}{45}$ (d) $\frac{8}{45}$

25. In City X, there were 900 residents. A survey was conducted in it regarding the favourite beverages of the residents. The results of the survey are partially conveyed in the table below.

Beverages	Number of Residents Liking it/them
Only tea	350
Only coffee	250
Both tea and coffee	200

Find the probability that a resident chosen at random likes only tea or only coffee.

(a) $\frac{2}{3}$ (b) $\frac{5}{9}$

(c) $\frac{7}{9}$ (d) $\frac{4}{9}$

PRACTICE QUESTIONS

Level 3

26. Find the probability that a non-leap year contains exactly 53 Mondays.

(a) $\frac{6}{7}$ (b) $\frac{1}{7}$

(c) $\frac{52}{365}$ (d) None of these

27. Two dice were rolled simultaneously. Find the probability that the sum of the numbers on them was a two digits prime number.

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{1}{6}$

28. Three biased coins were tossed 800 times simultaneously. The outcomes are given in the table below partially.

Outcome	No head	One head	Two heads
Frequency	120	280	x

If the occurrence of two heads was thrice that of all heads. Find x.

(a) 150 (b) 240

(c) 300 (d) 360

29. In the Question 27, find the probability that the sum of the numbers on the dice was a perfect cube.

(a) $\frac{5}{36}$ (b) $\frac{7}{36}$

(c) $\frac{2}{9}$ (d) $\frac{1}{6}$

30. A three digits number was chosen at random. Find the probability that it's hundreds digit, tens digit and units digit are consecutive integers in descending order.

(a) $\frac{1}{75}$ (b) $\frac{4}{225}$

(c) $\frac{2}{225}$ (d) $\frac{1}{45}$

31. x = ABCDEFGHIJ…Z. Find the probability of a letter selected from those in odd positions of x being a vowel.

(a) $\frac{5}{13}$ (b) $\frac{6}{13}$

(c) $\frac{7}{13}$ (d) $\frac{8}{13}$

32. In a bag, there are 2 red balls, 3 green balls and 4 brown balls. Find the probability of drawing a ball at random being red or green.

(a) $\frac{5}{9}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{4}{9}$

33. Year X is not a leap year. Find the probability of X containing exactly 53 Sundays.

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{1}{14}$

34. A three digits number was chosen at random. Find the probability that it is divisible by both 2 and 3.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{8}$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. HH, HT, TH or TT
2. 1, 2, 3, 4, 5 or 6
3. HT and TH
4. 1
5. 3
6. 0 and 1
7. $\frac{n}{m}$
8. 2
9. 0
10. 8

Short Answer Type Questions

11. $\frac{343}{500}, \frac{157}{500}$
12. $\frac{1}{7}$
13. $\frac{1}{2}$
14. $\frac{1}{6}$
15. $\frac{63}{100}$

Essay Type Questions

16. $\frac{121}{250}$
17. $\frac{1}{10}$
18. $\frac{38}{75}$
19. $\frac{1}{2}$
20. $\frac{43}{50}$

CONCEPT APPLICATION

Level 1

1. (b) **2.** (d) **3.** (a) **4.** (d) **5.** (d) **6.** (d) **7.** (a) **8.** (a) **9.** (c) **10.** (c)

Level 2

11. (c) **12.** (a) **13.** (b) **14.** (c) **15.** (a) **16.** (d) **17.** (c) **18.** (d) **19.** (b) **20.** (c)
21. (c) **22.** (c) **23.** (a) **24.** (b) **25.** (a)

Level 3

26. (b) **27.** (b) **28.** (c) **29.** (a) **30.** (c) **31.** (a) **32.** (a) **33.** (a) **34.** (b)

CONCEPT APPLICATION

Level 1

1. P(March or September)

$$= \frac{2}{\text{Total number of months in a year}}.$$

2. $P(\text{getting a tail}) = \dfrac{\text{Number of times tail occured}}{\text{Number of times coin tossed}}.$

3. P(getting a prime number)

$$= \frac{\text{Sum of frequences of getting prime numbers}}{\text{Total number of times dice rolled}}.$$

4. P(getting a number which is not a prime)

$$= \frac{\text{Number of numbers which are not prime from 51 to 100}}{\text{Total numbers from 51 to 100}}.$$

5. (i) Probability of selecting a vowel

$$= \frac{\text{Number of vowels}}{\text{Total number of letters}}.$$

(ii) The word contains 3 vowels, *viz.*, O, I, E.

(iii) The total number of letters in the word are 6.

6. P(the attendance of the class more than 75%)

$$= \frac{\text{Number of classes with more than 75\% attendance}}{\text{Total number of classes}}.$$

7. $P(\text{units place is 8}) = \dfrac{\text{Frequency of 8}}{\text{Sum of frequencies}}.$

8. P(a bag chosen is with > 10 kg rice)

$$= \frac{\text{Number of bags with weight} > 10\text{ kg}}{\text{Total number of bags}}.$$

9. (i) Total number of three digits numbers = 900. Total number of three digits even numbers = 450.

(ii) The first and the last three digits numbers, which are multiples of 2 are 100 and 998.

(iii) Count the numbers in the above case.

(iv) The total number of three digits numbers are 900.

(v) Apply the formula to find the required probability.

10. P(choosing a multiple of 3)

$$= \frac{\text{Number of two digit numbers divisible by 3}}{\text{Total number of two digit numbers}}.$$

Level 2

12. (i) Total years = 7

(ii) Favourable years = 3

(iii) In the years 2000–01, 2002–03 and 2003–04, he scored more than 3000 runs.

13. P(a person chosen with no opinion on political leaders) $= \dfrac{\text{Number of persons with no opinion}}{\text{Total number of persons surveyed}}.$

14. (i) Find the numbers which are divisible by 10 in between 101 and 500.

(ii) There are a total of 400 numbers from 101 to 500.

(iii) The numbers which end with 0 are 110, 120, 130, …, 200, 210, …, 500.

(iv) Count the above numbers and apply the formula.

15. (i) Probability of drawing a pencil

$$= \frac{\text{Number of pencils}}{\text{Total number of items in the bag}}.$$

(ii) The bag contains a total of 22 articles.

16. DABC is the required sequential order.

17. BADC is the required sequential order.

18. $P(\text{converting into a goal}) = \dfrac{4}{10} = \dfrac{2}{5}.$

19. If August starts with Tuesday it will have 5 Tuesdays out of 31 days.

$\therefore P(\text{not a Tuesday}) = \dfrac{26}{31}.$

20. $P(\text{batsman getting out}) = \dfrac{3}{27} = \dfrac{1}{9}.$

$\therefore P(\text{batsman not getting out}) = 1 - \dfrac{1}{9} = \dfrac{8}{9}.$

21. $P(\text{Monday}) = \dfrac{5}{30} = \dfrac{1}{6}.$

22. Required Probability = Probability (it being January) + Probability (it being June)

$$= \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

23. The possible perfect squares which can be obtained are 1 and 4. Required probability = probability (obtaining 1 or 4) = Probability (obtaining 1) + probability (obtaining 4)

$$= \frac{150}{800} + \frac{75}{800} = \frac{225}{800} = \frac{9}{32}.$$

24. The two digits multiples of 7 are 14, 21, 28, …, 98.

Number of such numbers = 14 − 1 = 13. (first 14 multiples except 7)

There are 90 two digits numbers.

$\therefore$ Required probability $= \frac{13}{90}$.

25. Required probability = Probability (resident liking only tea) + probability (resident liking only coffee)

$$= \frac{350}{900} + \frac{250}{900} = \frac{600}{900} = \frac{2}{3}.$$

Level 3

26. (i) On a non-leap year, every day repeats 52 times with one day left.

(ii) A non-leap year has 365 days, i.e., (52 weeks + 1 day).

(iii) The extra one day should be Monday.

27. Let the numbers on the dice be a and b, a, $b \le 6$.

If $a + b$ was a two digits prime number, it must be 11.

In this case, $(a, b) = (5, 6)$ or $(6, 5)$

$\therefore$ Required probability $= 2\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{18}$.

28. Let the frequency of all heads be y.

$$\frac{x}{800} = 3\left(\frac{y}{800}\right), \text{ i.e., } x = 3y \qquad (1)$$

$$120 + 280 + x + y = 800, \text{ i.e., } x + y = 400$$

From Eq. (1) $\Rightarrow x + \frac{x}{3} = 400$

$x = 300$.

29. In the solution of Question 27, $1 \le a$, $b \le 6$.

$2 \le a + b \le 12$.

If $a + b$ was a perfect cube, $a + b$ must be 8.

$\therefore$ $(a, b) = (2, 6)$ or $(3, 5)$ or $(5, 3)$ or $(4, 4)$ or $(6, 2)$.

$\therefore$ Required probability $= 5\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{5}{36}$.

30. Let the number be xyz.

If x, y and z are consecutive integers in descending order, xyz = 987 or 876 or 765 or 654 or 543 or 432 or 321 or 210.

Required probability $= \frac{8}{900} = \frac{2}{225}$.

31. The vowels are A, E, I, O and U. In x, the positions of A, E, I, O and U are 1, 5, 9, 15 and 21 respectively. A total of 13 odd positions are present in x, of which 5 are occupied by vowels.

$\therefore$ Required probability $= \frac{5}{13}$.

32. There are 2 red balls and 3 green balls. The number of possible outcomes = 2 + 3 = 5.

The total number of cases = 2 + 3 + 4 = 9

The required probability $= \frac{5}{9}$.

33. A non-leap year has 365 days, i.e., 52 weeks and 1 day. In the first 52 weeks of that year, there will be 52 Sundays. The 1st day of the 53rd week would be the last day of that year. The day of the week on this day would be the same as the day of the week on the 1st day of that year. If the first day of a non-leap year is a Sunday, that year will have 53 Sundays. Otherwise it will have 52 Sundays.

Required probability (first day of a week being a Sunday) $= \frac{1}{7}$.

35. Let the number be x, $100 \le x \le 999$.

A number divisible by both 2 and 3 must be divisible by the LCM of (2, 3) = 6.

Least value of x divisible by 6 = 102 = 6(17)

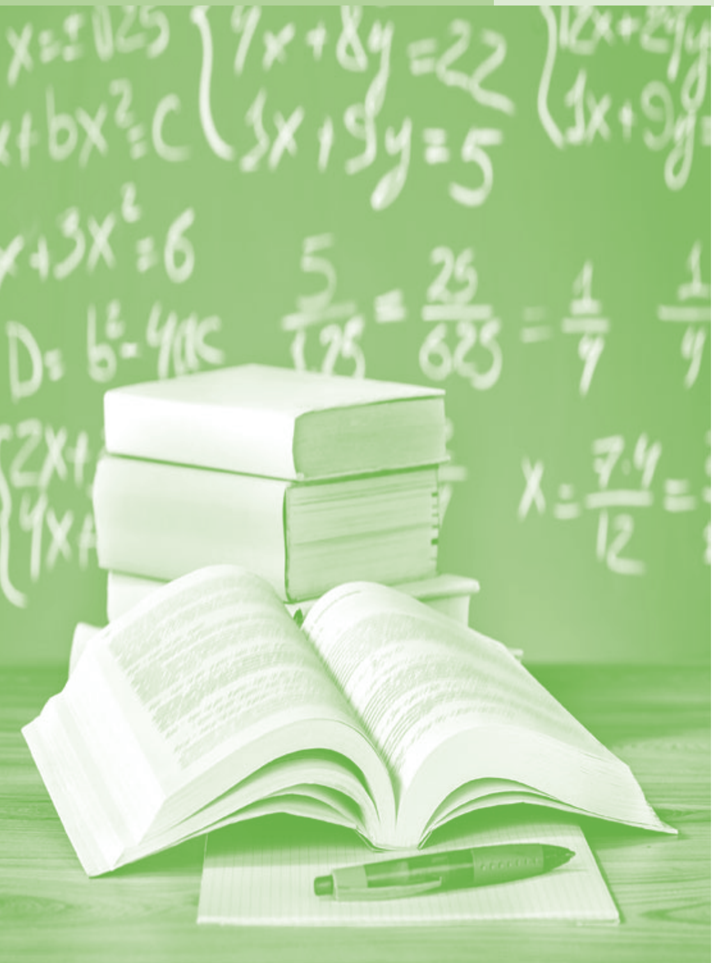
Greatest value of x divisible by 6 = 996 = 6(166)

There are 150 values of x divisible by 6.

Required probability $= \frac{150}{900} = \frac{1}{6}$ (There are 900 three digits numbers).

Chapter 11

Banking and Computing



BANKING (PART I)

REMEMBER

Before beginning this chapter, you should be able to:

- Understand the basic terms related to money
- The concept of bank as a financial institution

KEY IDEAS

After completing this chapter, you should be able to:

- Learn about remittance of funds, safe deposits in lockers, and public utility services
- Understand different types of deposit accounts including savings bank accounts, current accounts, and term deposit accounts
- Study different types of cheques and parties dealing with cheques
- Aware on the functions of loans, and the meaning of overdrafts
- Calculate interests on bank savings accounts
- Study hire purchase and instalment scheme

INTRODUCTION

Before people began using money, purchase and sale of goods used to happen through exchange of goods. This system is called, 'Barter system'. In this system, there was no uniform valuation of the traded goods. To establish a standard, value of all goods were converted to monetary units. This ensured payment of justified price for the goods purchased.

Once the monetary system became a standard method of value exchange, the necessity to ensure safety of money came into existence. With an intention to safeguard money and to facilitate availability of money in the society, the banking system gradually developed.

Among the various types of services offered by banks, taking deposits and providing loans are the basic ones. Apart from these, banks render the following types ancillary services.

Remittance of Funds

Banks help in transferring money from one place to another in a safe manner. They do this by issuing demand drafts, money transfer orders, and telegraphic transfers. Banks also issue traveller's cheques in home currency and also in foreign currency. This helps travellers minimizing the risk of theft or loss of money while travelling in a different location. A traveller's cheque can be easily converted to cash. With the advent of technology, money transfer has become easy through Internet and phone banking.

Safe Deposit Lockers

Banks provide safety lockers to customers to safely preserve their valuables. Customers can store their valuables, like gold ornaments, important documents in bank lockers by paying a small amount of rent charged by the bank.

Public Utility Services

Through bank accounts, customers can pay their telephone bills, electricity bills, insurance premium, and several other services.

DEPOSIT ACCOUNTS

Bank deposit accounts are meticulously designed to meet the various types of financial requirements of the customers. These are planned based on the financial capabilities of the customers. At present, banks in Indian offer the following types of deposit accounts.

Savings Bank Account

An Indian individual, either resident or non-resident, can open a savings bank account with a minimum balance of ₹500. The minimum balance may vary from bank to bank. A passbook is issued to the customer. It contains all the particulars of the transactions and the balance. Such an account can be opened in joint names also. It is known as a 'joint account'. If one of the joint account holders is a minor, the following guidelines are applicable: A minor who is at least 10-year-old, can open an account in a bank or a post office. However, the minimum age to open an account and to operate an account differs from bank to bank and post office. In a post office, the minimum age to open and operate a savings account is 10 years. If the minor's minimum age to operate an account is less than his/her minimum age, a guardian can operate the minor's account. A savings bank account carries certain amount of interest compounded half-yearly. The rate of interest varies from bank to bank. It may also vary from time to time. Cheque books are issued to an account-holder against a requisition slip duly filled up and signed

by the person. If a customer operates his/her account through cheques, then it is known as 'cheque-operated account'.

Depositing Money in the Bank Accounts

Money can be deposited in a bank account either by cash or through a duly filled pay-in-slip or challan. Pay-in-slips can be used for payment through cash or cheque.

Demand Draft

Money can be deposited through demand drafts (i.e., bank drafts). A person willing to send money to another person may purchase a bank draft.

A bank draft is an order issued by a bank to its specified branch or to another bank (if there is a tie-up) to make payment of the amount to the party, in whose name the draft is issued.

The purchaser of the draft specifies the name and address of the person to whom the money is being sent, which is written on the bank draft. The payee can encash the draft by presenting it at the specified branch or bank.

Withdrawal of Money from Saving Bank Account

Money deposited in these accounts can be withdrawn by using withdrawal slips or cheques. A specimen of a cheque is given below.

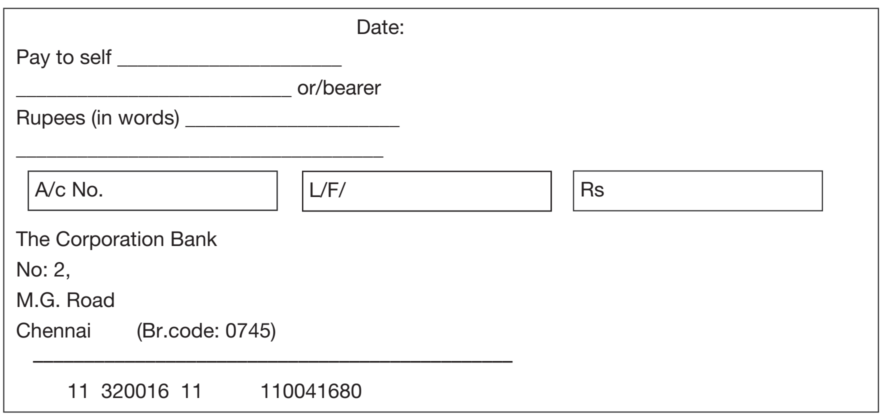

Date:
Pay to self ____________________
________________________ or/bearer
Rupees (in words) ___________________
____
A/c No. | L/F/ | Rs
The Corporation Bank
No: 2,
M.G. Road
Chennai (Br.code: 0745)
11 320016 11 110041680

Figure 11.1

Cheque books are issued only to those account-holders who fulfill certain special requirements, such as maintenance of minimum balance, updated information related to the account.

TYPES OF CHEQUES

Bank cheques are classified into two types. These are as follows:

Bearer Cheque

A bearer cheque can be encashed by anyone who possesses the cheque, though the person's name is not written on the cheque. There is a risk of wrong a person getting the payments.

If the word 'bearer' is crossed-out in a cheque, then the person whose name appears on it can alone encash. This type of cheque is known as 'order cheque'.

Crossed Cheque

When two parallel lines are drawn at the left-hand top corner of a cheque, it is called ‘crossed cheque’. The words ‘A/C Payee’ may or may not be written between the two parallel lines. The payee has to deposit crossed cheque in his/her account. The collecting bank collects the money from the drawer’s bank, and it is credited to the payee’s account.

Bouncing of Cheques

If an account-holder issues a cheque for an amount exceeding the balance in his account, the bank refuses to make payment. In such a situation, the cheque is said to be a dishonoured one. This is known as bouncing of cheque. If a cheque bounces, the issuer of the cheque is liable for prosecution under the Negotiable Instruments Act, 1887.

Safeguards to be Taken While Maintaining ‘Cheque-operated Accounts’

1. Immediately after receiving a cheque book, a customer verifies if all the leaves are serially arranged and printed with correct numbers.
2. Blank cheques should not be issued to anybody except the account holder.
3. Any changes, alterations, corrections made while filling a cheque, should be authenticated with full signature.
4. The amount on a cheque has to be written in words and figures legibly.
5. The amount of the cheque should be written immediately after the printed words ‘Rupees’ or ‘Rs’. Also, the word ‘only’ should be mentioned after the amount in words.
6. A cheque becomes outdated or stale after six months from the date of issue. Hence, it should be presented within six months from the date of issue.

Parties Dealing with a Cheque

Drawer

The account-holder who writes a cheques and signs on it to withdraw money is called ‘drawer’ of the cheque.

Drawee

The bank on whom a cheque is drawn is called ‘drawee bank’ as it pays the money.

Payee

The party to whom the amount of cheque is payable is called the ‘payee’. The payee has to affix his/her signature on the back of the cheque.

Any savings bank account-holder can withdraw money from his/her account using a withdrawal form, a specimen of which is given below.

State Bank of India

----------------------- Branch

Date:

Name of the account holder: ______________

Account No. | | | | | | | |

Note: This form is not a cheque.

Payment will be rejected if this form is not submitted along with the pass book.

Please pay self/ourselves only.

Rupees ________________________________ only

and debit the amount from my/our above savings bank account.

Rs

Token No.	PAY CASH	
Scroll No.	Passing officer	Signature of the customer

Figure 11.2

Banks impose restrictions on the number of times of withdrawal of money from savings bank accounts. A violation of such restriction attracts a nominal charge. The interest on savings bank accounts are paid half-yearly by taking the minimum balance for each month as the balance for that entire month. 'Minimum balance' is the least of all the balances left in the account beginning from the 10th to the previous day of that month.

Example: The following table shows the particular of the closing balances of a savings account during the month of March, 2006:

Date	Closing Balance
5th March	₹1800-00
10th March	₹2400-00
18th March	₹3500-00
25th March	₹1700-00
31st March	₹2500-00

From the given table, it can be observed that the closing balance on 25th March, i.e., ₹1700, is the minimum closing balance between 10th of March and the last day of March. This is the minimum balance for the month of March.

The monthly minimum balances for every six months are calculated. Based on these, the interest for six months is calculated. Most of the banks add the interest to the existing balance once in every half year. That is, on June 30th and December 31st.

However, the periodicity of interest calculation differs between banks and post offices.

Calculation of Interest on Savings Accounts in Banks

The monthly minimum balances from January to the end of June are added. This total amount is called the 'product' in banks. Interest is calculated on this product and added to the opening

balance on July 1st. In the same manner, the interest for the next half year is calculated and added to the opening balance on January 1st.

In savings account, interest is calculated by maintaining the following steps:

1. The least of the balances from the 10th day of a month to the last day of the month is considered as the balance for that month.
2. The sum of all these monthly balances is considered as the principle for calculating interest.
3. Interest $= \dfrac{\text{Principle} \times \text{Rate of interest}}{12 \times 100}$

EXAMPLE 11.1

The following is an extract of the savings bank pass book of Mrinalini who holds an account with Corporation Bank.

Calculate the interest accrued on the account at the end of June, 2005 at 5% per annum.

		Amount Withdrawn		Amount Deposited		Balances
Date	Particulars	₹	P	₹	P	₹ P
7-1-2005	Balance B/F					8400 00
10-1-2005	By cash			12,500	00	20,900 00
31-1-2005	To cheque No. 3541	6500	00			14,400 00
15-2-2005	By cash			3500	00	17,900 00
13-3-2005	To cheque No. 3543	2800	00			15,100 00
25-3-2005	By cheque			2000	00	17,100 00
3-4-2005	To cheque No. 3544	1400	00			15,700 00
18-4-2005	To cheque No. 3545	3500	00			12,200 00
21-5-2005	By cash			5400	00	17,600 00
15-6-2005	To cheque No. 3546	6000	00			11,600 00
21-6-2005	To cheque No. 3547	2000	00			9600 00
15-7-2005	By cash			3500	00	13,100 00

SOLUTION

The minimum balance (in ₹)

for January = 14,400

February = 14,400

March = 15,100

April = 12,200

May = 12,200

June = 9600

77,900

The product is ₹77,900.

$\therefore$ Interest $= \dfrac{\text{Product} \times \text{Rate}}{100 \times 12} = \dfrac{77{,}900 \times 5}{100 \times 12} = ₹324.58$

Current Account

Current account convenient for business people, companies, government offices, and various other institutions requiring frequent and large amounts of monetary transactions. Banks do not give any interest on these accounts, but the operation of these accounts is flexible. There is no restriction on amounts deposited or withdrawn (i.e., on the number of transactions) as savings bank accounts.

Term Deposit Accounts

Term deposit accounts are of two types:

1. Fixed deposit accounts
2. Recurring deposit accounts

Fixed Deposit Accounts

Customers can avail the facility of depositing a fixed amount of money for a definite period of time. As the time period is fixed, banks give a higher rate of interest on these accounts. If money is withdrawn from these accounts before the specified time period, banks pay lesser interest than what was agreed upon. As this discourages premature withdrawal, banks rely more on these funds. The rate of interest payable varies with the period for which money is deposited in these accounts. However, it varies from bank to bank. The rates of interest offered by a bank on fixed deposits are as follows:

Time Period	Rate of Interest (%) per annum
15 days and upto 45 days	5.25
46 days and upto 179 days	6.50
180 days to less than 1 year	6.75
1 year to less than 2 years	8.00
2 years to less than 3 years	8.25
3 years and above	8.50

Recurring Deposit Accounts

Recurring deposit accounts help customers accumulate large amounts through small deposits. These accounts facilitate depositing a fixed amount per month for a time span of 6 months to 3 years, and above. This time period is called 'maturity period'.

The following table gives an idea about how the principal amounts are taken to calculate the interest in recurring deposit accounts.

Date	Deposit	Principal Amount on which Interest is to be Paid
1-4-2006	₹3000	₹3000
1-5-2006	₹3000	₹6000
1-5-2006	₹3000	₹9000

Recurring deposit accounts are helpful to people with low earnings. They can save large amounts through regular and fixed savings. A person who opens this account deposits an initially agreed amount each month. At the end of the maturity period, the cumulative amount with interest, which is called the 'maturity amount', is paid to the account-holder. The rates of interest payable on these accounts are same as those payable on fixed deposit accounts.

Recurring deposit interest is calculated by applying the following formula.

We know that $1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$

If a man deposits ₹k per month, for n months at r% per annum, then simple interest $=₹\left[k\times\dfrac{n(n+1)}{2}\times\dfrac{1}{12}\times\dfrac{r}{100}\right]$

EXAMPLE 11.2

Govind opened a bank account on 1-4-2006 by depositing ₹3000. He deposited ₹1000 on 11-04-2006 and withdrew ₹500 on 15-4-2006. Compute the interest paid by the bank for the month of April, if rate of interest is 4% per annum.

SOLUTION

Balance as on 1-4-2006 = ₹3000

as on 11-4-2006 = ₹4000

as on 15-4-2006 = ₹3500

The minimum balance for the month of April = ₹3000

$\therefore$ Interest paid by the bank for the month of April $=\dfrac{3000\times1\times4}{1200}=₹10$

EXAMPLE 11.3

Rajan makes fixed deposit of ₹8000 in a bank for a period of 2 years. If the rate of interest is 10% per annum compounded annually, find the amount payable to him by the bank after two years.

SOLUTION

The amount of fixed deposit = ₹8000

R = 10% per annum and $n = 2$

$\therefore$ The amount returned by the bank $=P\left[1+\dfrac{r}{100}\right]^n$

$$=8000\left[1+\frac{10}{100}\right]^2$$

$$=8000[1+0.1]^2=8000\times1.21=₹9680$$

EXAMPLE 11.4

Mahesh deposits ₹600 per month in a recurring deposit account for 2 years at 5% per annum. Find the amount he receives at the time of maturity.

SOLUTION

Here, $P = ₹600$

$N = 2\times12$ months and $R = 5\%$ per annum

$$\text{SI} = P \times \frac{n(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100}$$

$$= 600 \times \frac{24(25)}{2} \times \frac{1}{12} \times \frac{5}{100} = 750$$

$\therefore$ Total amount = (24 × 600) + 750

= 14,400 + 750 = ₹15,150.

LOANS

Bank loans can be classified into the following three categories:

1. Demand loans
2. Term loans
3. Overdrafts (ODs)

Demand Loans

The borrower has to repay the loans on demand. The repayment of the loan has to be done within 36 months from the date of disbursement of the loan. The borrower has to execute a demand promissory note in favour of the bank, promising that he would repay the loan unconditionally as per the stipulations of the bank.

Term Loans

The borrower enters into an agreement with the bank regarding the period of loan and mode of repayment, number of installments, etc. The repayment period is generally more than 36 months. These loans are availed by purchasers of machinery, build houses, etc.

Overdrafts (ODs)

A current account holder enters into an agreement with the bank which permits him to draw more than the amount available in his account, but upto a maximum limit fixed by the bank. These loans are availed by traders.

Calculation of Interest on Loans

Interest on loans is calculated on daily product basis. Once in every quarter the loan amount is increased by that amount.
Daily product = Balance × Number of days it has remained as balance.

$$\text{Interest} = \frac{\text{Sum of daily products} \times \text{Rate}}{100 \times 365}.$$

Note If the loan is fully repaid, the date on which it is repaid is not counted for calculation of interest. If the loan is partially repaid, the day of repayment is also counted for calculating the interest.

EXAMPLE 11.5

Ganesh takes a loan of ₹20,000 on 1-4-2005. He repays ₹2000 on the 10th of every month, beginning from May, 2005. If the rate of interest is 15% per annum, calculate the interest till 30-6-2005.

SOLUTION

Loan Amount	Loan Period	No. of Days	Daily Product
20,000	1.4.2005 to 10.5.2005	40	40 × 20,000 = ₹800,000
Repays ₹2000 on 11-5-2005 Balance = ₹18,000	11.5.2005 to 10.6.2005	31	31 × 18,000 = ₹558,000
Repays ₹2000 on 11-6-2005 Balance = ₹16,000	11.6.2005 to 30.6.2005	20	20 × 16,000 = ₹320,000

Total daily product (DP) = ₹1,678,000

$$\therefore \text{Interest} = \frac{\text{DP} \times \text{Rate}}{100 \times 365}$$

$$= \frac{1{,}678{,}000 \times 15}{100 \times 365} = ₹689.60$$

COMPOUND INTEREST

When interest is calculated on the principal amount as well as on interest, it is known as compound interest. The interest is added to the principal at regular intervals, quarterly or half yearly or yearly, and further interest is calculated on the increased principal thus obtained.

The formula to find out the amount payable when the interest is compounded annually is as follows:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where,

P = Principal
r = Rate of interest
n = Number of years

When interest is compounded k times a year, $A = P\left(1 + \frac{r}{k \times 100}\right)^{n \times k}$

When interest is compounded quarterly, $k = \frac{12}{3} = 4.$

When interest is compounded half-yearly, $k = \frac{12}{6} = 2,$ and so on.

HIRE PURCHASE AND INSTALMENT SCHEME

When a buyer does not have purchasing capacity, the seller allows the buyer to make part payments in monthly, quarterly, half-yearly or yearly instalments. This scheme is of two types:

1. Hire purchase scheme
2. Instalment scheme

Hire Purchase Scheme

In this scheme, the buyer is called the 'hirer' and the seller is called the 'vendor'. They enters into an agreement which is known as 'hire purchase agreement'.

Important Features of Hire Purchase Scheme

1. The hirer pays an initial payment known as down payment.
2. The vendor allows the hirer to take possession of the goods on the date of signing the agreement, but he does not transfer the ownership of the goods.
3. The hirer promises to pay the balance amount in instalments.
4. If the hirer fails to pay the instalments, the vendor can repossess the goods.
5. Once goods are repossessed, the hirer cannot ask for repayment of the instalments of money already paid. This money paid will be treated as rent for the period.

Instalment Scheme

Under instalment scheme, the seller transfers the possession as well as the ownership of the goods to the buyer. The buyer has the right to resell or pledge the goods, but he has to repay outstanding instalments.

Finding the Rate of Interest on Buying in Instalment Scheme

The formula used in calculating the rate of interest in instalment purchase is:

$$R = \frac{2400E}{n[(n+1)I - 2E]}$$

R = Rate of interest

E = Excess amount paid

n = Number of instalments

I = Amount of each instalment

E = Down payment + Sum of instalment amounts – Cash price

EXAMPLE 11.6

A television set is sold for ₹9000 on ₹1000 cash down followed by six equal instalments of ₹1500 each. What is the rate of interest?

SOLUTION

$n = 6$

$I = ₹1500$

$E = [1000 + 6 \times 1500 - 9000] = ₹1000$

$$\therefore R = \frac{2400E}{n[(n+1)I - 2E]} = \frac{2400 \times 1000}{6[(6+1)1500 - 2000]} = 47.1\%$$

EXAMPLE 11.7

Chetan deposited ₹60,000 in a fixed deposit account for 2 years at 20% per annum, interest being compounded annually. At the end of the 2nd year, he withdrew certain amount. Chetan deposited the remaining amount for another one year at the same rate of interest. At the end of the third year, his account balance was ₹46,080. Find the amount that he withdrew (in ₹).

(a) 36,000 **(b)** 24,000 **(c)** 54,000 **(d)** 48,000

SOLUTION

Let the sum withdrew be ₹x.

Chetan's balance at the end of the second year (in ₹)

$$= (60,000)\left(1+\frac{20}{100}\right)^2 = 60,000(1.2)^2$$

$$= 60,000(1.44) = 86,400$$

∴ ₹(86,400 − x) must have amounted to ₹46,080 at the end of the third year.

$$(84,600 - x)\left(1+\frac{20}{100}\right) = 46,080$$

$$86,400 - x = \frac{46,080}{1.2} = 38,400$$

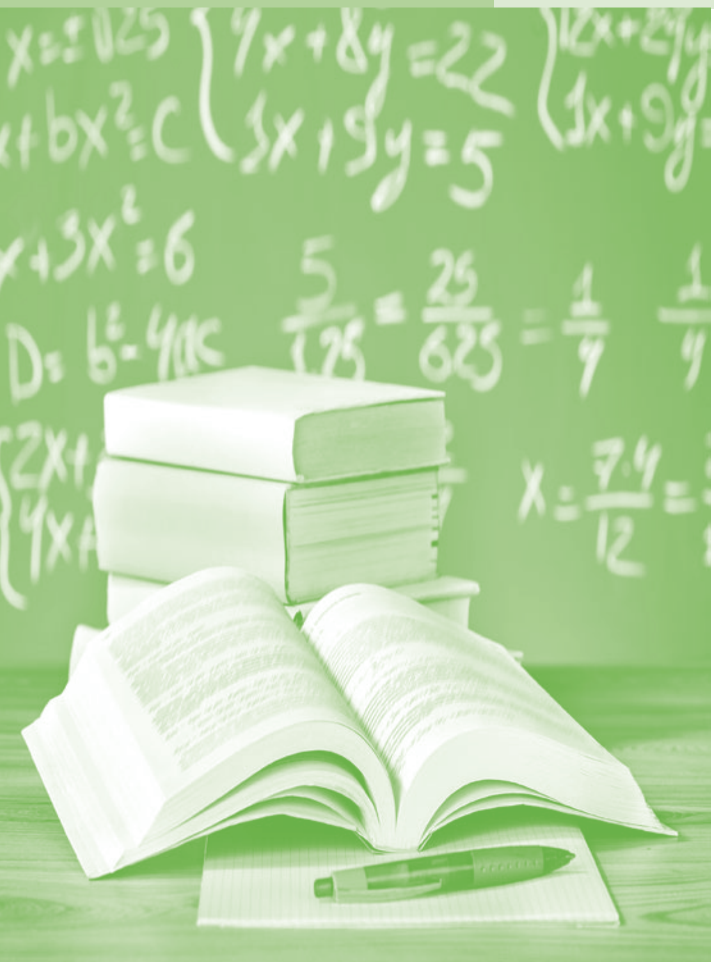
$$x = 48,000$$

∴ The sum withdrew is ₹48,000

Chapter

11

Banking and Computing



COMPUTING (PART II)

REMEMBER

Before beginning this chapter, you should be able to:

- State the concept of numbers
- Work on fundamental mathematical operators
- Review the concepts of computer technology

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the structure of a computer
- Learn about hardware and software
- Understand the basic algorithm
- Study different types of operators

INTRODUCTION

Computers are extensively used in various fields including banking, insurance, transportation, science and technology and entertainment, and so on. Complex tasks can be easily solved with the help of computers.

A computer is a multipurpose electronic device which is used to store information, process large amount of information and accomplish tasks with high speed and accuracy.

The basic concept of computer was developed by Charles Babbage in the 19th century. Since then, the architecture of computer has undergone many changes. Initially, vacuum tubes were used in computers. These were known as the first-generation computers. Later, vacuum tubes were replaced with transistors, and updating computers to the second generation. Small-scale integrated circuits were used in the third-generation computers. With rapid advancements in science and technology, very large-scale integrated (VLSI) circuits were fabricated. Computers in which VLSI circuits are used are known as the fourth-generation computers. The present day computers use VLSI circuits to achieve high speed, small size, better accuracy and memory. Today, many mini computers, such as laptops, note books, and personal digital assistant are also available in the market.

ARCHITECTURE OF A COMPUTER

A computer consists of three essential components. These are:

1. Input device
2. Central processing unit (CPU)
3. Output device

Input Device

It is a device through which data or instructions are entered in computers. Key board, mouse and joy stick are some examples of input device.

Central Processing Unit (CPU)

CPU is the most important component in a computer. It consists of:

1. Memory unit
2. Control unit
3. Arithmetic and logical unit (ALU)

Output Device

This devise is used to display the results of the operations performed by the computer. Monitor is an output device.

Block Diagram of a Computer

A computer receives instructions or processed data through input devices. Information is temporarily stored in the memory unit. The result is permanently stored in the storage devices. If any arithmetical operations are to be performed, then with the help of control unit and the ALU, performs the operations and stores the result in the memory. Finally, the results can be seen through output devices.

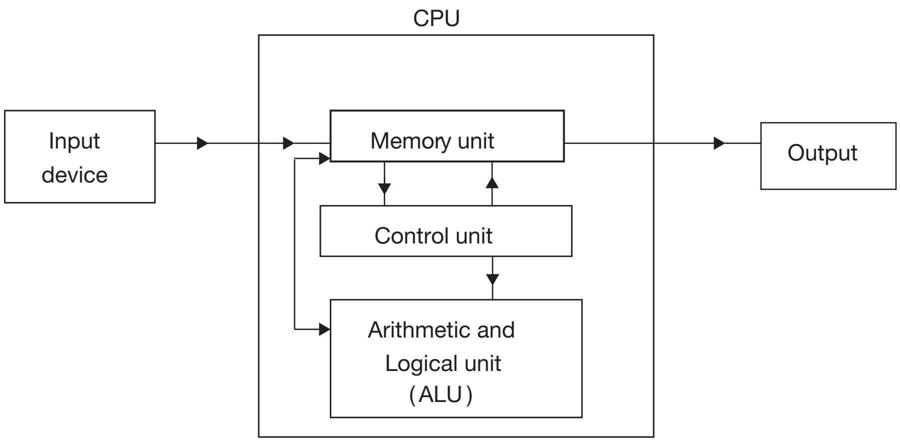


Figure 11.3

In all these processes, the control unit plays a major role. It controls the input and output devices, and also decodes instructions during the execution of a program.

Hardware

The three physical components, input device, CPU, and output devices are together referred as 'hardware' of a computer. All the touchable parts of a computer is named as hardware.

Software

To accomplish a particular task by using a computer, a set of instructions are written in a language that can be recognized by the computer. Any usual languages cannot be read and recognized by a computer. So we need to feed information on a computer by using a language called, 'programming language'. BASIC, PASCAL, C, C++, Java are some of the popular programming languages.

A set of instructions that are written in a language which can be recognized by a computer is called a 'program'. A set of programs is called 'software'.

To perform a task, a program has to be written. This program is fed into the memory of a computer by using an input device (i.e., a key board). The control unit reads instructions (given in program) from the memory and processes the data following the instructions. The result can be displayed by a device, such as monitor or printer. As discussed earlier, these devices are called 'output devices'.

Note that the arithmetic and logical unit (ALU) performs all the arithmetic and logical operations, such as addition, subtraction, multiplication, division, comparison under the supervision of the control unit (CU). The CU decodes the instructions to execute and the output unit receives results from the memory unit and converts the results into a suitable form in which the user can understand.

Algorithm

A comprehensive and detailed step-by-step plan or a design that is followed to solve a problem is known as algorithm. Thus, an algorithm is a set of systematic and sequential steps used in arriving at a solution to a problem.

For example, if you want to buy some articles from a grocery store, then the following steps are to be followed:

1. Make a list of the articles you intend to purchase.
2. Go to the grocery store.
3. Ask the storekeeper to give the pieces of articles, in the list you have.
4. List the price of the articles on a paper and add them to get the total amount of money you need to pay.
5. Verify whether you have received all the items.
6. Return home with the grocery items.

Steps 1 to 6 formed the algorithm for the task of buying grocery items. Even though it is a simple task, but we follow several steps, in a systematic way, to achieve the task. Similarly, to solve a task using a computer, first we need to make a list of the steps that are to be followed. Once an algorithm is ready, it can be represented through a flowchart.

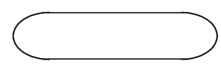
Flowchart

A flowchart is a pictorial representation of an algorithm. It distinctly depicts the points of input, decision-making, loops and output. Thus, with the help of a flowchart we can clearly and logically plan to perform a given task.

To draw a flowchart, we use certain symbols or boxes to represent the information appropriately. Following are the notations used in a flowchart.

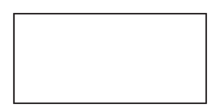
Terminal Box

This box indicates the start and the termination of the program.



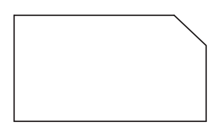
Operation Box

It is a rectangular box as shown in the adjacent figure. It is used to represent operations, such as addition, subtraction.



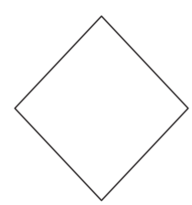
Data Box

This box is used to represent the data required to solve a problem, and information regarding the output of solution.



Decision Box

A diamond or rhombus-shaped box is used for making decisions. The points of decision can be represented by using this box. Usually, the answer to the decision is ‘yes’ or ‘no’. Once a flowchart is ready, we can translate it into a programming language and feed it into the memory. So, to accomplish a task on a computer the following steps are to be followed:



1. Identify and analyze the problem.
2. Design a systematic solution to the problem and write an algorithm.
3. Represent the algorithm in a flowchart.
4. Translate the flowchart into a program.
5. Execute the program and receive the output.

EXAMPLE 11.8

The principal and the rate of simple interest per month. Write an algorithm to calculate cumulative simple interest at the end of each year for 1 to 10 years, and draw a flowchart.

SOLUTION

Algorithm

Step 1: Read the values of principle (P), rate of interest (R), and time period (T).

Step 2: Take, $T = 1$.

Step 3: $\text{SI} = \dfrac{12 \times P \times T \times R}{100}$.

Step 4: Print the SI.

Step 5: If $T < 10$, then complete Steps 3, 4, 5.

Step 6: Otherwise, stop the program.

Step 7: Calculate, $T = T + 1$.

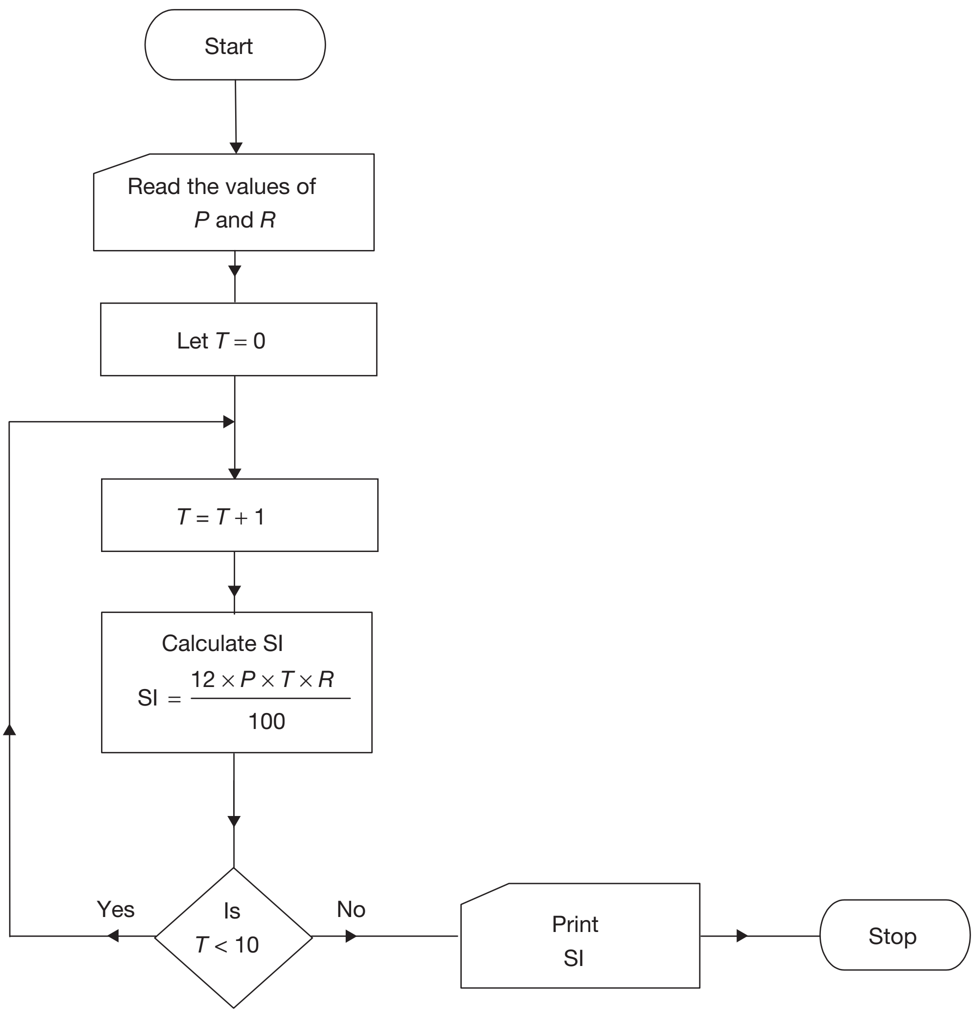


Figure 11.4

EXAMPLE 11.9

Write an algorithm and draw a flowchart to find the sum of first 50 natural numbers.

SOLUTION

Algorithm

Step 1: Set count = 1, sum = 0

Step 2: Add the count to sum

Step 3: Increase count by one, i.e., count = count + 1

Step 4: Check whether count is 51

Step 5: In Step (5) if count is 51, display sum, else Go to Step (2)

From the flowchart, we can observe that there is a loop among the Steps (boxes) 3, 4, and 5.

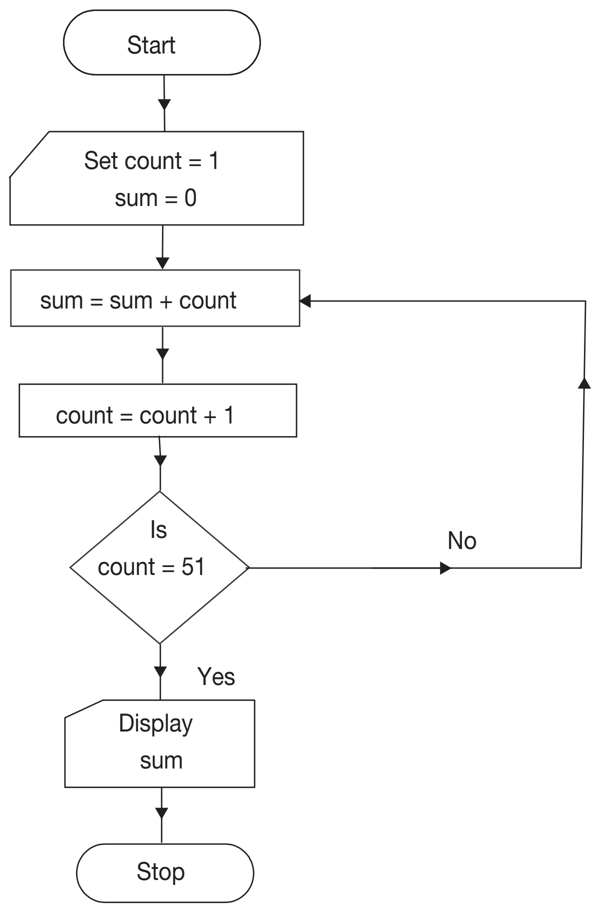


Figure 11.5

OPERATORS

Operators are used to perform various types of operations.

The different types of operators are as follows:

1. Shift operators
2. Logical operators

3. Relational operators
4. Arithmetic operators

Shift Operators

These are used to shift a character (numeric or alphabet) to the left or right side up to a given number of positions. It is denoted by the symbol << (left shift) or >> (right shift).

For example, << 2 means shift the given character to two positions left.

Logical Operators

The three logical operators are: AND, OR and NOT. AND expression is executed when both the conditions are true. OR expression is executed when any of the conditions is true. NOT expression is executed when condition is negated.

Relational Operators

The relational operators are equality (=), less than (<), greater than (>), less than or equal to (<=), greater than or equal to (>=) and not equal to (< >).

Examples:

1. $20 = 12 + 8$
2. $5 > 3$
3. $7 < 15$
4. 14 + 5 < >10 + 6

Arithmetical Operators

Arithmetical operators are addition (+), subtraction (–), multiplication (*), division (/), exponentiation (∧) and parenthesis [( )].
Example: 2 + 3/6 – 5 * 4.
Computer performance is measured in three ways. These are as follows:

1. Storage Capacity
2. Processing Speed
3. Data transformed Speed

Storage Capacity is measured in Bits, Bytes, Kilobytes, Megabytes or Giga Bytes.

1 Byte = 8 bits
1 Kilo Byte (KB) = 1024 Bytes
1 Mega Byte (MB) = 1024 KB
1 Giga Byte (GB) = 1024 MB
1 Tera Byte (TB) = 1024 GB

Processing Speed is measured in Hertzes, Megahertzes, Gigahertzes. It explains about processor speed.

1 Megahertz = 1000 Htz
1 Megahertz = 1000 Mega Htz

Example: 800 MHz, 1.5 GHz. Finally, data transfer speed is measured in Bytes per second.

Example: 256 KB/Sec or 256 KBPS, 128 KB/Sec or 128 KBPS.

EXAMPLE 11.10

1. Read A, B, C, D

2. $S = (A * C) + (B * D)$

3. Print "total cost is Rs"; S

4. End

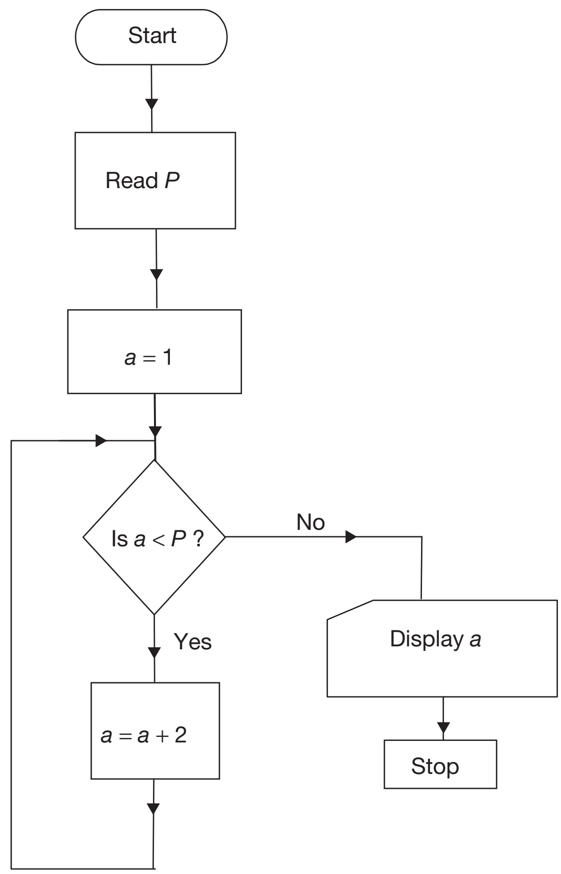
If D, C, B and A have values 2, 10, 4 and 13 respectively, then what is the output of the above algorithm?

(a) 72 **(b)** 138 **(c)** 66 **(d)** None of these

HINT

Use Step to obtain the output.

EXAMPLE 11.11



In the above chart if $P = 6$, then what is the output?

(a) 8 **(b)** 5 **(c)** 7 **(d)** 9

HINT

Repeat the loop until the condition, $a < P$, is satisfied.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Deposit-taking and Money-lending are the main functions of ______.
2. The business of receiving deposits and lending money is carried out by ______.
3. ______ are given by banks on rent to keep valuables in safe custody.
4. The purpose of ______ account is to encourage the habit of saving.
5. The deposits and withdrawals are recorded in a small note book known as ______.
6. If a cheque is issued without having minimum balance in the account, the cheque will be ______. (dishonoured/passed).
7. There is no restriction on the amount deposited/withdrawn or on the number of withdrawals in a ______ account.
8. Banks provide ______ cheques and ______ exchange to the tourists.
9. In big cities, sales tax and income tax are paid through ______.
10. Under hire purchase scheme, the buyer is called ______.
11. Money can be withdrawn from a savings account by the account-holder either by filling the withdrawal form or a ______.
12. The rate of interest paid for the money kept in the current account is ______.
13. The formula for simple interest if ₹P per month is deposited each month for n months at R% per annum is ______.
14. In ______ account, the depositor is paid a lump sum payment after the expiry of the fixed period during which the account holder deposits small amounts at regular intervals (i.e., monthly).
15. In ______ bank account, the interest is calculated on the sum of the minimum balance present between the 10th and the end of the month.
16. Why is Charles Babbage known as the father of computer?
17. What are the basic units of CPU?
18. The present day computers are known as ______ machines.
19. What are the four fundamental arithmetic operations?
20. The box in which START is written while drawing a flowchart is known as ______.
21. The box indicating the type of decision is known as ______.
22. Which of the following are the characteristics of a computer?

    (a) Speed

    (b) Accuracy

    (c) Storage

    (d) All the above
23. The method of solving a problem in computers is known as ______.
24. Computer works according to instructions. This set of instructions is called ______.
25. What are the types of the boxes used in the flowcharts?
26. The pictorial representation of the algorithm of a problem is known as ______.
27. Is a decision box needed for the following statement? "How much is that apple?" (Yes/No)
28. What is the input data in the problem? How many apples can you buy, if one apple costs ₹10, if you have ₹20?
29. "A marathon can complete a lap of 10 kms in 1 hour. What is the average speed? Pick the input data.
30. The communication with hardware parts of the computer is possible through ______.

Short Answer Type Questions

31. Mention the various types of accounts provided by banks.

32. Details of Mohan's savings bank account are given below.

Date	Amount Withdrawn	Amount Deposited
2-8-2099		₹2000
10-9-2099	₹400	
15-9-2099		₹4000
16-10-2099	₹1000	
30-11-2099	₹1000	

Calculate the sum, for which he earns interest from August, 1999 to November, 1999.

33. Anand makes a fixed deposit of ₹16,000 in a bank for 3 years. If the rate of interest is 10% per annum compounded yearly, then find the maturity value.

34. Rajeshwar makes a fixed deposit of ₹6000 for 1 year. If the rate of interest is 10% per annum and compounded half yearly, then find the total amount that he received after one year.

35. Subhash deposited ₹5000 and it becomes ₹5900 in 1 year under simple interest. If he deposited ₹10,000 with the same rate of interest for 1 year under simple interest, then what is the maturity amount?

36. A man deposits ₹64,000 at a certain rate of interest compounded yearly. If that amount becomes ₹81,000 in two years, then find the rate of interest.

37. Abhijit opened an account with ₹40,000 on 1-1-2006. His transactions are as follows:

(a) On the January 10, he withdrew 20% of the amount that he deposited.

(b) On the February 10, he withdrew 40% of the balance amount.

(c) On March 10, he drew 5% of the balance amount.

If Abhijit closes his account on 1-4-2006, then find the total amount he would receive if the bank paid interest at 4% per annum (approximately).

38. Rohit purchased a washing machine in an instalment scheme. It is sold for ₹8000 cash, or ₹1000 cash down followed by equal instalments of ₹1800 each. Find the rate of interest?

39. Write an algorithm for evaluating $x \times y - z + l \div m + l - n$.

40. Write an algorithm for finding the length of the side of a rhombus when the length of its diagonals is given.

41. Draw a flowchart to determine whether a given number is even or odd.

42. Draw a flowchart to pick the smallest among the two numbers.

43. Write an algorithm to find the first 10 multiples of a given number.

44. Write an algorithm to find the factors of a given number.

45. Execute a flowchart to find the product of first 100 even numbers.

Essay Type Questions

46. Details of Shashi's savings bank account are given below:

Date	Amount Deposited	Amount Withdrawn
1-12-2098	₹800	
04-1-2099	₹1200	
30-1-2099		₹400
10-2-2099	₹900	
9-3-2099	₹600	
28-4-2099		₹200

Calculate the total interest earned by him up to 30-4-1999.

The rate of interest is as follows:

(i) 4% per annum up to 28-2-1999.

(ii) 3% per annum from 01-3-1999 to 30-4-1999.

PRACTICE QUESTIONS

47. Krishna deposits ₹500 per month in a recurring deposit account for 20 months at 4% per annum. Find the interest he receives at the time of maturity.

48. Draw a flowchart to compute the sum of 10-terms of the series $S_n = \dfrac{n(n^2 - 1)}{2}$.

49. Write an algorithm to find the arithmetic mean (AM) of the given five numbers.

50. Write an algorithm to arrange a given set of five numbers in descending order.

CONCEPT APPLICATION

Level 1

1. Reena opened an account on 2-3-2006 by depositing ₹8000. She deposits ₹3000 on the 15th of every month and withdraws ₹2000 on the 20th of every month. If she closed her account on 19-7-2006, then what is the total amount she received from the bank on closing the account, if the bank paid an interest of 4% per annum? (approx.)

   (a) ₹15,427 (b) ₹15,127
   (c) ₹15,227 (d) ₹15,327

2. Ramu opened a savings bank account with a bank on 3-4-2005 with a deposit of ₹500. He deposited ₹50 on 12-4-2005 and, thereafter, neither deposited nor withdrew any amount. The amount on which he would receive interest for the month of April, 2005 is ______.

   (a) ₹500 (b) ₹50
   (c) ₹550 (d) None of the above

3. Rahul opened a savings bank account with a bank on 1-1-2006 with a deposit of ₹1000. He deposited ₹100 on 9-1-2006 and, thereafter, he neither deposited nor withdrew any amount in February, 2006. The amount, on which he would receive interest for the month of February, 2006 is ______.

   (a) ₹1000 (b) ₹100
   (c) ₹1100 (d) 0

4. Shiva makes a fixed deposit of ₹15,000 with a bank for 1 year 6 months. If the rate of interest is 8% per annum compounded half yearly, then the amount which Shiva receives at the end of this period is ______.

   (a) ₹16,812.90 (b) ₹16,872.96
   (c) ₹17,872.96 (d) ₹18,872.96

5. Pasha deposited ₹20,000 on 1-1-2006 to open a savings account. He withdrew ₹1000 on the 10th of every month. He closed his account on 6-6-2006. What was the interest he received while closing the account if the bank paid interest at 4% per annum? (approx.)

   (a) ₹283 (b) ₹192
   (c) ₹384 (d) ₹252

6. Rahul opened a savings bank account with a bank on 1-2-2006 with a certain amount. Interest is credited to his account at the end of June and December every year, and the rate of interest is 5% per annum. If the sum of the minimum balance up to the end of June is ₹2000, then the interest that Rahul gets at the end of June, 2006 is ______.

   (a) ₹8.01 (b) ₹8.33
   (c) ₹8.20 (d) ₹8.40

7. A company, named, Infosys, paid to its employee Arshya ₹15,000 through a bank on the 2nd of every month. The company opened the savings account with ICICI Bank on 2-7-2006 by depositing her first salary. She used to withdraw ₹4000 for her expenses on the 15th of every month. If she closed her account on 8-11-2006, then on what amount did she receive interest?

   (a) ₹121,000 (b) ₹111,000
   (c) ₹120,000 (d) ₹110,000

8. Sanjay makes a fixed deposit of ₹20,000 with a bank for 2 years. If the rate of interest is 8% per annum, then the amount which he receives at the time of maturity is ______.

   (a) ₹21,328 (b) ₹22,328
   (c) ₹23,328 (d) ₹24,328

9. Murthy makes a fixed deposit of ₹20,000 with a bank for 100 days. If the rate of interest is 5%, then find the amount he will receive on maturity of his fixed deposit.

(a) ₹20,273.97 (b) ₹21,273.97

(c) ₹22,273.97 (d) ₹23,273.97

Direction for questions 10–15: These questions are based on the following data:

Khalique joined a company, named, ADP on 1-6-2006. The company directly credits his salary to his savings bank account in the UTI Bank as follows.

(i) On every month, the company pays ₹10,000 towards basic salary.

(ii) After completion of every 3 months (from the date of joining) on the 16th, it pays ₹3500 towards medical expenses.

(iii) Every month, on the 15th the company pays ₹1200 towards traveling expenses (this will be paid after the first salary is credited into his account).

(iv) After completion of 1 year, it pays a bonus of ₹12,000.

Khalique did not draw any money till 1-11-2006 and closed the account on 2-11-2006.

10. What is the total amount for which the bank will pay interest?

(a) ₹500,000 (b) ₹100,000

(c) ₹110,700 (d) ₹120,900

11. If the bank pays an interest of 4% per annum, then what is the total interest he would receive from the bank?

(a) ₹169 (b) ₹269

(c) ₹469 (d) ₹369

12. If he draws ₹5000 on 5-10-2006 from the bank and the remaining data is the same, then what is the interest paid to him by the bank on 1-11-2006? (Interest Rate = 4%)

(a) ₹462 (b) ₹392

(c) ₹352 (d) ₹823

13. If he closes the account on August 31, then what is the total amount paid by the bank? (including interest)

(a) ₹21,159 (b) ₹21,000

(c) ₹21,400 (d) ₹21,304

14. If the bank pays interest at different rates for different periods, as follows:

(i) 4% per annum up to the date 1-9-2006

(ii) 3% per annum from 2-9-2006 to 31-12-2006

Then the total interest is _______.

(a) ₹285 (b) ₹310

(c) ₹290 (d) ₹303

15. A loan of ₹18,900 is to be paid back in two equal half-yearly instalments. If the interest is compounded half yearly at 20% per annum, then the interest is (in rupees).

(a) 2880 (b) 3000

(c) 3178 (d) 3380

16. Nikhil opened a recurring deposit account with The State Bank of India for 3 years. The bank paid him ₹20,220 on maturity. If the rate of interest is 8% per annum, then the amount that Nikhil deposited per month is _______.

(a) ₹300 (b) ₹400

(c) ₹500 (d) ₹600

17. A fan is sold for ₹900 cash, or ₹400 cash down payment followed by ₹520 after two months at simple interest. The annual rate of interest is:

(a) 25% (b) 30%

(c) 24% (d) 23%

18. Kiran has a cumulative term deposit account of ₹600 per month at 8% per annum. If he receives ₹24,264 at the time of maturity, then the total time for which the account was held is _______.

(a) 12 months

(b) 24 months

(c) 36 months

(d) 46 months

Direction for questions 19–21: These questions are based on the following data:

Kavitha opened an account with a bank on 1-4-2006 by depositing ₹80,000.

PRACTICE QUESTIONS

The particulars of her withdrawals are given below.

(a) 20% of the total amount on 15-4-2006.

(b) 30% of the remaining balance on 15-5-2006.

(c) 10% of the remaining balance on 15-6-2006.

She closed her account on 1-8-2006.

19. What is the total amount on which interest is paid till 1-8-2006?

(a) ₹160,000

(b) ₹189,440

(c) ₹198,000

(d) ₹200,000

20. If interest is paid @ 3% simple interest per annum, then the total interest (approximately) is __________.

(a) ₹440 (b) ₹500

(c) ₹490 (d) ₹474

21. What was the balance in her account as on 31-7-2006?

(a) ₹64,000 (b) ₹44,800

(c) ₹40,320 (d) None of these

22. At the time of closing the account, what amount did the bank pay her, when interest was paid at 3% simple interest per annum?

(a) ₹40,550

(b) ₹40,794

(c) ₹40,820

(d) ₹40,800

23. Ranjith opened a savings bank account in a bank on 11-2-2006 with a deposit of ₹5000. He deposited ₹1000 on 20-2-2006 and, thereafter, he neither deposited nor withdrew any amount. Find the interest received for the month of February 2006 at the rate of 4% per annum.

(a) ₹50/3

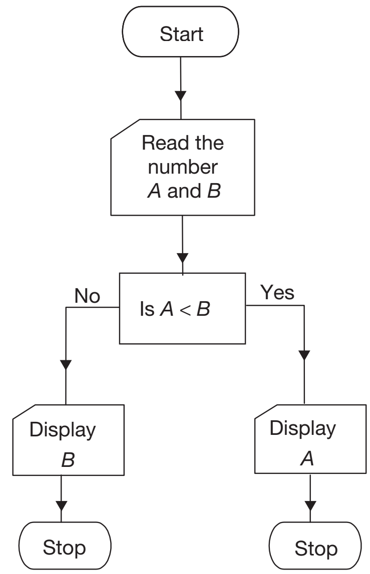
(b) ₹20

(c) ₹15

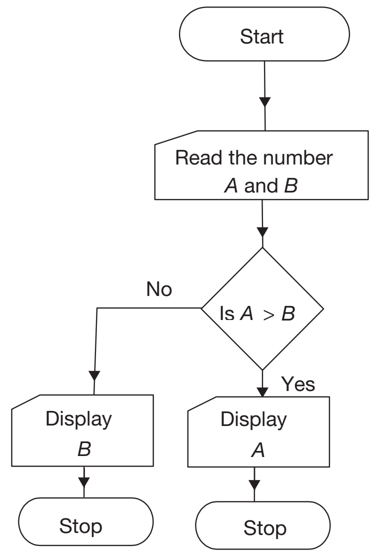
(d) He did not receive any interest.

24. Which of the following is an appropriate flowchart to find the greater number between the two numbers A and B?

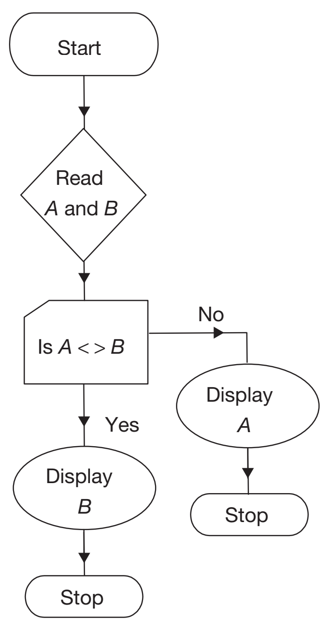
(a)



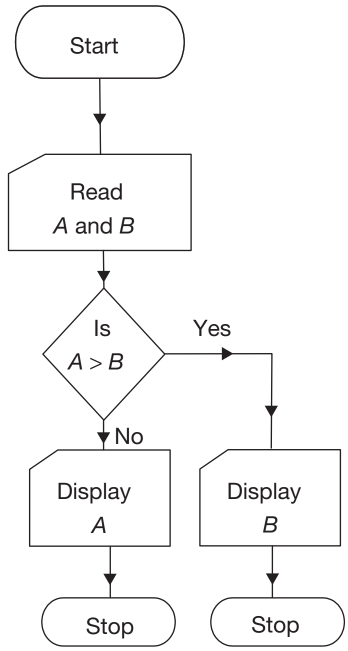
(b)



(c)



(d)



25. Which of the following problems has a loop in the flowchart drawn to solve the problem?

(a) Given cost price and selling price of an article, we need to find the gain or loss.

(b) Given a set of 100 natural numbers, we need to find the largest among the numbers.

(c) Given two numbers A and B, we need to find the sum and the product of the numbers.

(d) Given two numbers x and y, we need to find its geometric mean.

26. What will be the output of the following algorithm?

(A) Take $C = 25°$

(B) $F = C * (9/5) + 32$

(C) Print F

(D) End

(a) 47° (b) 97°

(c) 77° (d) 67°

27. Evaluate the expression, as performed by a computer: $720 - 7 \times 88 + \dfrac{111}{37} - \dfrac{256}{16}$.

(a) 58 (b) 85

(c) 119 (d) 91

28. Write an algorithm to find the area of a rectangle.

(a) (i) Read length (l) and breadth (b)
(ii) Find the area by using $A = l \times b$
(iii) Display the area

(b) (i) Find the area by using $A = 2(l + b)$
(ii) Display the area
(iii) Read length (l) and breadth (b)

(c) (i) Read length (l) and breadth (b)
(ii) Find the area by using $A = 2(l + b)$
(iii) Display the area

(d) (i) Read length (l) and breadth (b)
(ii) Display the area
(iii) Find the area by using $A = l \times b$
(iv) Display length and breadth

29. Evaluate the following expressions as a computer does.

I. $\dfrac{d \times a}{b - c \times e}$

II. $\dfrac{r}{(p \times q) - c \times e}$

The values of I and II are:

(a) $\dfrac{ad}{b} - ce, \dfrac{r}{pq} - ce$

(b) $\dfrac{d}{ab} - ce, \dfrac{rq}{p} - ce$

(c) $\dfrac{ad - ce}{b}, \dfrac{r}{pq - ce}$

(d) $\dfrac{ad}{(b - c)e}, \dfrac{r}{pq - ce}$

30. ______ are used to connect variables and constants to form expressions.

(a) Statements

(b) Basic keywords

(c) Operators

(d) Input/output statements

Level 2

Direction for questions 31 and 32: These questions are based on the following data:

Ravi opened a savings bank account with a bank on 1-1-2006.

Date	Deposit	Withdrawal
1-1-2006	₹1500	–
10-1-2006	–	₹400
20-1-2006	₹10,000	–
21-2-2006	–	₹5000
30-4-2006	₹4100	–
6-6-2006	₹2000	
10-9-2006	–	₹4000
21-9-2006	–	₹4000
24-10-2006	₹8000	
24-12-2006	–	₹1000

31. Find the sum of the eligible monthly balances for which interest is calculated at the end of December?

(a) ₹95,000

(b) ₹98,000

(c) ₹100,000

(d) ₹92,000

32. Compute the interest till the end of December at the rate of 4% per annum approximately.

(a) ₹327

(b) ₹753

(c) ₹300

(d) ₹792

33. A folio from the savings bank account of Mr Chetan is given below. The simple interest at 4% per annum from 3-1-2003 up to 1-6-2003 is ______. (approx.)

Date	Particular	Withdrawal (₹)	Deposit (₹)	Balance (₹)
3-1-2003	B/F	–	–	24,000
16-1-2003	To Cheque	5000	–	19,000
11-3-2003	By Cash	–	10,000	29,000
17-4-2003	By Transfer	–	8000	37,000
6-5-2003	By Cheque	–	7653	44,653
19-5-2003	By Cash	3040	–	41,613

(a) ₹440

(b) ₹425.38

(c) ₹450.49

(d) ₹460

34. Mr Ramu has his savings bank account with Andhra Bank. Given are the entries in his pass book.

Date	Particular	Withdrawal (₹)	Deposit (₹)	Balance (₹)
3-1-2003	B/F	–	–	7635.00
17-1-2003	To Cheque	4527.00	–	3108.00
8-2-2003	By Cash	–	215.00	3323.00
14-3-2003	By Cash	–	602.00	3925.00
15-5-2003	To Self	1580.00	–	2345.00
7-6-2003	To Cheque	219.00	–	2125.00
22-6-2003	By Cash	–	700.00	2825.00

The interest up to 30-6-2003 at $4\frac{1}{2}\%$ per annum is ________ (approx.)

(a) ₹60.70

(b) ₹65.70

(c) ₹68.06

(d) ₹70.06

35. Calculate the total approximate interest earned by Ravi up to 31-12-2006. The rates of interest which change from time to time are as follows:

(i) 5% per annum up to 3-8-2006

(ii) 4.5% per annum from 1-9-2006 to 31-12-2006

(a) ₹395

(b) ₹405

(c) ₹400

(d) ₹385

PRACTICE QUESTIONS

36. Sameena has a savings bank account with a State Bank branch. The entries in her pass book for the month of May are as follows.

As on 1-5-2006, balance is ₹22,200.

On 5-5-2006, amount deposited is ₹8800.

On 9-5-2006, amount deposited is ₹1000.

On 10-5-2006, amount drawn is ₹5000.

On 16-5-2006, amount deposited is ₹40,000.

Calculate the interest she earns for the month of May at the rate of 4% per annum.

(a) ₹100 (b) ₹223.30

(c) ₹90 (d) ₹256.60

37. Arshya makes a fixed deposit (FD) of ₹5000 for a period of 1 year. The rate of interest is 6% per annum compounded every four months in a year. Find the approximate maturity value of the FD.

(a) ₹5205 (b) ₹5000

(c) ₹5306 (d) ₹5400

38. If in a computer language:

+ means subtraction

− means addition

× means division

/ means multiplication,

then evaluate the expression $\frac{1000 + 89}{11 - 365 \times 5}$ as the computer performs when written in that language.

(a) 94 (b) 271

(c) −1726 (d) 2070

39. Which of the following is/are true?

(A) 1 KB = 1000 bytes

(B) 1 MHz = 1024 Htz

(C) Kbps is the unit used in measuring the memory of a computer.

(D) The pictorial representation, describing a method of solving a problem which is an algorithm.

(a) A, B, C

(b) All of the above

(c) A, B

(d) None of the above

40. (A) Read the values of P and Q.

(B) If P > Q, then K = P − Q

(C) If P < Q, then K = P + Q

(D) If P = Q, then K = P * Q

(E) Print the answer: K

(F) Stop

If the input values of P and Q are 10 and 25, then what is the output of the above algorithm?

(a) 250 (b) 15

(c) 35 (d) 2

41. Which of the following statements can be filled in the decision box?

(A) Is it hot?

(B) How much is the cost of an apple?

(C) Is the price less than ₹200?

(D) You are going to movie, aren't you?

(a) B, C, D (b) A, C

(c) A, C, D (d) All of the above

42. If three distinct numbers x, y and z are given and we need to find the largest number among the three, then an appropriate flowchart written to accomplish this task consists of ______.

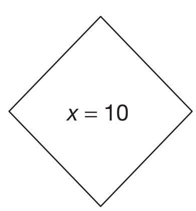
(a) at least 1 decision box

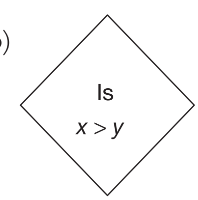
(b) at least 2 decision boxes

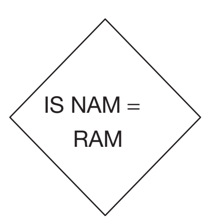
(c) only 1 decision box

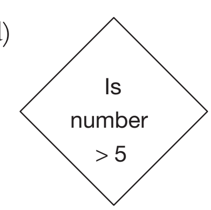
(d) no decision box

43. Which of the following is incorrect while writing flowchart?

(a)


(b)


(c)


(d)


44. Which of the following is an appropriate algorithm to add two numbers A and B?

(a) (i) Read the two numbers A and B
(ii) Display the sum of the numbers
(iii) Add the two numbers

(b) (i) Read the two numbers A and B
(ii) Add the two numbers A and B
(iii) Display the sum

(c) (i) Display the sum of the numbers
(ii) Read the two numbers A and B
(iii) Add the numbers A and B

(d) (i) Add the two numbers A and B
(ii) Read the two numbers A and B
(iii) Display the sum

45. Ganesh opened a savings bank account with a bank on 5-7-2007 with a deposit of ₹8000. He deposited 40% of his initial deposit on 9-8-2007. He closed his account on 7-10-2007. Find the amount on which he received interest for the month of August (in ₹).

(a) 8000 (b) 9400
(c) 10,600 (d) 11,200

46. A briefcase can be sold for ₹600, or for a certain amount of cash down payment followed by a payment of ₹309 after 3 months. If the rate of interest is 12% per annum, find the cash down payment (in ₹).

(a) 300 (b) 302
(c) 298 (d) 297

47. Bala opened a savings bank account with a bank on 4-1-2007, where interest is credited at the year end. The sum of the minimum balance held by Bala up to the end of December 2007 was ₹4800. He earned an interest of ₹2 per month that year. Find the annual rate of interest.

(a) 6% per annum (b) 4.5% per annum
(c) 5% per annum (d) 4% per annum

48. Rohan opened a savings bank account with Indian Bank on 2-5-2006. His initial balance was ₹650. He withdrew ₹150 on 11-5-2006, and, thereafter, he neither deposited nor withdrew any amount during that month. Find the amount on which he would receive interest for that month (in ₹).

(a) 150 (b) 650
(c) 500 (d) None of these

49. Bala opened a fixed deposit account for 2 years by making a deposit of ₹12,000. The rate of interest was 20% per annum, interest being compounded annually. Find the total interest paid by the bank (in ₹).

(a) 4800 (b) 5280
(c) 5760 (d) 6240

50. Evaluate the following expression as performed by a computer.

$$540 - 9 \times 58 + \frac{301}{43} - \frac{324}{18}$$

(a) 7 (b) 9
(c) 11 (d) 13

51. What is the output of the following algorithm?

Step 1: Take $F = 104$
Step 2: $C = 5/9 * F\text{-}(160/9)$
Step 3: Print C
Step 4: End

(a) 38 (b) 40
(c) 42 (d) 44

52. Which of the following can be filled in a decision box?

(A) Is it raining?
(B) How much is your monthly income?
(C) Did you go to Sunil's birthday party?
(D) Did you pay more than ₹300 to buy this watch?

(a) A, B, C (b) B, C ,D
(c) A, B, D (d) A, C, D

53. Evaluate the following expressions as performed by a computer.
$900 - 71 \times 13 + 546 \div 7$

(a) 55 (b) 35
(c) 65 (d) 75

54. If the input values of A and B are 18 and 12, then what is the output of the algorithm below?

Step 1: Read the values of A and B

Step 2: If $A \geq B$, then $C = A - B$.
Otherwise $C = B - A$

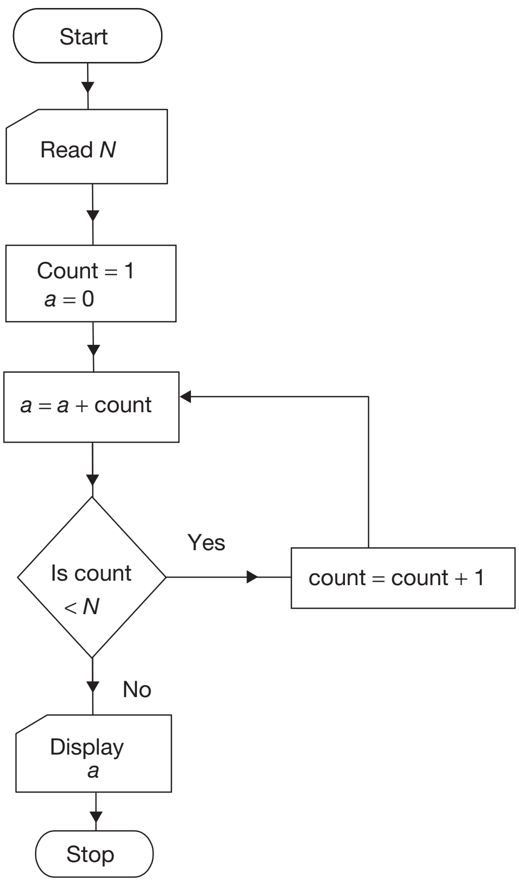
Step 3: Print C

Step 4: End

(a) 8 (b) −6

(c) 6 (d) 10

Level 3

55. 

In the above flowchart, if $N = 10$, then what is the output?

(a) 55 (b) 50

(c) 60 (d) None of these

56. A man deposited a certain amount for 3 years at compound interest. The bank gave him a statement that for the first year the amount was ₹1000, for the second year it was ₹$(1000 + x)$ and for the third year it was ₹$\left(1000 + \frac{41x}{20}\right)$. Find the rate of interest given by the bank. (Interest compounded annually).

(a) 2% per annum (b) 3% per annum

(c) 4% per annum (d) 5% per annum

57. A man earns money from different businesses. He opened his account on 4-2-2005 with ₹500. In every month, on the 8th, he deposits ₹10,000, and every month, on the 27th, he draws ₹5000. Beginning from February, on the 15th of every alternate month, he deposits ₹2000. On March 15, he withdraws ₹1500. Calculate the sum on which he earns interest at the end of April 2005.

(a) ₹22,500 (b) ₹37,500

(c) ₹36,500 (d) ₹11,500

58. Naveen opened a bank account by depositing ₹80,000 for 2 years at 10% per annum, compound interest compounded annually. At the end of two years, he withdrew a certain amount and the remaining amount is deposited for the 3rd year. At the end of the 3rd year, he withdrew the total balance of ₹99,000 from the bank. What amount did he withdraw at the end of the 2nd year?

(a) ₹8200 (b) ₹4500

(c) ₹6800 (d) ₹10,000

59. Rohit opened a fixed deposit of ₹25,000 with a bank for one years. If the rate of interest is 10% per annum, interest being compounded half yearly, then find the amount he received at the end of this period approximately (in ₹).

(a) 28,821 (b) 28,153

(c) 28,916 (d) 28,941

60. Anil opened a recurring deposit account with Axis Bank for n years. He deposited ₹800 in every month. The bank paid him ₹10,784n on maturity. The rate of interest was 8% per annum. Find n.

(a) 3 (b) 2.5

(c) 2 (d) 3.5

61. Mr Mohan lent a sum of ₹50,600 at 20% per annum compound interest, interest being compounded annually. It had to be repaid in two equal half-yearly installments. Find the interest on the sum (in ₹).

(a) 18,220 (b) 19,680

(c) 15,640 (d) 20,880

62. Ashok opened a fixed deposit with certain amount in a bank. The sum quadrupled in 8 years at $R\%$ per annum, interest being compounded annually. Find the time in which it will become 32 times itself at $R\%$ per annum, interest being compounded annually (in years).

(a) 20 (b) 24

(c) 16 (d) 28

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. banks
2. banks
3. Lockers
4. Savings bank
5. Pass book
6. Dishonoured
7. Current
8. Travellers, foreign
9. Banks
10. Hirer
11. Cheque
12. 0%
13. $P \times \dfrac{n(n+1)}{2} \times \dfrac{1}{12} \times \dfrac{R}{100}$
14. Recurring deposit
15. Savings
16. Because all the ideas of Babbage which he proposed were incorporated in the computer.
17. Memory, central unit, and arithmetic and logical unit.
18. Newmann
19. Addition, subtraction, multiplication, and division.
20. Terminal box
21. Decision box
22. (a), (b) and (c)
23. Programming
24. Program
25. Rectangular or operations box, decision box, terminal box, and the data box or input output box
26. Flowchart
27. No
28. One apple costs ₹10, you have ₹20.
29. Lap of 10 km, 1 lap takes 1 hour.
30. Software

Short Answer Type Questions

31. (i) Savings bank account
    (ii) Current account
    (iii) Term or fixed deposit account
    (iv) Recurring deposit account
32. ₹11,800
33. ₹21,296
34. ₹6615
35. ₹11,800
36. 12.5% per annum
37. ₹18,471
38. 35.08%

Essay Type Questions

46. ₹31.33
47. ₹350

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (d)
21. (c)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (c)

Level 2

31. (b)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (a)	**39.** (d)	**40.** (c)
41. (c)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (a)	**47.** (a)	**48.** (c)	**49.** (b)	**50.** (a)
51. (b)	**52.** (d)	**53.** (a)	**54.** (c)						

Level 3

55. (a)	**56.** (d)	**57.** (c)	**58.** (c)	**59.** (d)	**60.** (a)	**61.** (c)	**62.** (a)

CONCEPT APPLICATION

Level 1

1. (i) The minimum balance for the month of March is ₹8000.

 (ii) The minimum balance for the month of April is ₹9000.

 (iii) Similarly, find the minimum balances of May, June and July. Also, find the sum of all the minimum balances from March to July.

 (iv) Calculate simple interest on the above sum at 4% per annum, for 5 months.

 (v) Use Amount = Principal + Interest.

2. Interest will be calculated for the minimum balance beginning from the 10th day of the month.

3. Find the minimum balance between the 10th day and the last day of the month.

4. Use the formula to find compound interest.

5. (i) Find the minimum balance of each month from January to June. Find the sum of all such minimum balance.

 (ii) Calculate simple interest on the above sum at 4% per annum.

6. Use the formula to find simple interest.

7. (i) Minimum balance for the month of July is ₹11,000.

 (ii) Minimum balance for the month of August is ₹(11,000 + 1500 − 4000), i.e., ₹22,000.

 (iii) Similarly, calculate the minimum balance for the remaining month.

 (iv) Add all such minimum balances

8. Use the formula to find compound interest.

9. Find the interest paid by the bank.

10. Find the minimum balance for different months.

11. Use the formula to find simple interest.

12. Find the minimum balance for different months.

13. Find the minimum balance till August.

14. Find the minimum balance till 1-9-06. Similarly, find the minimum balance from 2-9-2006 to 31-12-06.

15. Use the concept of compound interest.

16. Use the concept of recurring deposit account.

17. Use the formula to find simple interest.

18. Use the concept of cumulative term deposit account.

19. Find the minimum balance for different months.

20. Use the formula to find simple interest.

21. Find the minimum balances up to the month of July.

22. Amount = Balance + Interest

23. (i) Find the minimum balance held between the 10th day and the last day of the month.

 (ii) Calculate the minimum balance. Let it be P.

 (iii) $\text{SI} = \dfrac{P \times T \times R}{100}$

Level 2

31. (i) Find the minimum balance for different months.

 (ii) Calculate the sum of all the minimum balance of each month.

32. (i) Find the minimum balance for different months.

 (ii) Calculate the sum of all the minimum balance of each month.

 (iii) Calculate $\text{SI} = \dfrac{P \times T \times R}{100}$.

33. (i) Use the formula to find SI.

 (ii) $\text{SI} = \dfrac{P \times T \times R}{100}$

34. (i) Find the minimum balance for different months.

(ii) Calculate the sum of all the minimum balance of each month.

(iii) Calculate, $\text{SI} = \frac{P \times T \times R}{100}$.

35. (i) Find the minimum balances till August and calculate interest. Similarly, find from September to December.

(ii) Find the minimum balance till August and calculate the interest.

(iii) Find the minimum balance from 1-9-2006 to 31-12-2006 and calculate the interest.

36. (i) Find the minimum balance till 10-5-06.

(ii) Calculate, $\text{SI} = \frac{P \times T \times R}{100}$.

37. (i) Use the formula to find the amount.

(ii) Maturity value $= P\left(1 + \frac{R}{100}\right)^n$.

38. (i) Change the signs following the given directions.

(ii) Apply the BODMAS rule.

39. 1 KB = 2^{10} Bytes; 1 MB = 2^{20} Bytes

1 KHz =10^3 Hz; 1 MHz =10^6 Hz

40. As $P < Q$, $K = P + Q$.

43. Diamond box is used for decision making.

44. Read the options carefully and decide the correct output for $p = 6$.

45. Minimum balance for August (in ₹)

$$= 8000 + \frac{40}{100}(8000) = 8000 + 3200 = 11,200.$$

46. The cash price of the briefcase = ₹600

Let the down payment be ₹x.

Principal amount for each month = ₹$(600 - x)$

∴ Total principal for 3 months (in ₹) = $3(600 - x)$

Interest (in ₹) = $x + 309 - 600 = x - 291$

$$\therefore x - 291 = \frac{3(600 - x)12}{1200}$$

$$\Rightarrow 100x - 29,100 = 1800 - 3x$$

$$\Rightarrow 103x = 30,900$$

$$\Rightarrow x = 300$$

47. Let the annual rate of interest be R% per annum. Total interest for that year (in ₹) = (2) (12) = 24.

$$24 = \frac{(4800)(R)}{(100)(12)}$$

$$R = 6$$

48. Balance of Rohan after withdrawal (in ₹) = 650 – 150 = 500.

Minimum closing balance between 10-5-2006 and the last day of 2006 is ₹500.

∴ Interest would be received on ₹500.

49. Maturity amount (in ₹) $= 12,000\left(1 + \frac{20}{100}\right)^2$

$$= 12,000(1.2)^2 = 12,000(1.44) = 17,280.$$

Total interest (in ₹) = 17,280 – 12,000 = 5280.

50. 540 – 9 × 58 + 301/43 – 324/18 = 540 – 9 × 58 + 7 – 18% (∵ 301/43 = 7 and 324/18 = 18)

= 540 – 522 + 7 – 18 = 547 – 540 = 7.

51. $C = \frac{5}{9} \times 104 - \left(\frac{160}{9}\right) = \left(\frac{520}{9}\right) - \left(\frac{160}{9}\right)$

$$= \left(\frac{360}{9}\right) = 40$$

52. A decision box must contain a question for which the answer is either 'yes' or 'no'.

∴ (A), (C) and (D) can be filled in a decision box (∵ answer for (b) cannot be 'yes' or 'no').

53. 900 – 71 × 13 + 546 ÷ 7

= 900 – 923 + 546 ÷ 7

= 900 – 923 + 78

= –23 + 78 = 55.

54. A = 18 and B = 12

As A > B

∴ C = A – B = 6

∴ Output = 6.

Level 3

55. Repeat the loop on the condition count < N.

56. (i) Find the interest on ₹x.

(ii) Use the formula, amount: $(A) = P\left(1+\dfrac{r}{100}\right)^n$

57. Find the minimum balance of each month and add them.

58. (i) Find the amount after 2 years.

(ii) Find the amount after 2 years of depositing by

$$A = P\left(1+\frac{R}{100}\right)^n$$

(ii) Subtract the amount which is withdrawn from A.

(iii) By using $A = \left(1+\dfrac{R}{100}\right)^n$ get the value of amount withdrawn at the end of the 2nd year.

59. Required amount (in ₹) $= 25000\left(1+\dfrac{10}{2(100)}\right)^3$

$$= 25000\left(\frac{21}{20}\right)^3 = 28940.625 \cong 28941.$$

60. Time period = $12n$ months.

$$\text{SI (in ₹)} = 800\,\frac{(12n)(12n+1)}{2}\left(\frac{1}{12}\right)\left(\frac{8}{100}\right)$$

$$= 32n(12n+1)$$

Amount paid by the bank (in ₹)

$= (12n)(800) + 32n(12n + 1)$

$= 9600n + 32n(12n + 1)$

Given, $9600n + 32n(12n + 1)$

$\Rightarrow 32n(12n + 1) = 10{,}784n = 1184n$

$\Rightarrow 12n + 1 = 37 \Rightarrow n = 3$

61. Let the first and second installment be ₹f and ₹s, respectively. Let the present values of ₹f and ₹s be ₹P_f and ₹P_s.

$$P_f + P_s = 50600 \qquad (1)$$

$$P_f\left(1+\frac{20}{100}\right) = f$$

$$P_s\left(1+\frac{20}{100}\right)^2 = s$$

$1.2P_f = f$ and $1.44P_s = s$; $f = s$

$\therefore\ 1.2P_f = 1.44P_s$

$\therefore\ P_f = 1.2P_s$

$(1) \Rightarrow 2.2P_s = 50600$

$P_s = 23000$

$P_f = 27600$

$f = 33120$

Interest (in ₹) $= f + s - 50600 = 2f - 50600 = 15640$

62. Let the sum be ₹P.

$$P\left(1+\frac{R}{100}\right)^8 = 4P$$

$$\left(1+\frac{R}{100}\right)^8 = 4$$

$$1+\frac{R}{100} = 4^{1/8}$$

Let the required time be N years.

$$P\left(1+\frac{R}{100}\right)^N = 32P$$

$$\left(1+\frac{R}{100}\right)^N = 32$$

$$\left(4^{1/8}\right)^N = 32$$

$$(2^2)^{N/4} = 2^5$$

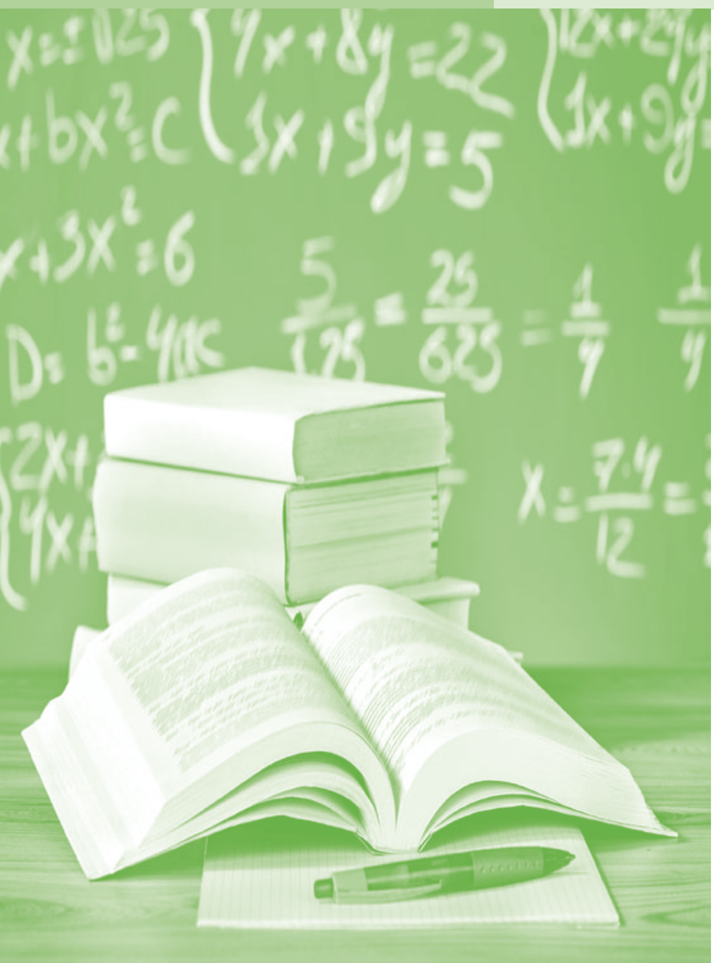
$$2^{N/4} = 2^5$$

$$\frac{N}{4} = 5 \Rightarrow N = 20$$

HINTS AND EXPLANATION

Chapter 12

Geometry



REMEMBER

Before beginning this chapter, you should be able to:

- State the simple definitions on plane, lines, angles, etc.
- Use the constructions of plane figures

KEY IDEAS

After completing this chapter, you should be able to:

- State the axioms and postulates
- Know different types of angles, like adjacent, linear pair, vertically opposite, complementary and supplementary angles
- Review triangles, their types, properties and congruency
- Know types, properties of quadrilaterals
- Know some theorems on triangles, like mid-point theorem, basic proportionality theorem
- Understand Pythagoras's theorem and its converse
- Construct polygons, triangles, quadrilaterals and circles

INTRODUCTION

In this chapter, we will revise the definition of a point, line, collinear points and angles. We will also learn the properties of parallel lines and construction related to lines and angles.

Further, we will discuss the types of triangles, concurrent lines, properties of triangles, similarity and congruence of triangles. In addition to the constructions of triangles, we will also focus on various types of quadrilaterals as well as other polygons.

Finally, we will revise the definition of a circle and study the theorems based on chords and angles. We will conclude with the constructions of circles.

Let us begin with some basic concepts.

Point A point is that which has no part.

Plane A plane is a surface which extends indefinitely in all directions. For example, the surface of a table is a part of a plane. A blackboard is a part of a plane.

LINE

A line is a set of infinite points. It has no end points. It is infinite in length. Figure 12.1 shows line l that extends to infinity on either side. If A and B are any two points on l, we denote line l as $\overleftrightarrow{AB}$, read as line AB.

Figure 12.1

Note A line has infinite length.

Line Segment

A line segment is a part of a line. The line segment has two end points and it has a finite length.

Figure 12.2

In Fig. 12.2, AB is a line segment. It is a part of line l, consisting of the points A, B and all points between A and B. Line segment AB is denoted as $\overline{AB}$. A and B are the end points of AB.

Ray

A ray has one end point and it extends infinitely on the other side.

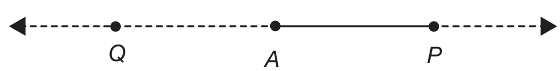


Figure 12.3

In Fig. 12.3, AP is a ray which has only one end point A. A ray with end point A is denoted as $\overrightarrow{AP}$ which is different from $\overrightarrow{AP}$ (but $\overline{AP}$ is similar to $\overline{QA}$). If A lies between P and Q, $\overrightarrow{AP}$ and $\overrightarrow{AQ}$ are said to be opposite rays.

Collinear Points

Three or more points lying on the same line are collinear points.

Coplanar Lines

Two lines lying in a plane are coplanar lines.

Intersecting Lines

Two lines which have a common point are known as intersecting lines. In Fig. 12.4, l_1 and l_2 are intersecting lines.

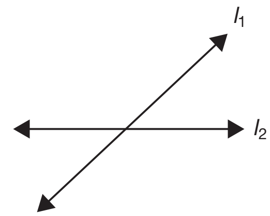


Figure 12.4

Concurrent Lines

Three or more lines passing through a common point are known as concurrent lines.

ANGLE

Two rays which have a common end point form an angle.

In Fig.12.5, $\overrightarrow{OQ}$ and $\overrightarrow{OP}$ are two rays which have O as the common end point and an angle with a certain measure is formed. The point O is called the vertex of the angle and $\overrightarrow{OQ}$ and $\overrightarrow{OP}$ are called the sides or arms of the angle. We denote this angle as $\angle QOP$ or $\angle POQ$ (or sometimes as $\angle O$). We observe that $\angle QOP = \angle POQ$ (These are two ways of representing the same angle). Angles are measured in a unit called degrees. This unit is denoted by a small circle placed above and to the right of the number. Thus $x°$ is read as x degrees.

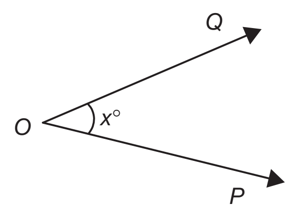


Figure 12.5

The angle formed by two opposite rays is called a straight angle. We define the unit of degree such that the measure of a straight angle is 180°.

Types of Angles

Let the measure of an angle be x.

1. If $0° < x < 90°$, then x is called an **acute angle**.

Example:

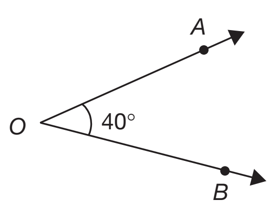


Figure 12.6

In the figure given above (Fig.12.6), $\angle AOB$ is an acute angle.

2. If $x = 90°$, then x is called a **right angle**.

Example:

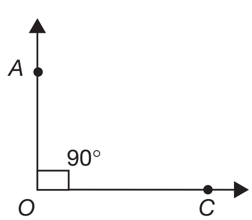


Figure 12.7

In the figure given above (Fig.12.7), $\angle AOC$ is a right angle.

3. If $90° < x < 180°$, then x is called an **obtuse angle**.

Example:

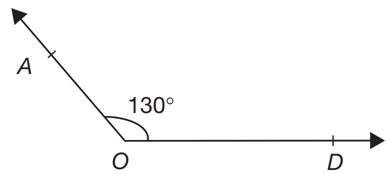


Figure 12.8

In the figure given above (Fig.12.8), $\angle AOD$ is an obtuse angle.

4. If $x = 180°$, then it is the **angle of a straight line**.

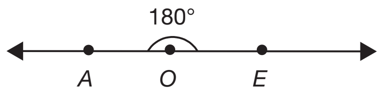


Figure 12.9

In the Fig.12.9, $\angle AOE = 180°$, $\overline{AE}$ is a straight line and O is a point on the line AE.

5. If both the rays coincide, then the angle formed is a **zero angle**.

Perpendicular Lines

Two intersecting lines making an angle of 90° are called perpendicular lines.

In Fig. 12.10, l_1 and l_2 are perpendicular lines. We write $l_1 \perp l_2$ and read it as l_1 is perpendicular to l_2.

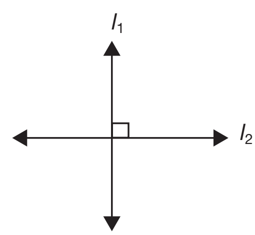


Figure 12.10

Perpendicular Bisector

If a line divides a line segment into two equal parts and is also perpendicular, then it is a perpendicular bisector.

Complementary Angles

When the sum of two angles is 90°, then the two angles are said to be complementary angles.

Example: If $x + y = 90°$, x and y are the complementary angles.

Supplementary Angles

When the sum of two angles is 180°, then the two angles are called supplementary angles.

Example: If $a + b = 180°$, then a and b are called supplementary angles.

Bisector of an Angle

If a line divides an angle into two angles of equal magnitudes, then it is the bisector of that angle.

In Fig. 12.11, $\overrightarrow{OC}$ is the bisector of $\angle AOB$.

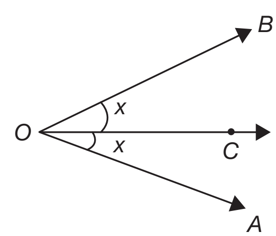


Figure 12.11

Adjacent Angles

If two angles have a common end point and a common side and the other two sides lie on either side of the common side, they are said to be adjacent angles.

In Fig. 12.12, $\angle AOD$ and $\angle DOC$ are adjacent angles. $\angle DOC$ and $\angle COB$ are also adjacent angles.

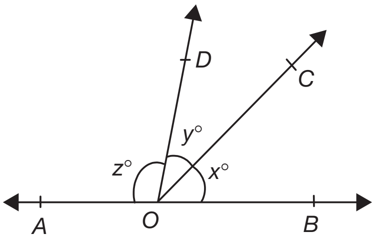


Figure 12.12

Linear Pair

In a pair of adjacent angles, if the non-common sides are opposite rays, then the angles are said to form a linear pair. Alternately, if O is a point between A and B, P is a point not on $\overleftrightarrow{AB}$, then $\angle AOP$ and $\angle POB$ forms a linear pair. The angles of a linear pair are supplementary.

Vertically Opposite Angles

When two lines intersect each other at a point, four angles are formed. Two angles which have no common arm are called vertically opposite angles.

In Fig. 12.13, $\overleftrightarrow{AB}$ and $\overleftrightarrow{PQ}$ intersects at O. $\angle AOP$ and $\angle BOQ$ are vertically opposite angles. $\angle POB$ and $\angle QOA$ are also vertically opposite angles.

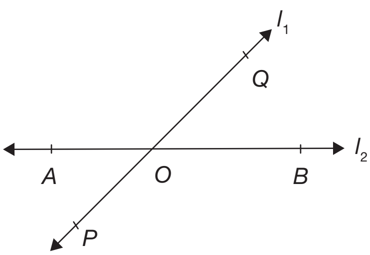


Figure 12.13

Note Vertically opposite angles are equal.

PARALLEL LINES

Two co-planar lines that do not have a common point are called parallel lines.

In Fig. 12.14, l_1 and l_2 are parallel lines. We write $l_1 \parallel l_2$ and read as l_1 is parallel to l_2.

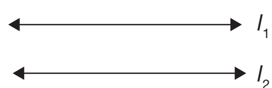


Figure 12.14

Properties of Parallel Lines

1. The perpendicular distance between two parallel lines is equal everywhere.
2. Two lines lying in the same plane and perpendicular to the same line are parallel to each other.
3. If two lines are parallel to the same line, then they are parallel to each other.
4. One and only one parallel line can be drawn to a given line through a given point which is not on the given line.

Transversal

A straight line intersecting a pair of straight lines in two distinct points is a transversal for the two given lines.

Let l_1 and l_2 be a pair of lines and t be a transversal.
As shown in Fig. 12.15, a total of eight angles are formed.

1. $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are exterior angles and $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are interior angles.
2. ($\angle 1$ and $\angle 5$), ($\angle 2$ and $\angle 6$), ($\angle 3$ and $\angle 7$) and ($\angle 4$ and $\angle 8$) are pairs of corresponding angles.

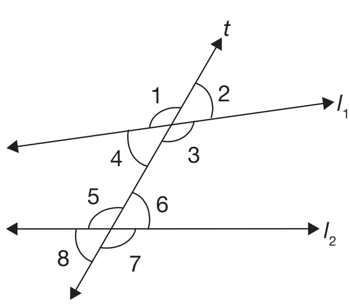


Figure 12.15

3. ($\angle 1$ and $\angle 3$), ($\angle 2$ and $\angle 4$), ($\angle 5$ and $\angle 7$) and ($\angle 6$ and $\angle 8$) are pairs of vertically opposite angles.
4. ($\angle 4$ and $\angle 6$) and ($\angle 3$ and $\angle 5$) are pairs of alternate interior angles.
5. ($\angle 1$ and $\angle 7$) and ($\angle 2$ and $\angle 8$) are pairs of alternate exterior angles.

If l_1 and l_2 are parallel, then we can draw the following conclusions:

1. Corresponding angles are equal, i.e., $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.
2. Alternate interior angles are equal, i.e., $\angle 4 = \angle 6$ and $\angle 3 = \angle 5$.
3. Alternate exterior angles are equal, i.e., $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$.
4. Exterior angles on the same side of the transversal are supplementary, i.e., $\angle 1 + \angle 8 = 180°$ and $\angle 2 + \angle 7 = 180°$.
5. Interior angles on the same side of the transversal are supplementary, i.e., $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$.

Intercepts

If a transversal t intersects two lines l_1 and l_2 in two distinct points P and Q respectively, then the lines l_1 and l_2 are said to make an intercept PQ on t.

In Fig. 12.16, $\overline{PQ}$ is an intercept on t.

A pair of parallel lines make equal intercepts on all transversals which are perpendicular to them.

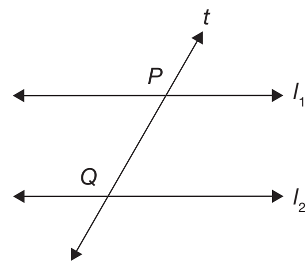


Figure 12.16

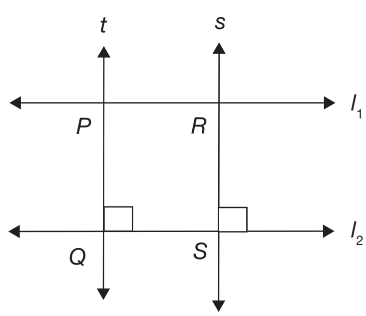


Figure 12.17

In the above figure, l_1 and l_2 are parallel lines. Transversals t and s are perpendicular to them. The intercepts PQ and RS are equal.

Equal Intercepts

If three parallel lines make equal intercepts on one transversal, then they will make equal intercepts on any other transversal as well.

In Fig. 12.18, l_1, l_2 and l_3 are parallel lines. They make intercepts AB and BC on transversal t and intercepts PQ and QR on transversal s.

If $AB = BC$, then $PQ = QR$.

Now, let us consider four parallel lines l_1, l_2, l_3 and l_4.

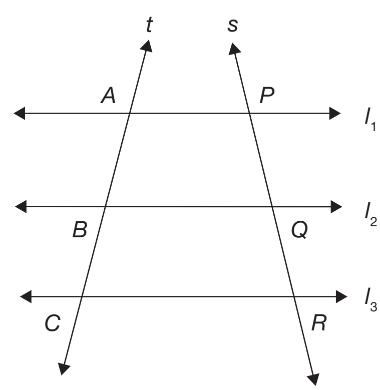


Figure 12.18

In Fig. 12.19, l_1, l_2, l_3 and l_4 are four parallel lines making intercepts on the transversals t and s.

The lines, l_1, l_2, l_3 and l_4, make intercepts AB, BC and CD on transversal t and intercepts PQ, QR and RS on transversal s. If $AB = BC = CD$, then $PQ = QR = RS$. Hence, we can say that if three or more parallel lines make equal intercepts on a transversal, they will also make equal intercepts on any other transversal. This is known as **equal intercepts property**.

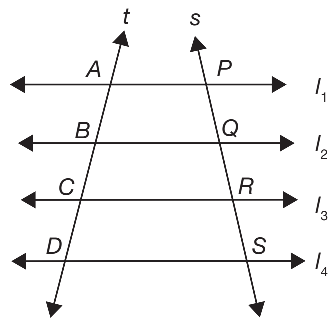


Figure 12.19

TRIANGLES

A triangle is a simple three-sided closed plane figure. The point of intersection of any two sides of a triangle is called a vertex. Hence, there are three vertices in a triangle. For example, in ΔABC, the vertices are A, B and C.

When one of the sides is produced (as shown in the Fig. 12.20), the angle ($\angle ABD$) thus formed is called the exterior angle and the angles $\angle BAC$ and $\angle ACB$ are called its interior opposite angles.

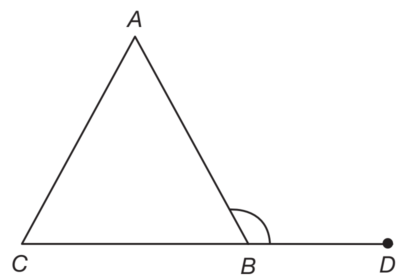


Figure 12.20

Types of Triangles

1. **Based on sides:**
   (i) *Scalene triangle:* A triangle in which no two sides are equal.
   (ii) *Isosceles triangle:* A triangle in which a minimum of two sides are equal.
   (iii) *Equilateral triangle:* A triangle in which all the three sides are equal.
2. **Based on angles:**
   (i) *Acute-angled triangle:* A triangle in which each angle is less than 90°.
   (ii) *Right-angled triangle:* A triangle in which one of the angles is equal to 90°.
   (iii) *Obtuse-angled triangle:* A triangle in which one of the angles is greater than 90°.

A triangle in which two sides are equal and one angle is 90°, is an isosceles right triangle. The hypotenuse is $\sqrt{2}$ times of each equal side.

Important Properties of Triangles

1. The sum of the angles of a triangle is 180°.
2. The measure of an exterior angle is equal to the sum of the measures of its interior opposite angles.
3. If two sides of a triangle are equal, then the angles opposite to them are also equal.
4. If two angles of a triangle are equal, then the sides opposite to them are also equal.
5. Each angle in an equilateral triangle is equal to 60°.
6. In a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
7. The sum of any two sides of a triangle is always greater than the third side.
   In ΔABC (see Fig. 12.21):
   (i) $AB + BC > AC$

(ii) $BC + AC > AB$

(iii) $AB + AC > BC$

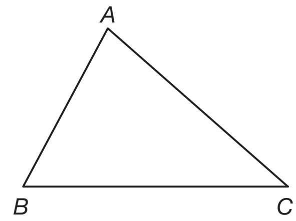


Figure 12.21

8. The difference of any two sides of a triangle is less than the third side.

In ΔABC (see Fig. 12.21):

(i) $(BC - AB) < AC$

(ii) $(AC - BC) < AB$

(iii) $(AC - AB) < BC$

9. In triangle ABC (see Fig. 12.21), if $\angle B > \angle C$, then the side opposite to $\angle B$ is longer than the side opposite to $\angle C$, i.e., $AC > AB$.

10. In triangle ABC (see Fig. 12.21), if $AC > BC$, then the angle opposite to side AC is greater than the angle opposite to side BC, i.e., $\angle B > \angle A$.

Congruence of Triangles

Two geometrical figures are congruent if they have the same shape and the same size.

The three angles of a triangle determine its shape and its three sides determine its size. If the three angles and the three sides of a triangle are respectively equal to the corresponding angles and sides of another triangle, then the two triangles are congruent. However, it is not necessary that each of the six elements of one triangle have to be equal to the corresponding elements of the other triangle in order to conclude that the two triangles are congruent.

Based on the study and experiments, the following results can be applied to establish the congruence of two triangles.

Side-Side-Side (SSS) Congruence Property

By the SSS congruence property, two triangles are congruent if the three sides of one triangle are respectively equal to the corresponding three sides of the other triangle.

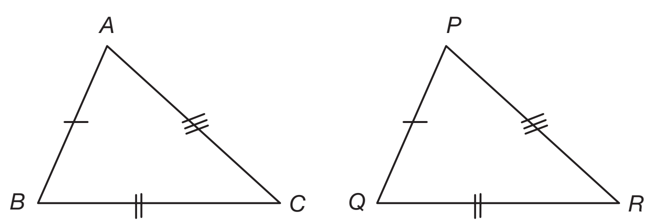


Figure 12.22

ABC and PQR are two triangles (see Fig. 12.22) such that $AB = PQ$, $BC = QR$ and $CA = RP$, then ΔABC is congruent to ΔPQR.

We write this as $\Delta ABC \cong \Delta PQR$.

Side-Angle-Side (SAS) Congruence Property

By the SAS congruence property, two triangles are congruent if the two sides and the included angle of one triangle are respectively equal to the corresponding two sides and the included angle of the other triangle.

If ABC and PQR are two triangles (see Fig. 12.23) such that $AB = PQ$, $BC = QR \angle ABC = \angle PQR$, then $\Delta ABC \cong \Delta PQR$.

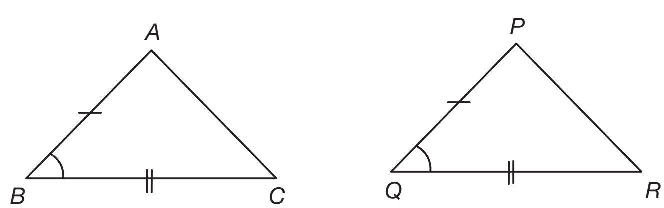


Figure 12.23

Angle-Side-Angle (ASA) Congruence Property

By the ASA congruence property, two triangles are congruent if any two angles and a side of one triangle are respectively equal to the corresponding parts of the other triangle.

In Fig. 12.24, $\angle BAC = \angle QPR$ and $\angle ABC = \angle PQR$ and $AB = PQ$. It follows that $\Delta ABC \cong \Delta PQR$.

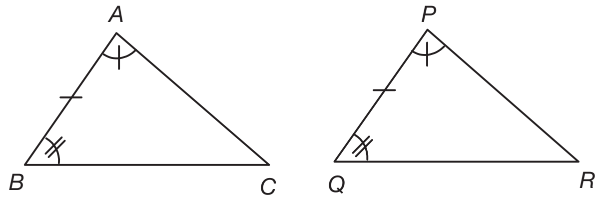


Figure 12.24

Right Angle-Hypotenuse-Side Congruency Property

The RHS congruence property states that two right triangles are congruent, if the hypotenuse and one side of a triangle are respectively equal to the hypotenuse and the corresponding side of the other right triangle.

In ΔABC and ΔPQR (see Fig. 12.25), $AB = PQ$, $AC = PR$ and $\angle ABC = \angle PQR = (90°)$, then $\Delta ABC \cong \Delta PQR$.

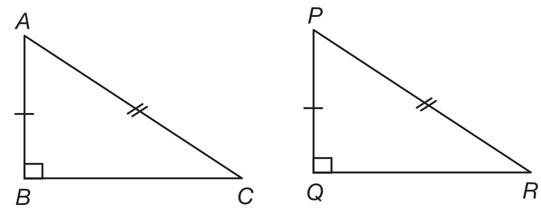


Figure 12.25

QUADRILATERALS

A closed figure bounded by four line segments is called a quadrilateral. A quadrilateral $PQRS$ has the following elements (see Fig. 12.26):

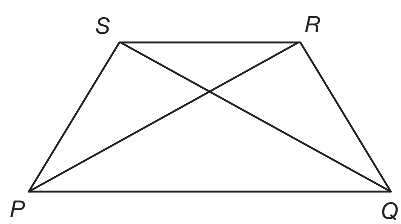


Figure 12.26

1. Four vertices P, Q, R and S.
2. Four sides $\overline{PQ}, \overline{QR}, \overline{RS}$ and $\overline{SP}$.
3. Four angles $\angle P, \angle Q, \angle R$ and $\angle S$.
4. Two diagonals $\overline{PR}$ and $\overline{QS}$.
5. Four pairs of adjacent sides $(\overline{PQ}, \overline{QR}), (\overline{QR}, \overline{RS}), (\overline{RS}, \overline{SP})$ and $(\overline{SP}, \overline{PQ})$.
6. Two pairs of opposite sides $(\overline{PQ}, \overline{RS})$ and $(\overline{QR}, \overline{PS})$.
7. Four pairs of adjacent angles $(\angle P, \angle Q)$ $(\angle Q, \angle R)$, $(\angle R, \angle S)$ and $(\angle S, \angle P)$.
8. Two pairs of opposite angles $(\angle P, \angle R)$ and $(\angle Q, \angle S)$.
9. The sum of the four angles of a quadrilateral is 360°.

Different Types of Quadrilaterals

Trapezium

In a quadrilateral, if two opposite sides are parallel to each other, then it is called a trapezium.

In Fig. 12.27, $\overline{AB} \parallel \overline{CD}$, hence $ABCD$ is a trapezium.

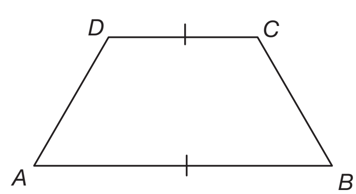


Figure 12.27

Parallelogram

In a quadrilateral if both the pairs of opposite sides are parallel, then it is called a parallelogram.

In Fig. 12.28:

1. $AB = CD$ and $BC = AD$.
2. $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$.

Hence, $ABCD$ is a parallelogram.

Note In a parallelogram, diagonals need not be equal, but they bisect each other.

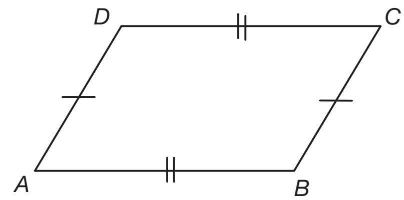


Figure 12.28

Rectangle

In a parallelogram, if each angle is a right angle (90°), then it is called a rectangle.

In Fig. 12.29, $\angle A = \angle B = \angle C = \angle D = 90°$, $AB = CD$ and $BC = AD$.

Hence, $ABCD$ is a rectangle.

Note In a rectangle, the diagonals are equal, i.e., $AC = BD$.

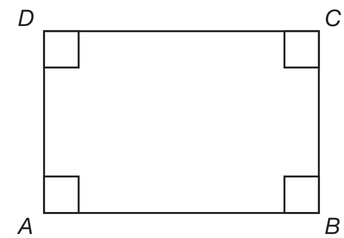


Figure 12.29

Rhombus

In a parallelogram, if all the sides are equal, then it is called a rhombus.

In Fig. 12.30, $AB = BC = CD = AD$, hence $ABCD$ is a rhombus.

Notes

1. In a rhombus, the diagonals need not be equal.
2. In a rhombus, the diagonals bisect each other at right angles, i.e., $AO = OC$, $BO = OD$ and $\overline{AC} \perp \overline{DB}$.

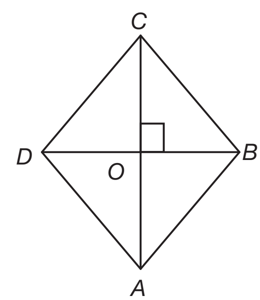


Figure 12.30

Square

In a rhombus, if each angle is a right angle, then it is called a square, or otherwise in a rectangle, if all the sides are equal, then it is called a square.

In Fig. 12.31, $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.

Hence, $ABCD$ is a square.

Notes

1. In a square, the diagonals bisect each other at right angles.
2. In a square, the diagonals are equal.

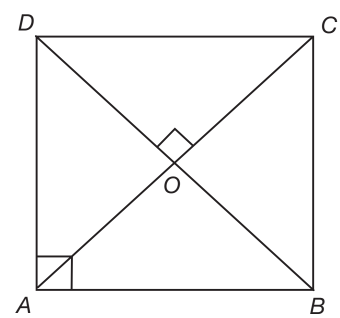


Figure 12.31

Isosceles Trapezium

In a trapezium, if the non-parallel opposite sides are equal, then it is called an isosceles trapezium.

In Fig. 12.32, $\overline{AB} \parallel \overline{CD}$ and $BC = AD$.

Hence, $ABCD$ is an isosceles trapezium.

$\Rightarrow \angle A = \angle B$ and $\angle C = \angle D$.

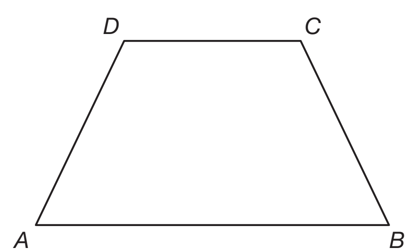


Figure 12.32

Kite

In a quadrilateral, if two pairs of adjacent sides are equal, then it is called a kite.

In Fig. 12.33, $AB = AD$ and $BC = CD$.
Hence, $ABCD$ is a kite.

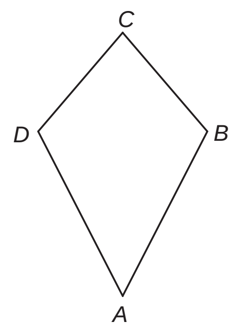


Figure 12.33

GEOMETRICAL RESULTS ON AREAS

1. Parallelograms on the same base and between the same parallels are equal in area.

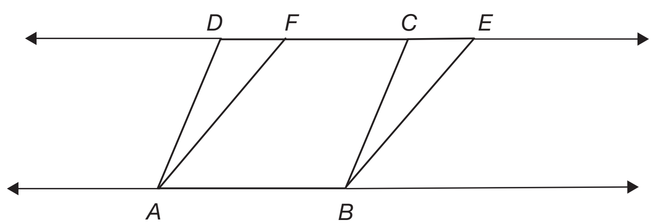


Figure 12.34

In Fig. 12.34, parallelogram $ABCD$ and parallelogram $ABEF$ are on the same base $\overline{AB}$ and between the same parallels $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\therefore$ Area of parallelogram $ABCD$ = Area of parallelogram $ABEF$.

Note A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

2. The area of a triangle is half the area of the parallelogram, if they lie on the same base and between the same parallels.

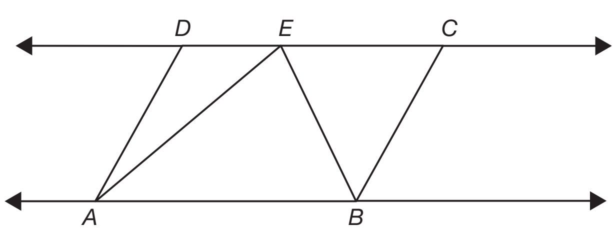


Figure 12.35

In Fig. 12.35, parallelogram $ABCD$ and ΔABE are on the same base $\overline{AB}$ and between the same parallels $\overline{AB}$ and $\overline{CD}$.

$\therefore$ Area of $\Delta ABE = \frac{1}{2}$ Area of parallelogram $ABCD$.

3. Triangles on the same base and between the same parallels are equal in area.

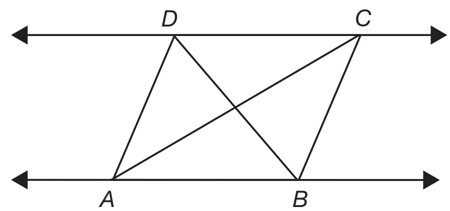


Figure 12.36

In Fig. 12.36, ΔABC and ΔABD are on the same base $\overline{AB}$ and between the same parallels $\overline{AB}$ and $\overline{CD}$.

$\therefore$ Area of ΔABC = Area of ΔABD.

Note Triangles with equal bases and between the same parallels are equal in area.

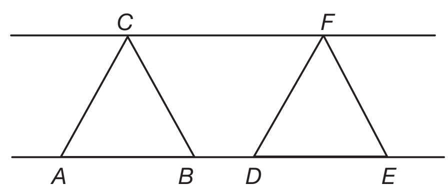


Figure 12.37

In Fig. 12.37, in ΔABC and ΔDEF, $AB = DE$ and $\overline{AE} \parallel \overline{CF}$.

$\therefore$ Area of ΔABC = Area of ΔDEF.

4. Triangles with equal bases and with equal areas lie between the same parallels.

In Fig. 12.38, if Area of ΔABC = Area of ΔABD, then $\overline{AB} \parallel \overline{CD}$.

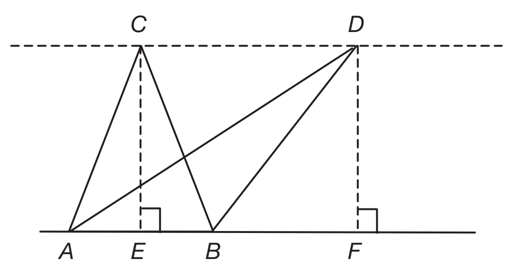


Figure 12.38

Note In this case, altitudes CE and DF are equal.

5. A diagonal of a parallelogram divides the parallelogram into two triangles of equal area.

In Fig. 12.39, diagonal $\overline{AC}$ divides parallelogram $ABCD$ into two triangles, ΔABC and ΔACD.

Here, area of ΔABC = area of ΔACD.

Similarly, diagonal BD divides the parallelogram into two triangles, ΔABD and ΔBDC.

Hence, the area of ΔABD = area of ΔBCD.

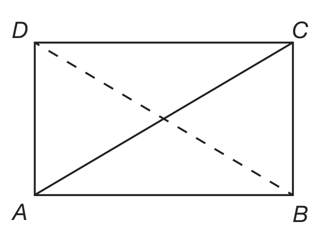


Figure 12.39

SOME THEOREMS ON TRIANGLES

Mid-point Theorem

In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and also half of it.

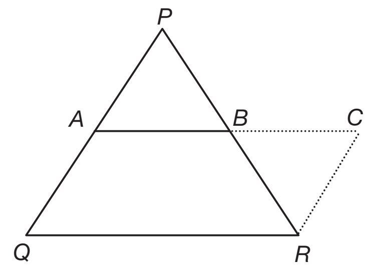


Figure 12.40

Given: In ΔPQR, A and B are the mid-points of $\overline{PQ}$ and $\overline{PR}$. respectively.

RTP: $\overline{AB} \parallel \overline{QR}$ and $AB = \frac{1}{2}QR$

Construction: Draw $\overline{RC}$ parallel to $\overline{QA}$ to meet produced $\overline{AB}$ at C (see Fig. 12.40).

Proof:

1. In ΔABP and ΔCBR,
   $\angle PBA = \angle RBC$ (vertically opposite angles)
   $\angle PAB = \angle RCB$ (alternate angles and $\overline{CR} \parallel \overline{PQ}$)
   $PB = BR$ (B is mid-point of PR).
   By AAS congruence property,
   $\Delta ABP \cong \Delta CBR$
   $\therefore PA = CR$ and $AB = BC$ (corresponding parts of congruent triangles)
   $$\Rightarrow AQ = CR. \qquad (\because PA = AQ)$$
   In quadrilateral $ACRQ$, $AQ = CR$ and $\overline{AQ} \parallel \overline{CR}$.
   $\therefore ACRQ$ is a parallelogram.
   $$\therefore \overline{AC} \parallel \overline{QR} \Rightarrow \overline{AB} \parallel \overline{QR}$$
2. $AC = QR$ (opposite sides of parallelogram)
   $$\Rightarrow QR = AB + BC$$
   $$\Rightarrow QR = 2AB \ (\because AB = BC)$$
   $$\Rightarrow AB = \frac{1}{2}QR.$$

Basic Proportionality Theorem

In a triangle, if a line is drawn parallel to one side of the triangle, then it divides the other two sides in the same ratio.

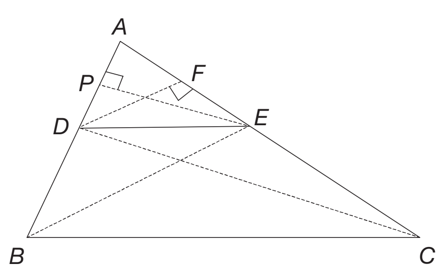


Figure 12.41

Given: In ΔABC, $\overline{DE}$ is drawn parallel to $\overline{BC}$.

RTP: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw $\overline{EP} \perp \overline{AB}$ and $\overline{DF} \perp \overline{AC}$. Join DC and BE (see Fig. 12.41).

Proof:

$$\frac{\text{Area of triangle } ADE}{\text{Area of triangle } BDE} = \frac{\frac{1}{2} \times AD \times PE}{\frac{1}{2} \times BD \times PE} = \frac{AD}{BD}$$

and $$\frac{\text{Area of triangle } ADE}{\text{Area of triangle } CDE} = \frac{\frac{1}{2} \times AE \times DF}{\frac{1}{2} \times EC \times DF} = \frac{AE}{EC}.$$

But, area of triangles BDE and CDE are equal. ($\because$ Two triangles lying on the same base and between the same parallel lines are equal in areas).

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}.$$

Similarly, it can be proved that $\frac{AB}{AD} = \frac{AC}{AE}$ and $\frac{AB}{BD} = \frac{AC}{CE}$.

Converse of Basic Proportionality Theorem

If a line divides two sides of a triangle in the same ratio, then that line is parallel to the third side.

In Fig. 12.42, $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \overline{DE} \parallel \overline{BC}$.

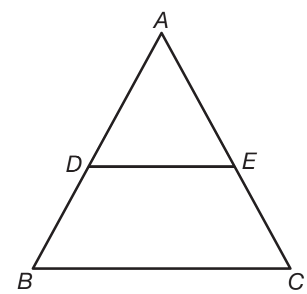


Figure 12.42

Note The intercepts made by three or more parallel lines on any two transversals are proportional.

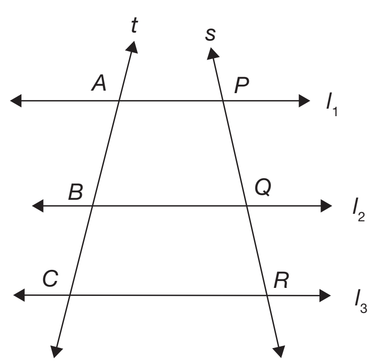


Figure 12.43

In Fig. 12.43, the parallel lines l_1, l_2 and l_3 make intercepts AB and BC on transversal t and intercepts PQ and QR on transversal s.

Then, $\frac{AB}{BC} = \frac{PQ}{QR}$.

EXAMPLE 12.1

Divide line segment $AB = 10$ cm into six equal parts.

SOLUTION

Step 1: Draw $AB = 10$ cm.

Step 2: Draw a ray AL such that AL does not coincide with AB.

Step 3: Mark six equal line segments AC, CD, DE, EF, FG and GH on ray AL with convenient length, using a compass.

Step 4: Join HB.

Step 5: Draw lines parallel to HB through the points C, D, E, F and G intersecting the line segment AB at P, Q, R, S and T respectively. The line segments AP, PQ, QR, RS, ST and TB are the required six equal parts of the line segment AB (see Fig. 12.44).

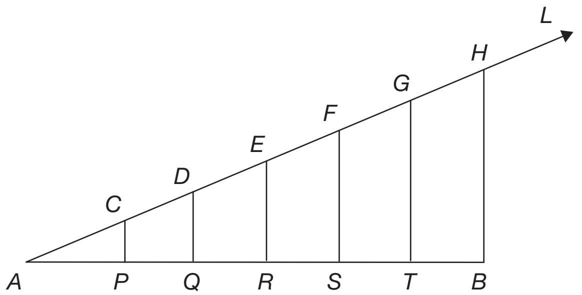


Figure 12.44

Similarity

Two figures are said to be congruent, if they have the same shape and same size. But the figures of the same shape need not have the same size. The figures of the same shape, but not necessarily of the same size are called similar figures.

Examples:

1. Any two line segments are similar (see Fig. 12.45).
2. Any two squares are similar.

Figure 12.45

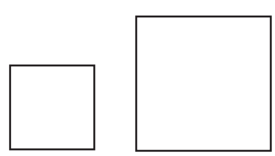


Figure 12.46

3. Any two equilateral triangles are similar.

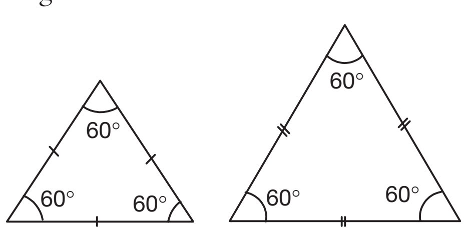


Figure 12.47

4. Any two circles are similar.

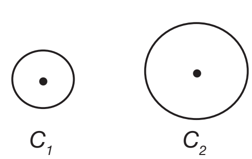


Figure 12.48

Two polygons are said to be similar to each other, if:

1. Their corresponding angles are equal, and
2. The lengths of their corresponding sides are proportional.

Note '~' is the symbol used for 'is similar to'.

If ΔABC is similar to ΔPQR, we denote it as $\Delta ABC \sim \Delta PQR$.
The relation 'is similar to' satisfies the following properties:

1. It is reflexive as every figure is similar to itself.
2. It is symmetric as, if A is similar to B, then B is also similar to A.
3. It is transitive as, if A is similar to B and B is similar to C, then A is similar to C.

$\therefore$ The relation 'is similar to' is an equivalence relation.

Criteria for Similarity of Triangles

In two triangles, if either corresponding angles are equal or the ratio of corresponding sides are proportional, then the two triangles are similar to each other.

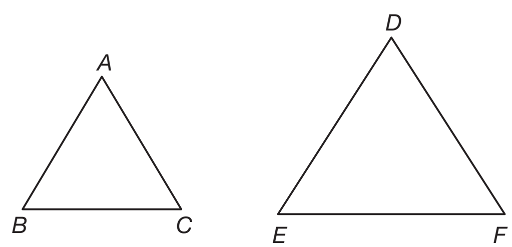


Figure 12.49

In ΔABC and ΔDEF (see Fig. 12.49),

1. If $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$. This property is called **AAA criterion**.
2. If $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$. This property is called **SSS criterion**.
3. If $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$. This property is called **SAS criterion**.

Results on Areas of Similar Triangles

1. The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides of the triangles. In Fig. 12.50, $\Delta ABC \sim \Delta DEF$.

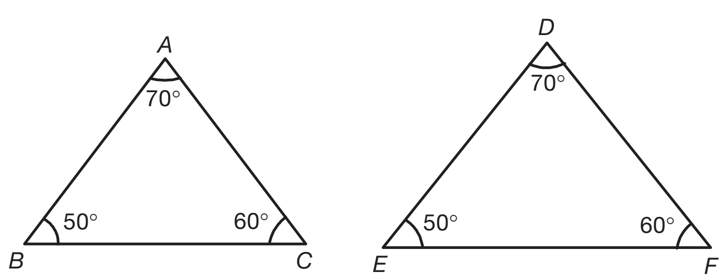


Figure 12.50

$$\Delta ABC \sim \Delta DEF \Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{FD^2}.$$

2. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

In Fig. 12.51, $\Delta ABC \sim \Delta DEF$ and AX, DY are the altitudes.

Then, $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AX^2}{DY^2}$.

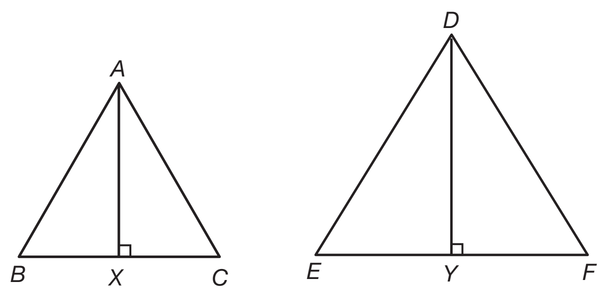


Figure 12.51

3. The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding medians.

In Fig. 12.52, $\Delta ABC \sim \Delta PQR$ and AD and PS are medians.

Then, $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \dfrac{AD^2}{PS^2}$.

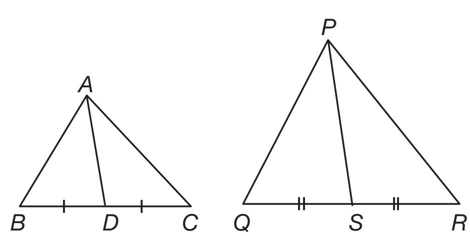


Figure 12.52

4. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding angle bisector segments.

In Fig. 12.53, $\Delta ABC \sim \Delta DEF$ and AP, DQ are bisectors of $\angle A$ and $\angle D$ respectively, then

$\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AP^2}{DQ^2}$.

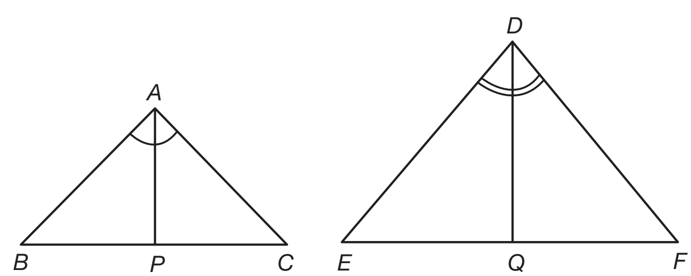


Figure 12.53

Pythagorean Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: In ΔABC, $\angle B = 90°$.

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $\overline{BP}$ perpendicular to $\overline{AC}$ (see Fig. 12.54).

Proof: In ΔAPB and ΔABC,

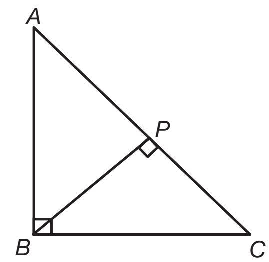


Figure 12.54

$\angle APB = \angle ABC$ (right angles)

$\angle A = \angle A$ (common)

$\therefore$ Triangle APB is similar to triangle ABC.

$$\Rightarrow \frac{AP}{AB} = \frac{AB}{AC}$$

$\Rightarrow AB^2 = (AP)(AC)$.

Similarly, $BC^2 = (PC)(AC)$

$\therefore AB^2 + BC^2 = (AP)(AC) + (PC)(AC)$

$AB^2 + BC^2 = (AC)(AP + PC)$
$AB^2 + BC^2 = (AC)(AC)$

$\Rightarrow AC^2 = AB^2 + BC^2$.

Hence, proved.

Converse of Pythagorean Theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

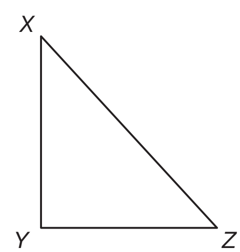


Figure 12.55

Given: In ΔXYZ, $XZ^2 = XY^2 + YZ^2$ (see Fig. 12.55).
To prove: $\angle Y = 90°$
Construction: Construct ΔMNO, such that $XY = MN$, $YZ = NO$ and $\angle N = 90°$ (see Fig. 12.56).
Proof: In ΔMNO, $\angle N = 90°$.

$\therefore OM^2 = MN^2 + NO^2$ (Pythagorean theorem)

$\Rightarrow OM^2 = XY^2 + YZ^2$ (construction) (1)

But, $XZ^2 = XY^2 + YZ^2$ (Given) (2)

From Eqs. (1) and (2), we get:

$$OM^2 = XZ^2 \Rightarrow OM = XZ$$

By SSS congruence criterion, we get:

$$\Delta XYZ \cong \Delta MNO$$

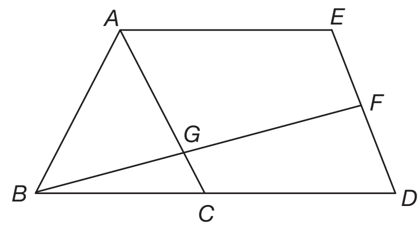
$$\therefore \angle Y = \angle N = 90°.$$

Hence, $\angle Y = 90°$.

EXAMPLE 12.2

In the given figure (not to scale), ABC is an isosceles triangle in which $AB = AC$. $AEDC$ is a parallelogram. If $\angle CDF = 70°$ and $\angle BFE = 100°$, then find $\angle FBA$.

(a) 30° **(b)** 40° **(c)** 50° **(d)** 80°



HINTS

(i) In a parallelogram, any pair of adjacent angles are supplementary.

(ii) $\angle EFG = \angle FGC = \angle AGB$ and $\angle ACB$

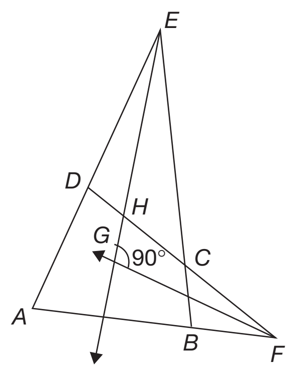
$= \angle ABC = \angle CDE. \left(\because \overline{AC} \parallel \overline{DE}\right)$

(iii) Use the above data and find $\angle BAC$, and then $\angle ABG$.

EXAMPLE 12.3

In the given figure, $ABCD$ is a cyclic quadrilateral, $\angle DAB = 50°$ and $\angle ABC = 80°$. $\overrightarrow{EG}$ and $\overrightarrow{FG}$ are the angle bisectors of $\angle DEC$ and $\angle BFC$. Find $\angle FHG$.

(a) 80° **(b)** 90° **(c)** 75° **(d)** 105°



SOLUTION

$\angle DAB = 50° \Rightarrow \angle DCE = 50°$ (exterior angle of a cyclic quadrilateral)

$\angle ABC = 80° \Rightarrow \angle EDC = 80°$ (exterior angle of a cyclic quadrilateral)

$\therefore$ In ΔDEC, $\angle DEC = 180° - (50° + 80°) = 50°$.

$\overline{EG}$ is the bisector of $\angle DEC$

$$\Rightarrow \angle DEH = \frac{50°}{2} = 25°.$$

$\therefore \angle DHE = 180° - (\angle HDE + \angle DEH) = 180° - (80° + 25°) = 75°$.

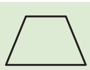
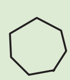
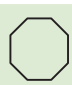
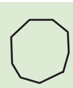
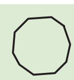
$\angle FHG = \angle DHE$ (vertically opposite angles) $\Rightarrow \angle FHG = 75°$.

POLYGONS

A closed plane figure bounded by three or more line segments is called a polygon.

1. Each line segment is called a side of the polygon.
2. The point at which any two adjacent sides intersect is called a vertex of the polygon.

Different polygons, the number of their sides and their names are given in the following table.

Number of Sides	3	4	5	6	7	8	9	10
Name of the polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Septagon	Octagon	Nonagon	Decagon
Corresponding figure								

Polygons can also be classified as

1. Convex polygons and
2. Concave polygons.

Convex Polygon and Concave Polygon

A polygon in which each interior angle is less than 180° is called a convex polygon (Fig. 12.57). Otherwise it is called concave polygon (Fig. 12.58).

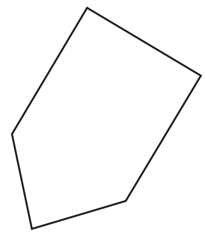


Figure 12.57

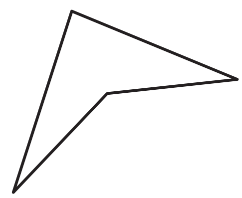


Figure 12.58

Note Unless otherwise mentioned, we refer to convex polygons simply as polygons.

Some Important Results on Polygons

1. **Diagonals of a polygon:** A line segment joining any two non-consecutive vertices of a polygon is called its diagonal.

   Note The number of diagonals of a polygon with n sides is given by $\dfrac{n(n-3)}{2}$.

EXAMPLE 12.4

Find the number of diagonals of a 10-sided polygon.

SOLUTION

Here, $n = 10$

$$\therefore \text{Number of diagonals } = \frac{10(10-3)}{2} = 35.$$

2. The sum of the interior angles of an n-sided polygon is $(2n - 4)\ 90°$ or $(2n - 4)$ right angles.

EXAMPLE 12.5

Find the sum of the interior angles of a polygon of 8 sides.

SOLUTION

Here, $n = 8$

$$\therefore \text{Sum of interior angles} = [(2)(8) - 4]90° = 1080°.$$

3. The sum of all the exterior angles of a polygon is 360°.
4. In a polygon, if all the sides are equal and all the angles are equal, then it is called a **regular polygon**.

   In case, all the sides are not equal, then the polygon is called an **irregular polygon**.
5. Each interior angle of regular polygon of n sides is $\left(\dfrac{(2n-4)90}{n}\right)^{\circ}$.

6. Each exterior angle of a regular polygon of n sides is $\left(\frac{360}{n}\right)^{\circ}$.

Note In case of a regular polygon, all the interior angles are equal and all the exterior angles are also equal.

7. The number of sides n of a regular polygon whose exterior angle x° is $= \left(\frac{360}{x}\right)^{\circ}$.

CONSTRUCTION OF TRIANGLES

In the earlier classes, we have learnt construction of triangles, when

1. all the three sides are given.
2. two sides and an angle are given.
3. two angles and an included side are given.
4. a side and the hypotenuse of a right triangle are given.

In the present section, we will discuss construction of triangles in some more cases.

1. **To construct a triangle when the base and the sum of the other two sides and one base angle are given.**

EXAMPLE 12.6

Construct a triangle ABC, in which base $BC = 6$ cm, $AB + AC = 9$ cm and $\angle ABC = 45°$.

SOLUTION

Step 1: Draw $BC = 6$ cm.

Step 2: Draw $\overrightarrow{BX}$, such that $\angle CBX = 45°$.

Step 3: With B as the centre and the radius as $AB + AC = 9$ cm, draw an arc to meet $\overrightarrow{BX}$ at D. Join CD.

Step 4: Draw the perpendicular bisector of $\overline{CD}$ to intersect $\overline{BD}$ at A. Join CA. ΔABC is the required triangle.

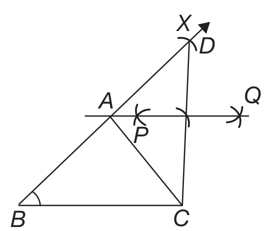


Figure 12.59

Proof: As AQ is the perpendicular bisector of CD, ACD is an isosceles triangle, hence $AD = AC$ and $AB + AC = AB + AD = BD = 9$ cm.

2. **Construction of a triangle when its perimeter and the base angles are given.**

EXAMPLE 12.7

Construct ΔABC, whose perimeter is 10 cm and base angles are 60° and 44°.

SOLUTION

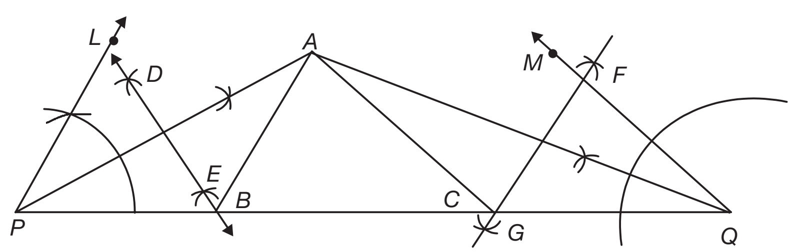


Figure 12.60

Step 1: Draw $PQ = AB + BC + CA = 10$ cm.

Step 2: Draw $\angle LPQ$ equal to $\angle B$ and $\angle MQP$ equal to $\angle C$.

Step 3: Draw the angle bisectors of $\angle LPQ$ and $\angle MQP$. Let these bisectors meet at A.

Step 4: Draw the perpendicular bisectors of $\overline{AP}$ and $\overline{AQ}$ to meet PQ at B and C respectively.

Step 5: Join AB and AC to form the required ΔABC.

3. **To construct a triangle when its base, one base angle and the difference of the remaining sides are given.**

EXAMPLE 12.8

Construct a triangle PQR, in which $QR = 4.5$ cm, $\angle Q = 44°$ and $PQ - PR = 2$ cm.

SOLUTION

Step 1: Draw a line segment $QR = 4.5$ cm.

Step 2: Draw $\overrightarrow{QX}$, such that $\angle RQX = 44°$.

Step 3: With Q as the centre and radius = 2 cm, draw an arc to intersect $\overrightarrow{QX}$ at S.

Step 4: Join SR and draw the perpendicular bisector of SR to intersect $\overrightarrow{QX}$ at P.

Step 5: Join P and R. PQR is the required triangle.

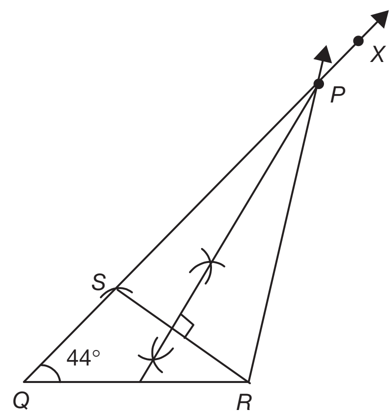


Figure 12.61

4. **Construction of a triangle, when the base, the angle at the vertex and the sum of the other two sides are given.**

EXAMPLE 12.9

Construct ΔPQR in which $QR = 2.1$ cm, $\angle P = 46°$ and $PQ + PR = 5.1$ cm.

SOLUTION

Step 1: Construct $\angle QS = PQ + PR = 5.1$ cm.

Step 2: Construct $\angle QSR = \frac{1}{2}(46°) = 23°$.

Step 3: With Q as the centre and the radius 2.1 cm ($QR = 2.1$ cm), draw an arc to intersect SX at R and R'.

Step 4: Join QR (or QR') and draw the perpendicular bisector of SR (or SR') to intersect $\overrightarrow{QS}$ at P (or P'). PQR (or $P'QR'$) is the required triangle.

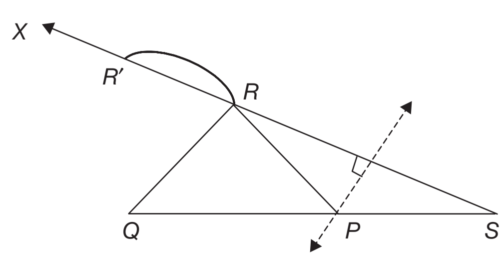


Figure 12.62

5. To construct a triangle when two sides and the median drawn on the third side are given.

EXAMPLE 12.10

Construct a triangle PQR, such that $PQ = 3.8$ cm, $QR = 4.3$ cm and the median from Q to PR is 3.5 cm.

SOLUTION

Step 1: Draw a line segment QR of length 4.3 cm.

Step 2: Draw the perpendicular bisector of $\overrightarrow{QR}$ and mark the mid-point of QR as S.

Step 3: Taking Q as the centre and radius equal to 3.5 cm, draw an arc.

Step 4: Taking S as the centre and radius = 1.9 cm $\left(\frac{1}{2}PQ\right)$, draw another arc intersecting the already drawn arc (in Step 3) at T.

Step 5: Join R and T and produce RT to X.

Step 6: With Q as the centre and radius = 3.8 cm, draw an arc to intersect $\overrightarrow{RX}$ at P. Join PQ. ΔPQR is the required triangle.

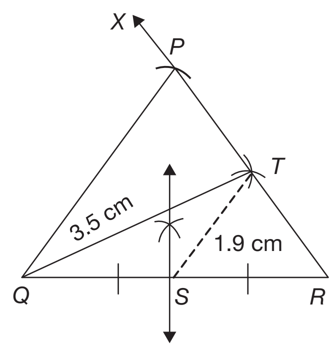


Figure 12.63

6. Construction of a triangle when the base, the angle between the other two sides and the difference of the other two sides are given.

EXAMPLE 12.11

Construct ΔPQR in which $QR = 3.4$ cm, $\angle P = 50°$ and $PQ - PR = 1.2$ cm.

SOLUTION

Step 1: Draw the ray, QX.

Step 2: Mark the point S on $\overrightarrow{QX}$, such that $QS =$ 1.2 cm.

Step 3: Construct an angle equal to

$$180 - \frac{1}{2}(180 - 50) = 115°$$

at S, such that $\angle QSY = 115°$.

Step 4: With Q as the centre and radius = 3.4 cm, draw an arc to meet SY at R.

Step 5: Join Q and R.

Step 6: Draw the perpendicular bisector of SR intersecting QX at P.

Step 7: Join PR. PQR is the required triangle.

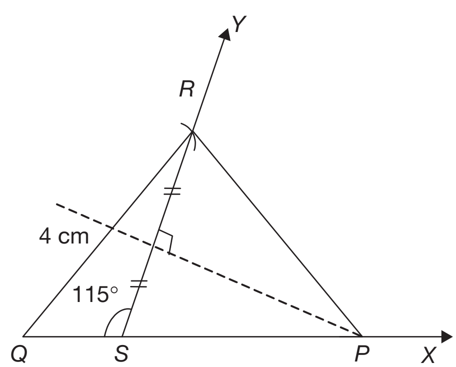


Figure 12.64

7. Construction of a triangle when the base, sum of the remaining sides and the difference of the base angles are given.

EXAMPLE 12.12

Construct ΔPQR in which $QR = 3.2$ cm, $PQ + PR = 5.9$ cm and $\angle R - \angle Q = 60°$.

SOLUTION

Step 1: Draw line segment $QR = 3.2$ cm.

Step 2: Draw $\overrightarrow{RX}$, such that

$$\angle QRX = \left(90° + \frac{\angle R - \angle Q}{2}\right) = 90° + 30° = 120°.$$

Step 3: Taking Q as the centre and a radius equal to 5.9 cm ($PQ + PR$), draw an arc intersecting $\overrightarrow{RX}$ at S.

Step 4: Join Q and S.

Step 5: Draw RY, such that

$$\angle QRY = \left(\frac{\angle R - \angle Q}{2}\right) = 30°,$$ intersecting QS at T.

Step 6: Draw the perpendicular bisector of $\overline{RT}$ intersecting $\overline{QS}$ at P.

Step 7: Join P and R. PQR is the required triangle.

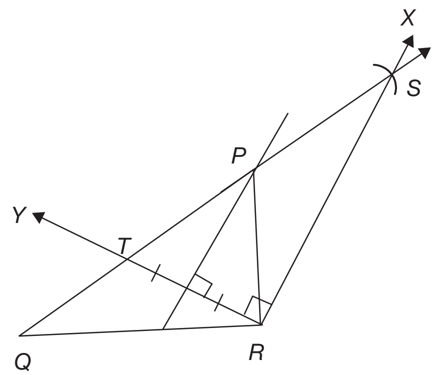


Figure 12.65

8. Construction of a triangle when the base, difference of the remaining two sides and the difference of the base angles are given.

EXAMPLE 12.13

Construct a triangle PQR in which $QR = 4.7$ cm, $PR - PQ = 2.3$ cm and $\angle Q - \angle R = 50°$.

SOLUTION

Step 1: Draw line segment $QR = 4.7$ cm.

Step 2: Draw $\overrightarrow{QX}$, such that

$$\angle RQX = \frac{1}{2}(\angle Q - \angle R) = 25°.$$

Step 3: With R as the centre and the radius = 2.3 cm, draw an arc intersecting $\overrightarrow{QX}$ at A.

Step 4: Join RA and produce $\overline{RA}$.

Step 5: Draw the perpendicular bisector of QA to intersect RA produced at P.

Step 6: Join PQ. PQR is the required triangle.

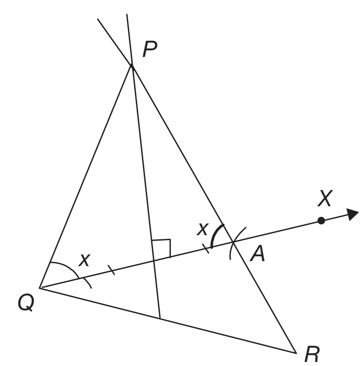


Figure 12.66

9. Construction of a triangle when two angles and the sum of the sides are given.

EXAMPLE 12.14

Construct ΔPQR in which $\angle P = 96°$, $\angle Q = 40°$ and $PQ + PR = 6$ cm.

SOLUTION

Step 1: Draw a line segment $\overline{RS} = 6$ cm.

Step 2: Draw $\overrightarrow{RX}$, such that $\angle SRX = 44°$.

Step 3: Draw $\overline{SY}$, such that $\angle RSY = \frac{1}{2}\angle P = 48°$, intersecting $\overrightarrow{RX}$ at Q.

Step 4: Draw the perpendicular bisector of QS to intersect RS at P.

Step 5: Join P and Q. PQR is the required triangle.

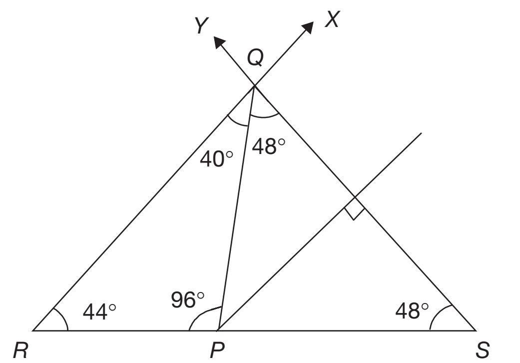


Figure 12.67

10. Construction of a triangle with two angles and the difference between two sides are given.

EXAMPLE 12.15

Construct a triangle PQR, such that $\angle P = 50°$, $\angle Q = 30°$ and $PQ - QR = 1.4$ cm.

SOLUTION

Step 1: Draw $\overrightarrow{PX}$.

Step 2: Locate the point S on $\overrightarrow{PX}$, such that $PS = 1.4$ cm.

Step 3: Draw $\overrightarrow{SY}$, such that $\angle PSY = 180 - \frac{1}{2}$

$(180° - \angle Q) = 180° - \frac{1}{2}(180° - 30°) = 105°$.

Step 4: Draw $\overrightarrow{PZ}$, such that $\angle SPZ = 50°$, intersecting $\overrightarrow{SY}$ at R.

Step 5: Draw perpendicular bisector of $\overline{RS}$ intersecting $\overrightarrow{PX}$ at Q.

Step 6: Join Q and R. PQR is the required triangle.

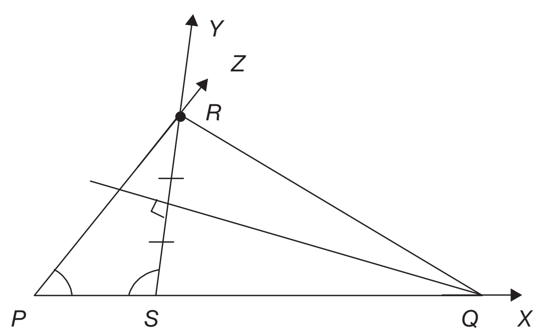


Figure 12.68

11. Construct a triangle equal to the area of a given convex quadrilateral.

Let $PQRS$ be the given quadrilateral.

Step 1: Join QS.

Step 2: Through R, draw a line parallel to $\overline{QS}$.

Step 3: Produce PQ to meet the parallel line drawn through R at T.

Step 4: Join ST. PST is the required triangle.

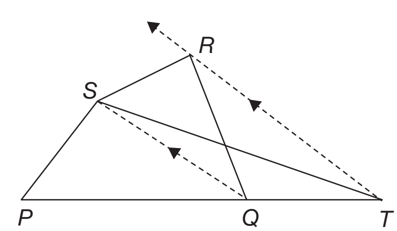


Figure 12.69

Justification

Ar(ΔQSR) = Ar(ΔQST) $\rightarrow$ Eq. 1. ($\therefore$ They lie on the same base $\overline{SQ}$, and lie between the same parallels).

Area of quadrilateral $PQRS = Ar(\Delta PQS)$ + Ar(ΔQSR).

= Ar(ΔPQS) + Ar(ΔQST) (From Eq. 1)

$\therefore$ Area of quadrilateral $PQRS$ = Area of ΔPST.

12. Construct a triangle equal to the area of a given pentagon.

Step 1: Construct a pentagon $ABCDE$.

Step 2: Join BD and AD.

Step 3: Draw lines parallel to $\overline{BD}$ and $\overline{AD}$ through C and E respectively to meet AB produced and BA produced at F and G. respectively.

Step 4: Join DF and DG to form ΔDGF which is equal to the area of the pentagon $ABCDE$.

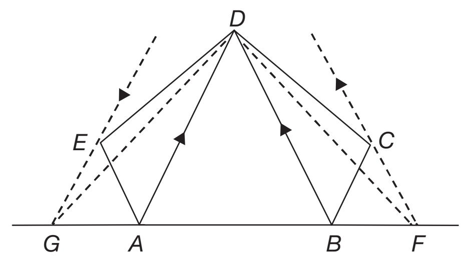


Figure 12.70

Justification

Area of ΔBDC = Area of ΔBDF

Area of ΔADE = Area of ΔADG ($\because$ They lie on the same bases, $\overline{BD}$ and $\overline{AD}$, and between the same parallels.)

Area of pentagon $ABCDE$ = Sum of the areas of ΔAED, ΔABD, ΔDBC

= Sum of the areas of ΔADG, ΔABD and ΔBDF

= Area of ΔGDF

$\therefore$ The area of the pentagon $ABCDE$ is equal to the area of ΔGDF.

CONSTRUCTION OF QUADRILATERALS

1. When four sides and one angle are given.

EXAMPLE 12.16

Construct a quadrilateral $ABCD$ in which $AB = 4.2$ cm, $\angle A = 80°$, $BC = 2.4$ cm, $CD = 3.3$ cm and $AD = 2.4$ cm.

SOLUTION

Step 1: Draw a line segment $AB = 4.2$ cm.

Step 2: Draw $\angle BAX = 80°$.

Step 3: Mark D on $\overrightarrow{AX}$, such that $AD = 2.4$ cm.

Step 4: Taking D as the centre and 3.3 cm as the radius, draw an arc, and taking B as the centre and 2.4 cm as radius, draw another arc to intersect the previous arc at C.

Step 5: Join CD and BC. $ABCD$ is the required quadrilateral.

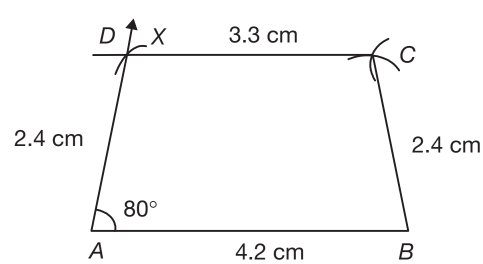


Figure 12.71

2. When three consecutive sides and two included angles are given.

EXAMPLE 12.17

Construct a quadrilateral $ABCD$ with $AB = 4$ cm, $BC = 2.8$ cm, $CD = 4$ cm, $\angle B = 75°$ and $\angle C = 105°$.

SOLUTION

Step 1: Draw a line segment $AB = 4$ cm.

Step 2: Draw $\overrightarrow{BX}$, such that $\angle ABX = 75°$.

Step 3: With B as the centre and a radius of 2.8 cm, draw an arc to cut $\overrightarrow{BX}$ at C.

Step 4: Draw $\overrightarrow{CY}$ which makes an angle 105° with $\overline{BC}$.

Step 5: Mark D on $\overline{CY}$, such that $CD =$ 4 cm.

Step 6: Join AD. $ABCD$ is the required quadrilateral.

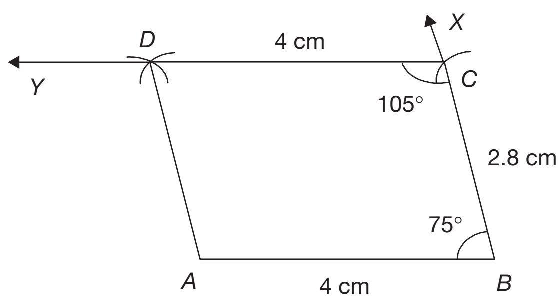


Figure 12.72

3. When four sides and one diagonal are given.

EXAMPLE 12.18

Construct a quadrilateral $ABCD$ in which $AB = 4.6$ cm, $BC = 2.6$ cm, $CD = 3.5$ cm, $AD =$ 2.6 cm and the diagonal $AC = 4.9$ cm.

SOLUTION

Step 1: Draw a line segment $AB = 4.6$ cm.

Step 2: With A and B as the centres, draw two arcs of radii 4.9 cm and 2.6 cm respectively to intersect each other at C.

Step 3: With C and A as the centres, draw two arcs of radii 3.5 cm and 2.6 cm respectively to intersect at D.

Step 4: Join BC, CD and AD to form quadrilateral $ABCD$.

$ABCD$ is the required quadrilateral.

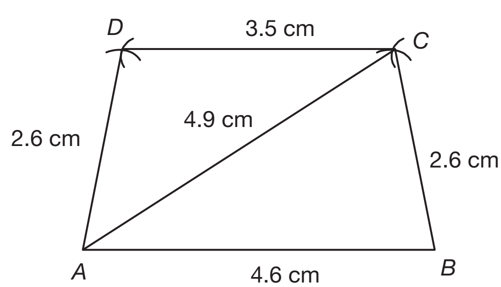


Figure 12.73

4. To construct a parallelogram, when two consecutive sides and the included angle are given.

EXAMPLE 12.19

Construct a parallelogram $ABCD$, when $AB = 4$ cm, $BC = 2.5$ cm and $\angle B = 100°$.

SOLUTION

Step 1: Draw line segment $AB = 4$ cm.

Step 2: Construct line BX, such that $\angle ABX = 100°$.

Step 3: Taking B as the centre and the radius = 2.5 cm, cut $\overline{BX}$ at the point C with an arc.

Step 4: Draw two arcs taking C and A as centres and 4 cm and 2.5 cm as radii respectively to intersect at D.

Step 5: Join AD and CD. $ABCD$ is the required parallelogram.

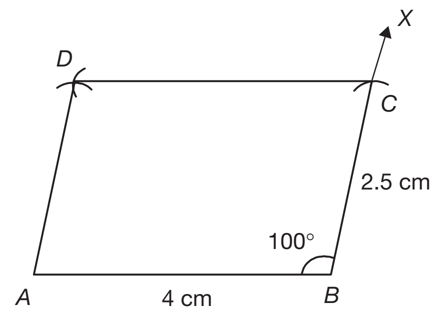


Figure 12.74

5. **Construction of a parallelogram when two adjacent sides and one diagonal are given.**

EXAMPLE 12.20

Construct a parallelogram $PQRS$, when $PQ = 3.7$ cm, $QR = 2.3$ cm and $PR = 4.8$ cm.

Step 1: Draw a line segment $PQ = 3.7$ cm.

Step 2: Draw an arc with P as the centre and a radius of 4.8 cm.

Step 3: With Q as the centre and $QR = 2.3$ cm, draw another arc to intersect the previous arc of Step 2 at R and join QR.

Step 4: With R as the centre, draw an arc of radius 3.7 cm.

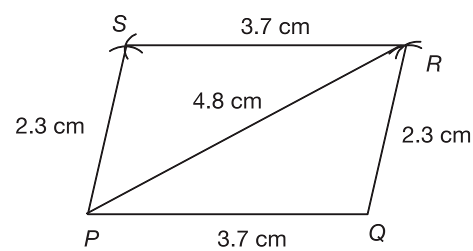


Figure 12.75

Step 5: With P as the centre, draw another arc of radius 2.3 cm to intersect the arc in Step 4 at S. Join RS and PS.

$PQRS$ is the required parallelogram.

6. **Construction of a parallelogram when both the diagonals and the angle between them are given.**

EXAMPLE 12.21

Construct a parallelogram $PQRS$ with $PR = 3$ cm, $QS = 4.2$ cm and the angle between the diagonals equal to 75°.

SOLUTION

Step 1: Draw the diagonal $PR = 3$ cm.

Step 2: Bisect PR to mark the mid-point of PR as O.

Step 3: Construct an angle of 75°at O, such that $\angle POX = 75°$.

Step 4: Taking O as the centre and radius $= \frac{1}{2}(QS) = \frac{1}{2} \times 4.2 = 2.1$ cm, draw arcs on the angular line constructed in Step 3 to cut at Q and S.

Step 5: Join PQ, QR, RS and SP to obtain the required parallelogram $PQRS$.

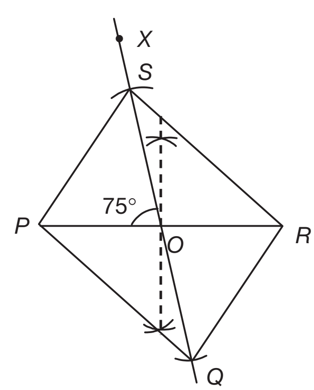


Figure 12.76

7. Construction of a rectangle when two adjacent sides are given.

EXAMPLE 12.22

Construct a rectangle $PQRS$ with $PQ = 5.2$ cm and $QR = 2.6$ cm.

Step 1: Draw $PQ = 5.2$ cm.

Step 2: At Q, construct a right angle, such that $\angle PQX = 90°$.

Step 3: Taking Q as the centre and 2.6 cm as radius, draw an arc to cut $\overrightarrow{QX}$ at R.

Step 4: With R and P as centres, draw two arcs with radii 5.2 cm and 2.6 cm respectively to cut each other at S. Join PS and RS. $PQRS$ is the required rectangle.

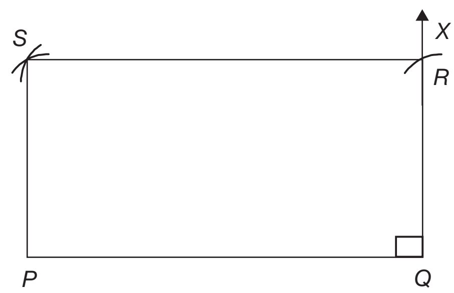


Figure 12.77

8. Construction of a rectangle when a side and a diagonal are given.

EXAMPLE 12.23

Construct a rectangle $PQRS$ with $PQ = 5.3$ cm and diagonal $PR = 5.8$ cm.

SOLUTION

Step 1: Draw a line segment $PQ = 5.3$ cm.

Step 2: At Q, construct $\angle PQX = 90°$.

Step 3: Taking P as the centre and 5.8 cm as radius, draw an arc to cut $\overrightarrow{QX}$ at R.

Step 4: With R and Q as centres, 5.3 cm and 5.8 cm respectively as radii, draw two arcs to intersect each other at S.

Step 5: Join RS and PS to form the required rectangle $PQRS$.

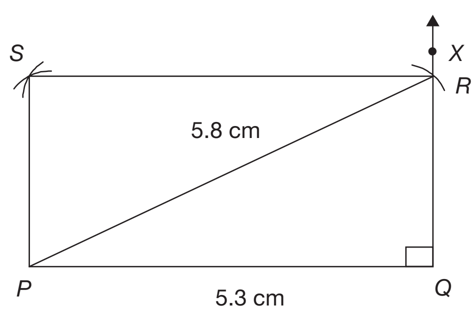


Figure 12.78

9. Construction of a rectangle when one diagonal and the angle between two diagonals are given.

EXAMPLE 12.24

Construct a rectangle $PQRS$, such that $PR = 5.2$ cm and the angle between the diagonals is 50°.

Step 1: Draw a line segment $PR = 5.2$ cm.

Step 2: Mark the midpoint of PR as O.

Step 3: Draw $\overleftrightarrow{XY}$ which makes an angle of 50° with $\overline{PR}$ at the point O.

Step 4: With O as the centre and with radius equal to $\frac{1}{2}(PR) = 2.6$ cm, cut $\overrightarrow{OX}$ and $\overrightarrow{OY}$ at S and Q respectively.

Step 5: Join PQ, QR, RS and PS to form the required rectangle $PQRS$.

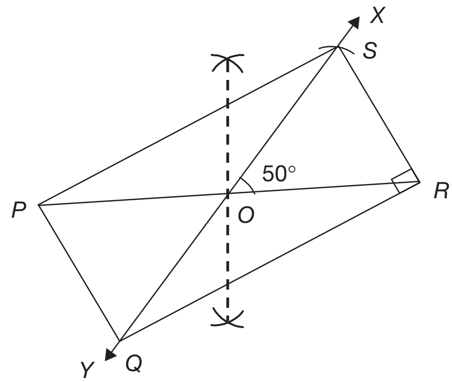


Figure 12.79

10. Construction of a square when one side is given.

EXAMPLE 12.25

Construct a square of side 3 cm.

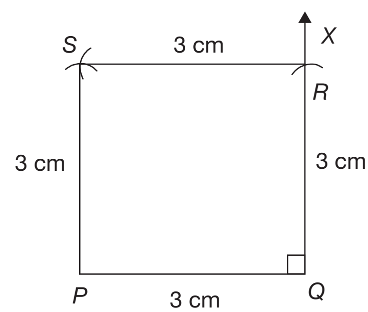


Figure 12.80

SOLUTION

Step 1: Draw a line segment $PQ = 3$ cm.

Step 2: Construct $\angle PQX = 90°$.

Step 3: Mark the point R on $\overrightarrow{QX}$, such that $QR = 3$ cm.

Step 4: With R and P as centres and with radii of 3 cm each draw two arcs to intersect each other at S.

Step 5: Join PS and RS to form the required square $PQRS$.

11. Construction of a square when a diagonal is given.

EXAMPLE 12.26

Construct a square with its diagonal as 4 cm.

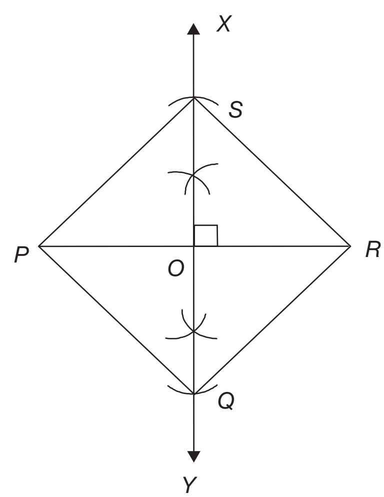


Figure 12.81

SOLUTION

Step 1: Draw a line segment $PR = 4$ cm.

Step 2: Draw perpendicular bisector XY of $\overline{PR}$ to bisect $\overline{PR}$ at O.

Step 3: Mark the points Q and S on $\overrightarrow{OY}$ and $\overrightarrow{OX}$, respectively, such that $OQ = OS = 2$ cm.

Step 4: Join PS, RS, PQ and QR to form the required square $PQRS$.

12. Construction of a rhombus when one side and one angle are given.

EXAMPLE 12.27

Construct a rhombus $PQRS$ with $PQ = 3.6$ cm and $\angle P = 50°$.

SOLUTION

Step 1: Draw a line segment $PQ = 3.6$ cm.

Step 2: Construct $\angle QPX = 50°$.

Step 3: Taking P as the centre and a radius equal to 3.6 cm, draw an arc to cut $\overrightarrow{PX}$ at S, such that $PS = 3.6$ cm.

Step 4: From Q and S, draw two arcs with radii 3.6 cm each to meet each other at R.

Step 5: Join QR and SR to form the required rhombus $PQRS$.

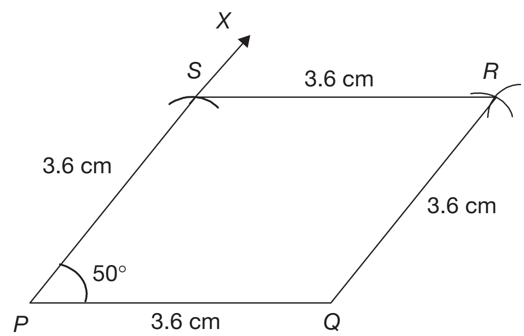


Figure 12.82

13. Construction of rhombus when one side and one diagonal are given.

EXAMPLE 12.28

Construct a rhombus $PQRS$, such that $PQ = 3.2$ cm and $PR = 4.2$ cm.

SOLUTION

Step 1: Draw a line segment $PQ = 3.2$ cm.

Step 2: Taking P as the centre and radius equal to 4.2 cm, draw an arc and taking Q as centre, radius as 3.2 cm draw another arc to cut the previous arc at R.

Step 3: With R and P as centres and the radii equal to 3.2 cm each, draw two arcs to meet at S.

Step 4: Join PS, RS and QR to form rhombus $PQRS$.

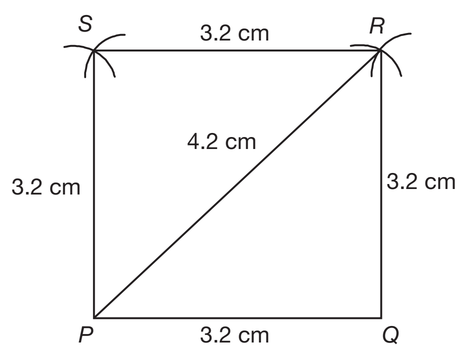


Figure 12.83

14. Construction of a rhombus when both the diagonals are given.

EXAMPLE 12.29

Construct a rhombus $PQRS$ with diagonal $PR = 3.4$ cm and $QS = 3.6$ cm.

SOLUTION

Step 1: Draw a line segment $PR = 3.4$ cm.

Step 2: Bisect PR and mark its mid-point as O.

Step 3: With O as the centre and radii 1.8 cm each, draw arcs on either sides of PR to cut perpendicular bisector of PR at Q and S.

Step 4: Join PS, PQ, QR and RS to form the required rhombus $PQRS$.

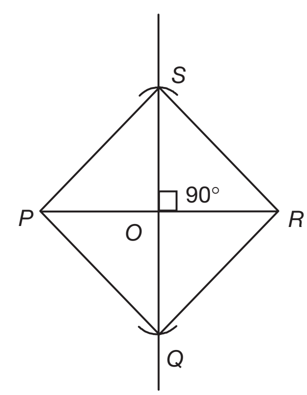


Figure 12.84

CIRCLES

In the lower classes, you have learnt some properties of a circle, like:

- A circle is a simple closed round figure.
- Three non-collinear points are required to construct a unique circle.
- The diameter of a circle is its longest chord.
- Equal chords are equidistant from its centre.
- The perpendicular from the centre of a circle to a chord in the circle bisects it.

Now, you shall learn more about circles.

Arc of a Circle

An arc is a part of a circle. In Fig. 12.85, the two parts of the circle between A and B are arcs. When the two arcs are unequal, the shorter one is called the minor arc and the longer one is called the major arc.

Note In Fig. 12.85, AQB is the minor arc and APB is the major arc.

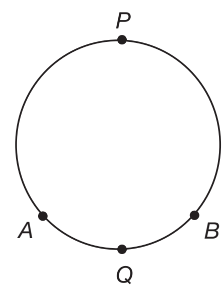


Figure 12.85

Semi-circle

The diameter of a circle divides it into two equal arcs. Each arc is called a **semi-circle**.

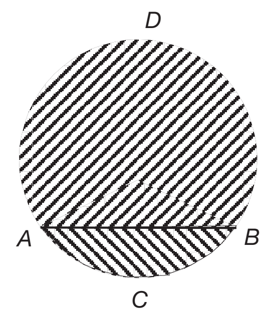


Figure 12.86

Segment of a Circle

A segment of a circle is the region between the arc of the circle and the chord joining the endpoints of the arc.

In Fig. 12.86, AB is a chord of the circle. Chord AB divides the circle into two regions. The smaller region is the **minor segment** and the larger region is the **major segment**. The minor segment lies between the minor arc ACB and chord AB. The major segment lies between the major arc ADB and the chord AB.

Congruence of Circles

Circles having equal radii are congruent. Two arcs of a circle (or of congruent circles), which subtend equal angles at the centre (or at the corresponding centres) are equal in length. Two chords of the same or different circles are said to be congruent, if they have the same length.

Arcs–Chords

If two arcs of a circle are congruent, then their corresponding chords are congruent. Conversely, if two chords of a circle are congruent, then their corresponding arcs are congruent (see Fig. 12.87).

1. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then chord $AB \cong$ chord CD.
2. If chord $AB \cong$ chord CD, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

Note This theorem holds good for two congruent circles also.

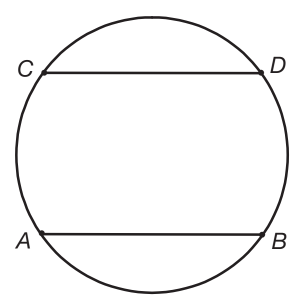


Figure 12.87

If two circles C_1 and C_2 are congruent, and PQ is a chord in C_1 and RS is a chord in C_2, such that $\overline{PQ} \cong \overline{RS}$, then $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$.

Theorem 1

The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at any point on the remaining part of the circle.

Given: In Fig. 12.88, $\angle AOB$ is subtended by the $\overset{\frown}{AB}$ at the centre O and $\angle ACB$ is subtended by the same arc AB at a point C on the remaining part of the circle.

The centre of the circle O may lie in the interior, the exterior or on $\angle ACB$.

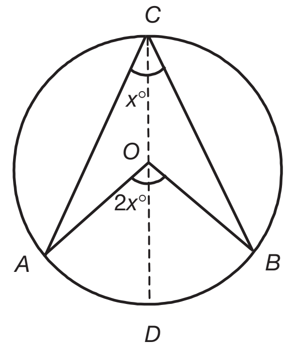


Figure 12.88

Case 1: Centre O lies in the interior of $\angle ACB$.

Case 2: Centre O lies on $\angle ACB$.

Case 3: Centre O lies in the exterior of $\angle ACB$.

Construction: Join C and O and produce CO to D.

Proof

Case 1: Centre O lies in the interior of $\angle ACB$.

In triangle OCA,

$\angle OCA = \angle OAC$ (Angles opposite to equal sides OC, OA, the radii of the circle.)

$\angle AOD = \angle OCA + \angle OAC$ (Exterior angle is equal to the sum of the interior opposite angles)

$= 2\angle OCA$.

Similarly, $\angle DOB = 2\angle OCB$

$\therefore\ \angle AOB = 2\angle OCA + 2\angle OCB$

$\Rightarrow \angle AOB = 2\angle ACB$.

Hence, proved.

Similarly, it can be proved in the other two cases mentioned above.

EXAMPLE 12.30

In Fig. 12.89, O is the centre of the circle. Find the angles of ΔABC.

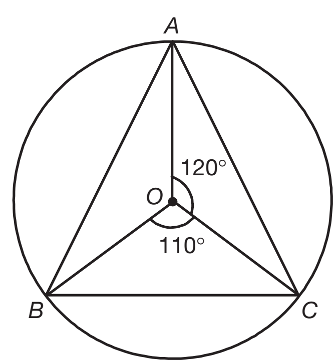


Figure 12.89

SOLUTION

We know that the sum of angles at a point is 360°.

$\therefore\ \angle AOB + \angle BOC + \angle COA = 360°$

$\angle AOB = 360° - (\angle BOC + \angle COA) = 360° - (110 + 120)°$.

$\therefore\ \angle AOB = 130°$.

An angle subtended by an arc at the centre of the circle is double the angle subtended by the same arc at any point on the remaining circle.

$\therefore \angle C = \frac{1}{2}\angle AOB = \frac{1}{2}(130°) = 65°$,

$\angle A = \frac{1}{2}\angle BOC = \frac{1}{2}(110°) = 155°$ and $\angle B = \frac{1}{2}\angle AOC = \frac{1}{2}(120°) = 60°$.

$\therefore$ The angles of the triangle ABC are $\angle A = 55°$, $\angle B = 60°$ and $\angle C = 65°$.

EXAMPLE 12.31

In Fig. 12.90, O is the centre of the circle. Find the value of x.

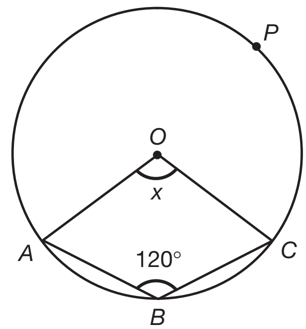


Figure 12.90

SOLUTION

In the given figure, reflex $\angle AOC$ is the angle subtended by $\overset{\frown}{APC}$.
$\angle ABC$ is the angle subtended by the $\overset{\frown}{APC}$ at point B on the remaining part of the circle.

$\therefore$ Reflex $\angle AOC = 2\angle ABC = 2(120°) = 240°$.

$\therefore$ $x = 360° - 240° = 120°$.

Theorem 2

Angles in the same segment of a circle are equal.

Given: O is the centre of the circle. A, B, C and D are the points on the circle as shown in Fig. 12.91.

To prove: $\angle ACB = \angle ADB$.

Proof: $\angle ACB = \frac{1}{2}\angle AOB$ (Angle subtended by an arc at the centre is double the angle subtended at any point on the remaining part of the circle.)

Also, $\angle ADB = \frac{1}{2}\angle AOB$ (Using the above rule).

$\therefore$ $\angle ACB = \angle ADB$.

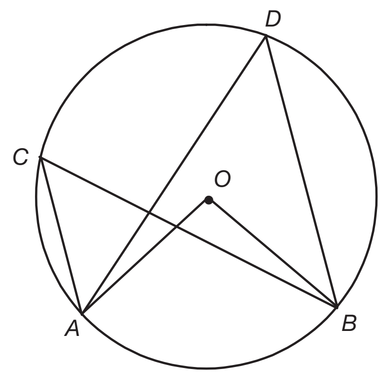


Figure 12.91

Theorem 3

An angle in a semi-circle is a right angle.

Given: AB is the diameter of the circle (see Fig. 12.92).

To prove: $\angle ACB = 90°$

Proof: $\overset{\frown}{ADB}$ is an arc, making an angle 180° at the centre O. (AOB is a straight line).

$$\therefore \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}(180°).$$

$\therefore$ $\angle ACB = 90°$.

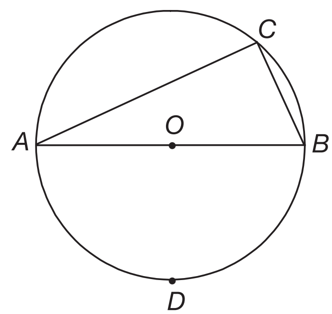


Figure 12.92

Corollary If an arc of a circle subtends a right angle at any point on the remaining part of the circle, it is a semi-circle.

Cyclic Quadrilateral

If all the four vertices of a quadrilateral lie on one circle, then the quadrilateral is called a cyclic quadrilateral.

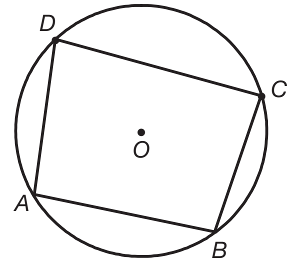


Figure 12.93

Notes

1. Opposite angles in a cyclic quadrilateral are **supplementary**.
2. In a quadrilateral, if the opposite angles are supplementary, then the quadrilateral is a cyclic quadrilateral.

   In Fig. 12.93, $ABCD$ is a cyclic quadrilateral.

   $$\angle A + \angle C = 180° \text{ and } \angle B + \angle D = 180°$$

3. Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

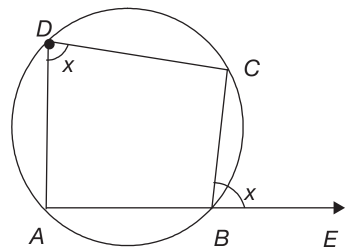


Figure 12.94

In Fig. 12.94, $ABCD$ is a cyclic quadrilateral.

AB is produced to E to form an exterior angle, $\angle CBE$ and it is equal to the interior angle at the opposite vertex, i.e., $\angle ADC$.

$\therefore \angle CBE = \angle ADC$

EXAMPLE 12.32

In Fig. 12.95, find the value of x.

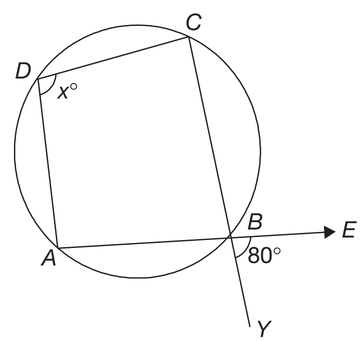


Figure 12.95

SOLUTION

In the given figure, $\angle CBE$ is an exterior angle which is equal to the opposite interior angle at the opposite vertex, $\angle ADC$.

$\therefore \angle CBE = \angle ADC$ (1)

$\angle CBE + \angle EBY = 180°$ ($\because$ linear pair)

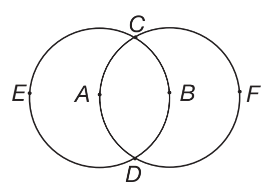
$\therefore \angle CBE = 180° - 80° = 100°$

$x° = \angle ADC = \angle CBE = 100°$.

EXAMPLE 12.33

A and B are the centres of the circles as shown in the given figure. The circles intersect at C and D. Find $\angle CED + \angle CFD$.

(a) 90° **(b)** 135° **(c)** 120° **(d)** 150°



HINTS

(i) Join AB, which is the radius of both the circles.

(ii) $AC = AD = BC = BD =$ radius.

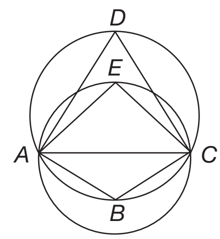
(iii) ΔABC and ΔABD are equilateral triangles. $\angle CAD = \angle BAD = 120°$.

(iv) $\angle CED = \angle CFD = \frac{1}{2}\angle CAD = \frac{1}{2}\angle CBD.$

(Angle at the centre is equal to half the angle at the circumference).

EXAMPLE 12.34

In the given figure, AC is the diameter of the circle on which the point E lies. A, B, C and D are concyclic. If $\angle ADC = 55°$, find the sum of $\angle DAE$ and $\angle DCE$.



(a) 35° **(b)** 55° **(c)** 45° **(d)** 65°

SOLUTION

$ABCD$ is a cyclic quadrilateral

$\Rightarrow \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$ (sum of the opposite angles of acyclic quadrilateral)

$\angle D = 55°$ (given)

$\Rightarrow \angle B = 180° - 55° = 125°$.

In quadrilateral $ABCE$,

$\angle A + \angle B + \angle C + \angle E = 360°$

$\therefore\ \angle A + \angle C + 125° + 90° = 360°$

($\therefore$ AC is the diameter)

$\therefore\ \angle A + \angle C = 360° - 215° = 145°$.

$\angle DAE + \angle DCE = \angle BAD + \angle BCD - (\angle BAE + \angle BCE)$

$= 180° - (145°) = 35°$.

Constructions Related to Circles

Construction 1: To construct a segment of a circle, on a given chord and containing a given angle.

EXAMPLE 12.35

Construct a segment of a circle with a chord of length 8.5 cm and containing an angle of 55° (θ).

SOLUTION

Step 1: Draw a line segment BC of the given length, 8.5 cm.

Step 2: Draw $\overrightarrow{BX}$ and $\overrightarrow{CY}$, such that $\angle CBX = BCY = \dfrac{180 - 2\theta}{2} = 35°$.

Step 3: Mark the intersection of $\overrightarrow{BX}$ and $\overrightarrow{CY}$ as O.

Step 4: Taking O as the centre and OB or OC as radius, draw $\overparen{BAC}$.

Step 5: In ΔBOC, $\angle BOC = 110°$

$$\Rightarrow \angle BAC = \left(\frac{1}{2}\right)\angle BOC = \left(\frac{1}{2}\right)(110°) = 55°.$$

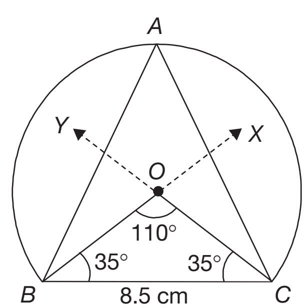


Figure 12.96

$\therefore\ \angle BAC = 55°$.

The region bounded by $\overparen{BAC}$ and $\overline{BC}$ is the required segment.

Construction 2: Construct an equilateral triangle inscribed in a circle of radius 3.5 cm.

Step 1: Draw a csircle of radius 3.5 cm and mark its centre as O.

Step 2: Draw radii $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$, such that $\angle AOB = \angle BOC = 120°$.

Step 3: Join AB, BC and CA, which is the required equilateral ΔABC in the given circle.

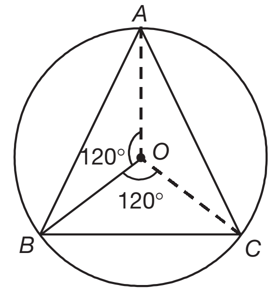


Figure 12.97

Construction 3: Construct an equilateral triangle circumscribing a circle of radius 3 cm.

Step 1: Draw a circle of radius 3 cm with centre O.

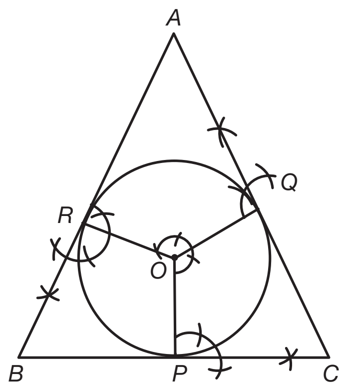


Figure 12.98

Step 2: Draw radii $\overline{OP}, \overline{OQ}$ and $\overline{OR}$, such that $\angle POR = \angle ROQ = 120°$.

Step 3: At P, Q and R draw perpendiculars to $\overline{OP}, \overline{OQ}$ and $\overline{OR}$ respectively to form ΔABC.

ΔABC is the required circumscribing equilateral triangle.

Construction 4: Construct a square inscribed in a circle of radius 3 cm.

Step 1: Draw a circle of radius 3 cm and mark the centre as O.

Step 2: Draw diameters $\overline{AC}$ and $\overline{BD}$, such that $\overline{AC} \perp \overline{BD}$.

Step 3: Join AB, BC, CD and DA. Quadrilateral $ABCD$ is the required square inscribed in the given circle.

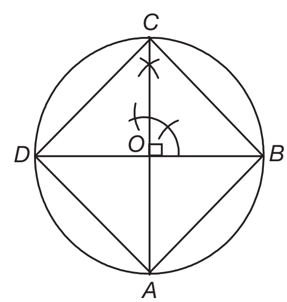


Figure 12.99

Construction 5: Construct a square circumscribing a circle of radius 2.5 cm.

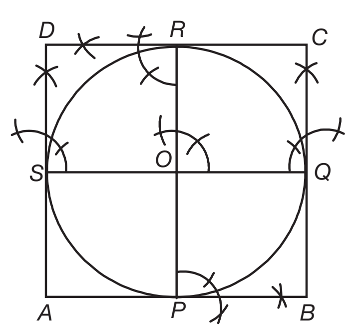


Figure 12.100

Step 1: Draw a circle of radius 2.5 cm.

Step 2: Draw two mutually perpendicular diameters $\overline{PR}$ and $\overline{SQ}$.

Step 3: At P, Q, R and S, draw lines perpendicular to OP, OQ, OR and OS to form square $ABCD$ as shown in Fig. 12.100.

Construction 6: Inscribe a regular hexagon in a circle of radius 3 cm.

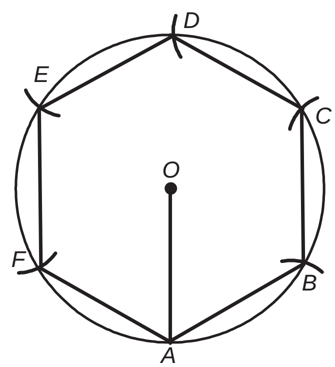


Figure 12.101

Step 1: Draw a circle of radius 3 cm taking the centre as O.
Step 2: Draw radius $\overline{OA}$. With radius equal to OA and starting with A as the centre, mark points B, C, D, E and F one after the other.
Step 3: Join A, B, C, D, E and F. Polygon $ABCDEF$ is the required hexagon.

Construction 7: Construct a regular hexagon circumscribing a circle of radius 3.5 cm.

Step 1: Draw a circle of radius 3.5 cm and mark its centre as O.

Step 2: Draw radii $\overline{OP}$, $\overline{OQ}$, $\overline{OR}$, $\overline{OS}$, $\overline{OT}$ and $\overline{OU}$, such that the angle between any two adjacent radii is 60°.

Step 3: Draw lines at P, Q, R, S, T and U perpendicular to $\overline{OP}$, $\overline{OQ}$, $\overline{OR}$, $\overline{OS}$, $\overline{OT}$ and $\overline{OU}$ respectively to form the required hexagon $ABCDEF$.

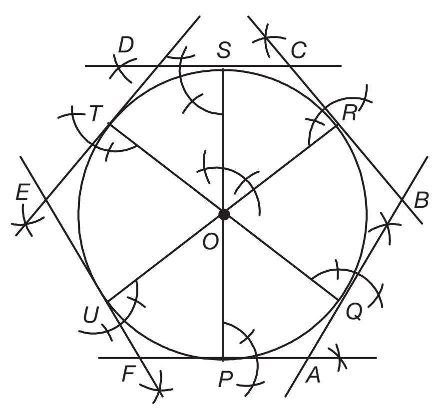


Figure 12.102

Construction 8: Draw the circum-circle of a given triangle.

Step 1: Draw ΔABC with the given measurements.

Step 2: Draw perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$ to intersect each other at S.

Step 3: Taking S as the centre and radius equal to AS or BS or CS, draw a circle. The circle passes through all the vertices A, B and C of the triangle.

$\therefore$ The circle drawn is the required circum-circle.

Note The circum-centre is equidistant from the vertices of the triangle.

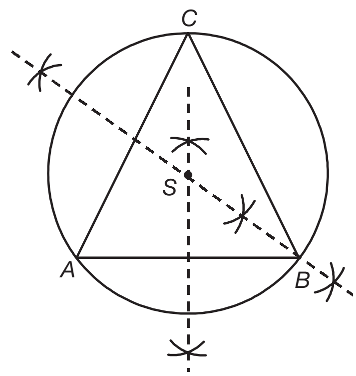


Figure 12.103

Construction 9: Construct the in-circle of a given triangle ABC.

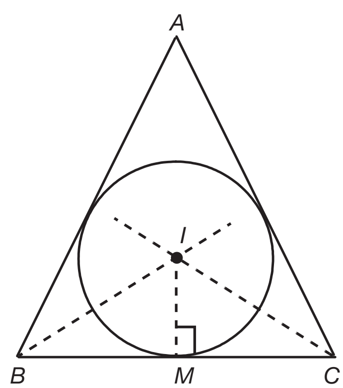


Figure 12.104

Step 1: Draw a ΔABC with the given measurements.

Step 2: Draw bisectors of $\angle B$ and $\angle C$ to intersect at I.

Step 3: Draw perpendicular $\overline{IM}$ from I onto $\overline{BC}$.

Step 4: Taking I as centre and IM as the radius, draw a circle.

This circle touches all the sides of the triangle. This is the in-circle of the triangle.

Note The in-centre is equidistant from all the sides of the triangle.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Can a triangle be formed by line segments of lengths a, b and c, such that $a > b - c$?

2. Can a triangle be formed by line segments of lengths a, b and c, such that $a = b - c$?

3. The areas of parallelograms on the same base and between the same parallel lines are ________.

4. In a regular polygon, are all the exterior angles equal?

5. Can the sum of the two angles of a triangle be less than the third angle?

6. If all the sides of a polygon are equal, then all its interior angles must be equal. Is the given statement true?

7. If a circle passes through four points, then the four points are said to be __________.

8. Two circles cannot intersect in more than two points. [True/False]

9. Two quadrilaterals of equal perimeters occupy equal areas. Is this statement always true?

10. Can a polygon have the sum of all its interior angles equal to 810°?

11. The exterior angle of a regular polygon is 60°. The number of sides of the polygon is _______.

12. A line l intersects a pair of parallel lines. The exterior angles on the same side of line l are in the ratio 5 : 4. The measure of the bigger angle of the two is __________.

13. When all the sides of a quadrilateral are equal, then it is either a _________ or a ________.

14. In a quadrilateral, $ABCD$, $\angle DAB + \angle BCD = 180°$, then the quadrilateral $ABCD$ is _________.

15. If four lines intersect in a plane, at the maximum how many triangles are formed?

16. If all the angles of a quadrilateral are equal, then all its sides must be equal. Is the above statement true?

17. If lines l_1, l_2 and l_3 pass through a point P, then they are called __________ lines.

18. In an isosceles right triangle ABC, $\angle B = 90°$ and $\overline{BD} \perp \overline{AC}$. Then BD = ______. $\left(\frac{AB}{2} / \frac{BC}{2} / \frac{AC}{2}\right)$.

19. The sum of all the altitudes in a triangle is ________ the sum of all the sides. (equal to/less than/greater than)

20. An angle is $\frac{2}{3}$ times its supplementary angle. What is the angle?

21. Are all the diagonals of a regular polygon always concurrent?

22. Two lines AB and CD intersect at point O. $\angle AOD : \angle BOD = 3 : 1$, then $\angle AOD$ = __________.

23. Are the diagonals of a regular polygon equal in length?

24. In a triangle ABC, if $\angle B > \angle C$, then $AB > AC$. Is the given statement true?

25. Can the length of the median in a triangle to a side be less than the corresponding altitude?

26. If d is the distance from a point P to the centre of the circle of radius r and $d - r > 0$, then the point P lies __________ the circle. (outside/inside/on)

27. If the longest chord of a circle is of length 14 cm, then the circumference of the circle is ________ cm.

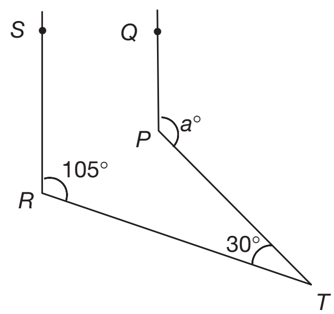
28. In a triangle ABC, D is a point on BC, such that AD is the shortest distance from A to BC and $\angle ADC = 70°$. Is it possible?

29. Can the exterior angle of a polygon be three times its interior angle?

30. If AB and CD are two chords of a circle and $AB > CD$, then the chord which is nearer to the centre is __________.

Short Answer Type Questions

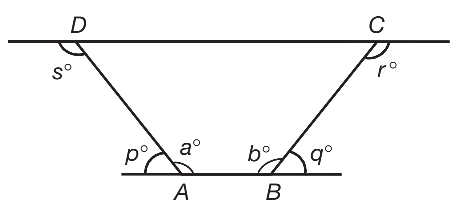
31. In the following figure, $PQ \parallel RS$. If $\angle TRS = 105°$, $\angle PTR = 35°$, $\angle QPT = a°$, find the value of a.



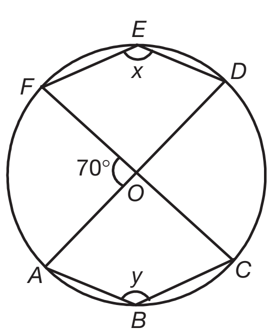
32. In triangle ABC, $\angle A < \angle B$, $\angle B > \angle C$ and $\angle A = 2\angle C$, prove that $AC > BC > AB$.

33. If two complementary angles are in the ratio 7 : 11, find the supplement of the bigger angle.

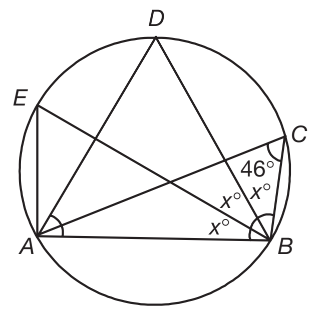
34. In the given quadrilateral $ABCD$, $p° + q° = 100°$, $a° = 140°$ and $r° = \frac{1}{2}(a° + q°)$. Find the angles $p°$, $q°$, $r°$ and $s°$.



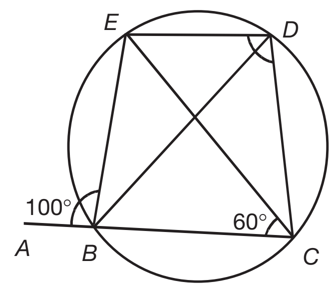
35. In the following figure, O is the centre of the circle. Find the value of $x + y$



36. In the following figure, BE is the diameter of the circle. Find the value of 'x' and $\angle DAB$.

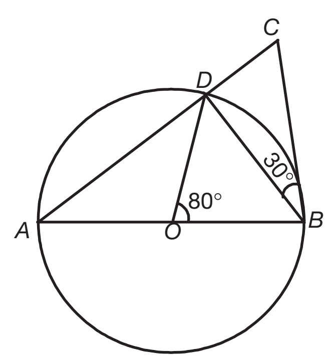


37. In the following figure, $\overline{CB}$ is produced to the point A. Find $\angle BDC$.

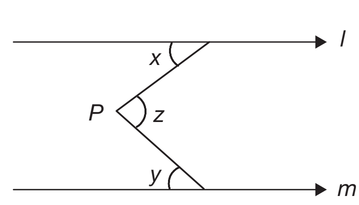


38. If the sum of the interior angles of a polygon is 2340°, then find the number of sides of the polygon.

39. In the following figure, if O is the centre of the circle and $\overline{AB}$ is the diameter, then find $\angle ACB$.

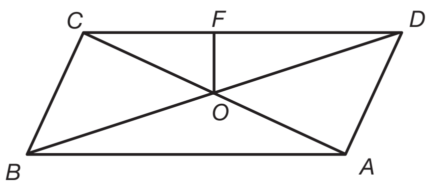


40.

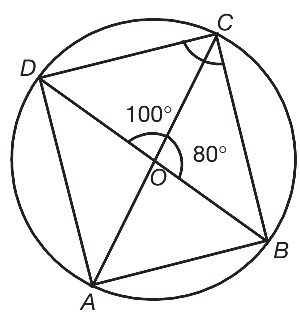


In the above figure, find the value of z, if x is two–third of y which is a complement of 45°.

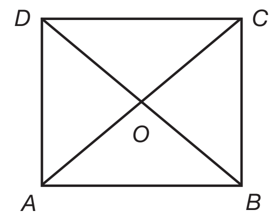
41. Find the length of the sides of the given parallelogram, if the perimeter of the parallelogram is 24 cm, the length of perpendicular $OF = 3$ cm, $OB = 5$ cm and $OC = \sqrt{18}$ cm.



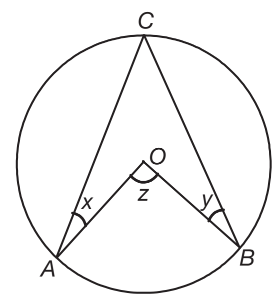
42. In the figure given below, $OD = OC = OB$. Find $\angle DCB$.



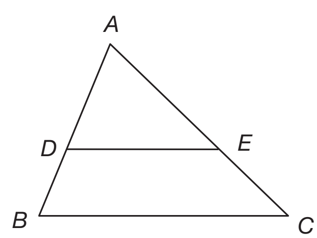
43. If the sum of the lengths of two diagonals of the given square $ABCD$ is 24 cm, then find the side of the square and the magnitude of $AO + OB$.



44. In the following figure, O is the centre of the circle. If $x = 40°$ and $x : y = 4 : 3$, find the value of z.

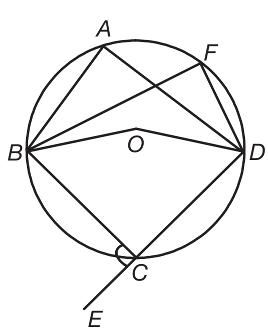


45. In the following figure, $AD = 5.6$ cm, $AE = (x + 1)$ cm, $AB = 8.4$ cm and $EC = (x - 1)$ cm, find AC. Given that $DE \parallel BC$.



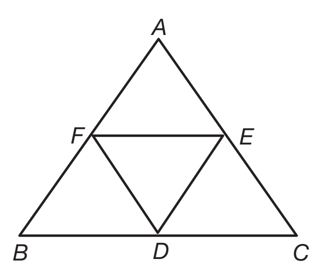
Essay Type Questions

46. $ABCD$ and $BFDC$ are cyclic quadrilaterals. $\overline{CD}$ is produced to E. If $\angle BCE = 45°$, then find $\angle BFD$.

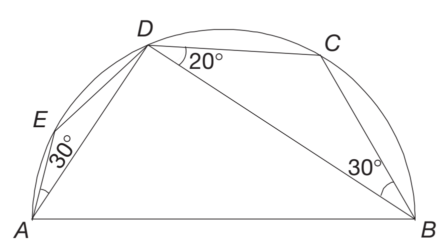


47. In the following triangle ABC, D, E and F are the mid-points of sides BC, CA and AB respectively.

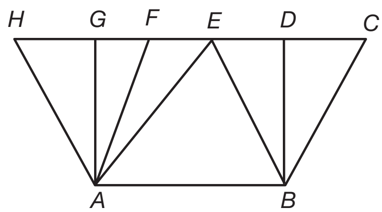
Prove that $\dfrac{AB - BC}{2} < AE < \dfrac{AB + BC}{2}$.



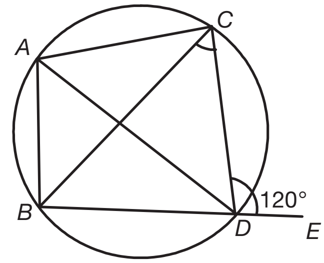
48. Find $\angle DBA$, $\angle DAB$ and $\angle AED$ in the following figure, where $ABCDE$ is a semi-circle.



49. In the following figure, $ABDG$ is a rectangle with AB = 10 cm and AG = 6 cm. Find the areas of parallelograms $ABCF$ and $ABEH$. Also find the area of ΔAEB.



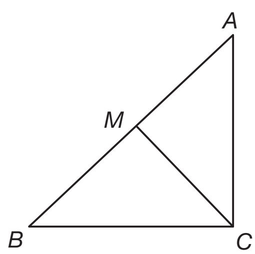
50. In the following figure, $\overline{BD}$ is produced to E. If $\overline{AD}$ is the angle bisector of $\angle A$, then find $\angle BCD$.



CONCEPT APPLICATION

Level 1

1. In the following figure, ΔABC is right-angled at C, and M is the mid-point of hypotenuse AB. If AC = 32 cm and BC = 60 cm, then find the length of $\overline{CM}$.

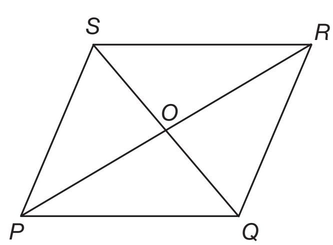


(a) 32 cm
(b) 30 cm
(c) 17 cm
(d) 34 cm

2. A cyclic polygon has n sides such that each of its interior angle measures 144°. What is the measure of the angle subtended by each of its side at the geometrical centre of the polygon?

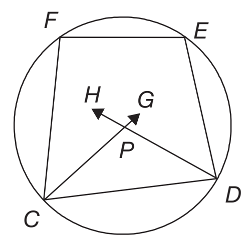
(a) 144° (b) 30°
(c) 36° (d) 54°

3. In the following, $PQRS$ is a rhombus, SQ and PR are the diagonals of the rhombus intersecting at O. If angle OPQ = 35°, then find the value of angle ORS + angle OQP.



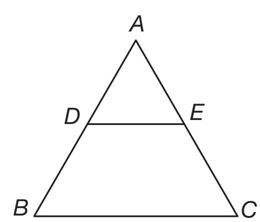
(a) 90° (b) 180°
(c) 135° (d) 45°

4. In the following, $CDEF$ is a cyclic quadrilateral. $\overline{CG}$ and $\overline{DH}$ are the angle bisectors of $\angle C$ and $\angle D$ respectively. If $\angle E$ = 100° and $\angle F$ = 110°, then find $\angle CPD$.



(a) 105° (b) 80°
(c) 150° (d) 90°

5. In the following figure, ABC is an equilateral triangle. DE is parallel to BC and equal to half the length of BC. If $AD + EC + CB$ = 24 cm, then what is the perimeter of triangle ADE?

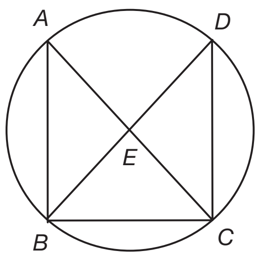


(a) 12 cm
(b) 16 cm
(c) 18 cm
(d) Cannot be determined

6. In ΔPQR, M and N are points on PQ and PR, respectively, such that $\overline{MN} \parallel \overline{QR}$. If $PM = x$, $PR = x + 9$, $PQ = x + 13$ and $PN = x - 2$, then find x.

(a) 10 (b) 11
(c) 13 (d) 15

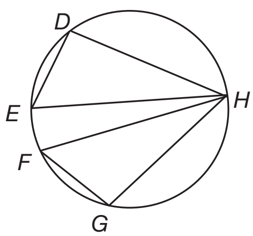
7. In the following figure (not to scale), the chords AC and BD intersect at E and $\angle BAE = \angle ECD + 20°$. If $\angle CDE = 60°$, find $\angle ABE$.



(a) 40° (b) 60°

(c) 80° (d) None of these

8. In the following figure, $\overline{DE}$ and $\overline{FG}$ are equal chords of the circle subtending $\angle DHE$ and $\angle FHG$ at the point H on the circle. If $\angle DHE = 23\frac{1}{2}°$, then find $\angle FHG$.



(a) $27\frac{1}{2}°$ (b) 30°

(c) $23\frac{1}{2}°$ (d) 60°

9. The bisectors of two adjacent angles in a parallelogram meet at a point P inside the parallelogram. The angle made by these bisectors at a point is ________.

(a) 180° (b) 90°

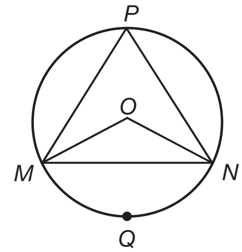
(c) 45° (d) None of these

10. If $x°$ is the measure of an angle which is equal to its complement and $y°$ is the measure of an angle which is equal to its supplement, then $\frac{x°}{y°}$ is______.

(a) 1 (b) 3

(c) 0.5 (d) 2

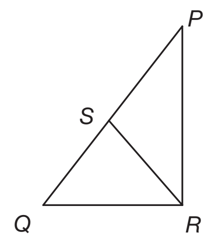
11. In the following figure, O is the centre of the circle. If $\angle MPN = 55°$, then find the value of $\angle MON + \angle OMN + \frac{1}{2}\angle MNO$.



(a) 145° (b) $162\frac{1}{2}°$

(c) $158\frac{1}{2}°$ (d) 180°

12. In the following figure, ΔPQR is right-angled at R and S is the mid-point of hypotenuse PQ. If $RS = 25$ cm and $PR = 48$ cm, then find QR.



(a) 7 cm (b) 25 cm

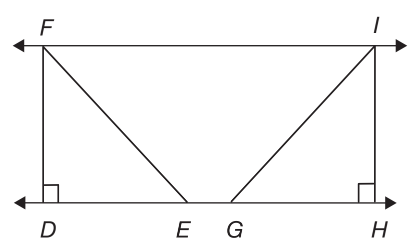
(c) 14 cm (d) Cannot be determined

13. In a cyclic quadrilateral $PQRS$, $PS = PQ$, $RS = RQ$ and $\angle PSQ = 2\angle QSR$. Find $\angle QSR$.

(a) 20° (b) 30°

(c) 40° (d) 50°

14. In the following figure, two isosceles right triangles, DEF and HGI are on the same base $\overline{DH}$ and $\overline{DH}$ is parallel to $\overline{FI}$. If $DE = GH = 9$ cm and $DH = 20$ cm, then the area of the quadrilateral $FEGI$ is _______.



(a) 99 cm^2 (b) 40.5 cm^2

(c) 81 cm^2 (d) 180 cm^2

15. A pole of height 14 m casts a 10 m long shadow on the ground. At the same time, a tower casts a 70 m long shadow on the ground. Find the height of the tower.

(a) 50 m (b) 78 m

(c) 90 m (d) 98 m

16. The angle subtended by a minor arc in its alternate segment is ________.

(a) acute (b) obtuse

(c) $90°$ (d) reflex angle

17. The number of diagonals of a regular polygon is 27. Then, each of the interior angles of the polygon is __________.

(a) $\left(\dfrac{500}{3}\right)^{\circ}$ (b) $140°$

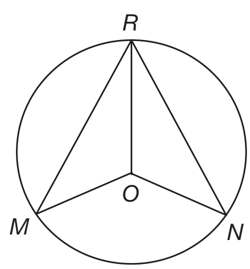
(c) $128°$ (d) $154°$

18. ABC is a triangle inscribed in a circle, AC being the diameter of the circle. The length of AC is as much more than the length of BC as the length of BC is more than the length of AB. Find $AC : AB$.

(a) 5 : 3 (b) 5 : 4

(c) 6 : 5 (d) 3 : 2

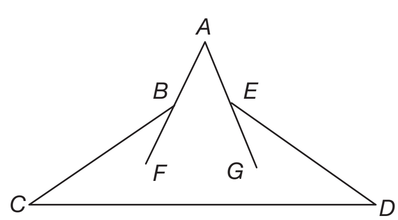
19. $\overset{\frown}{MN}$ is the arc of the circle with centre O. If $\angle MOR = 100°$ and $\angle NOR = 135°$, then $\dfrac{1}{2}\angle ORN + \dfrac{1}{4}\angle ORM$ is ________.



(a) $22\frac{1}{2}°$ (b) $40°$

(c) $125°$ (d) $21\frac{1}{4}°$

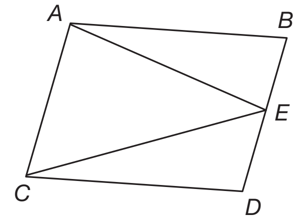
20. In the following figure (not to scale), $\angle BCD = 40°$, $\angle EDC = 35°$, $\angle CBF = 30°$ and $\angle DEG = 40°$. Find $\angle BAE$.



(a) $70°$ (b) $50°$

(c) $110°$ (d) $35°$

21. In the following figure (not to scale), $\overline{AB} \parallel \overline{CD}$. If $\angle BAE = 25°$ and $\angle DCE = 30°$, then find $\angle AEC$.



(a) $30°$ (b) $45°$

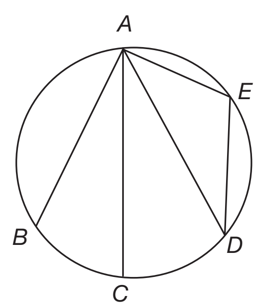
(c) $50°$ (d) $55°$

22. A tower of height 60 m casts a 40 m long shadow on the ground. At the same time, a needle of height 12 cm casts a x cm long shadow the ground. Find x.

(a) 6 (b) 8

(c) 10 (d) 14

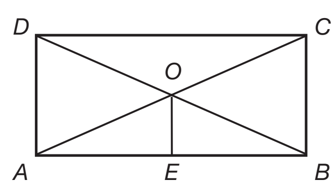
23. In the given figure, AC is the diameter. AB and AD are equal chords. If $\angle AED = 110°$, then find $\angle BAD$.



(a) $40°$ (b) $55°$

(c) $110°$ (d) $120°$

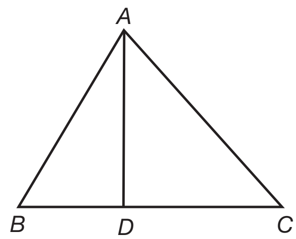
24. In the given rectangle $ABCD$, the sum of the lengths of two diagonals is equal to 52 cm and E is a point in AB, such that $\overline{OE}$ is perpendicular to $\overline{AB}$. Find the lengths of the sides of the rectangle, if $OE = 5$ cm.



(a) 24 cm, 10 cm (b) 12 cm, 10 cm

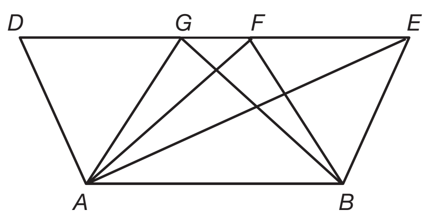
(c) 24 cm, 5 cm (d) 12 cm, 15 cm

25. In the following figure (not to scale), AD bisects $\angle BAC$. If $\angle BAD = 45°$ and ΔABC is inscribed in a circle, then which of the following is the longest?



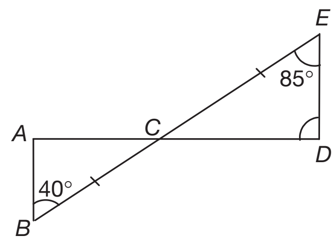
(a) AB (b) AD
(c) AC (d) BC

26. In the given figure, $\overline{AB} \parallel \overline{DE}$ and area of the parallelogram $ABFD$ is 24 cm². Find the areas of ΔAFB, ΔAGB and ΔAEB.



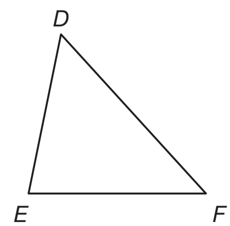
(a) 8 cm²
(b) 12 cm²
(c) 10 cm²
(d) 14 cm²

27. In the given figure, $\overline{AD}$ and $\overline{BE}$ intersect at C, such that $BC = CE$, $\angle ABC = 40°$ and $\angle DEC = 85°$. Find $\angle BAC - \angle CDE$.



(a) 45° (b) 125°
(c) 55° (d) 110°

28. In the given figure, DEF is a triangle. If DF is the longest side and EF is the shortest side, then which of the following is true?



(a) $\angle E > \angle D > \angle F$
(b) $\angle D < \angle F < \angle E$
(c) $\angle D < \angle E < \angle F$
(d) None of these

29. The ratio between the exterior angle and the interior angle of a regular polygon is 1 : 3. Find the number of the sides of the polygon.

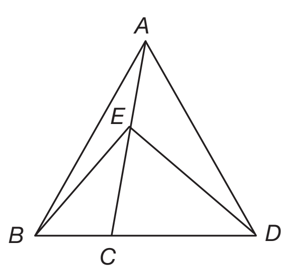
(a) 12 (b) 6
(c) 8 (d) 10

30. Find each interior and exterior angle of a regular polygon having 30 sides.

(a) 144°, 36° (b) 156°, 24°
(c) 164°, 16° (d) 168°, 12°

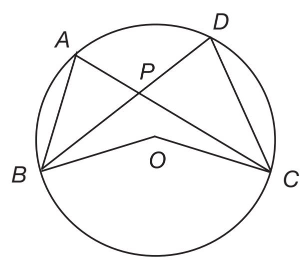
Level 2

31. If $BC : CD = 2 : 3$, $AE : EC = 3 : 4$ and $BC : AE = 2 : 3$, then find the ratio of the area of ΔECD to the area of ΔAEB.



(a) 2 : 1 (b) 2 : 3
(c) 3 : 5 (d) 4 : 3

32. In the given figure (not to scale), O is the centre of the circle. If $PB = PC$, $\angle PBO = 25°$ and $\angle BOC = 130°$, then find $\angle ABP + \angle DCP$.

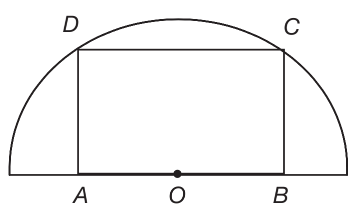


(a) 10°
(b) 30°
(c) 40°
(d) 50°

33. In a polygon, the greatest angle is 110° and all the angles are distinct in integral measures (in degrees) Find the maximum number of sides it can have.

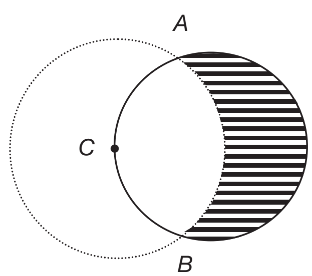
(a) 4 (b) 5
(c) 6 (d) 7

34. In the given figure, $ABCD$ is a rectangle inscribed in a semi-circle. If the length and the breadth of the rectangle are in the ratio 2 : 1. What is the ratio of the perimeter of the rectangle to the diameter of the semicircle?



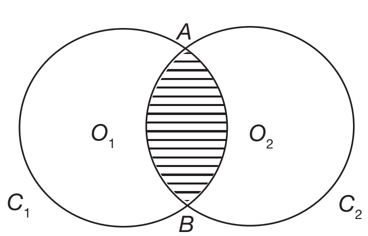
(a) $3:\sqrt{2}$ (b) $2:\sqrt{3}$
(c) $2:\sqrt{5}$ (d) $3:\sqrt{5}$

35. In the given figure, $\overline{AB}$ is the diameter of the circle with area π sq. units. Another circle is drawn with C as centre, which is on the given circle and passing through A and B. Find the area of the shaded region.



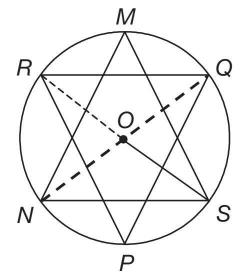
(a) $\frac{\pi}{3}$ sq. units (b) $\frac{2\pi}{3}$ sq. units
(c) 1 sq. units (d) 1.2 sq. units

36. In the following figure (not to scale), C_1 and C_2 are two congruent circles with centres O_1 and O_2. respectively. Each circle passes through the centre of the other circle. If the circumference of each circle is 2 cm, the perimeter of the shaded region is ________ cm.



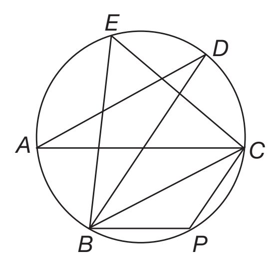
(a) $\frac{4}{3}$ (b) 1
(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

37. In the given figure (not to scale), the points M, R, N, S and Q are concyclic. Find $\angle PQR + \angle OPR + \angle NMS + \angle OSN$, if O is the centre of the circle.



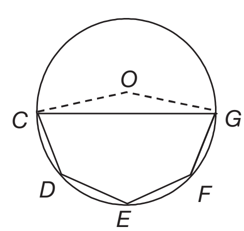
(a) 90° (b) 180°
(c) 270° (d) Data inadequate

38. In the given figure (not to scale), AC is the diameter of the circle and $\angle ADB = 20°$, then find $\angle BPC$.



(a) 50° (b) 70°
(c) 90° (d) 110°

39. In the following figure, O is the centre of the circle and $CD = DE = EF = GF$. If $\angle COD = 40°$, then find reflex $\angle COG$.



(a) 200° (b) 90°
(c) 80° (d) 160°

40. In the given figure (not to scale), AC is the median as well as altitude to BD. In ΔACE, AD is the median to CE. Which of the following is true?

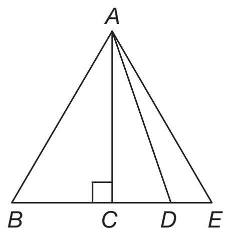
PRACTICE QUESTIONS

(a) $AB + CD > AE$

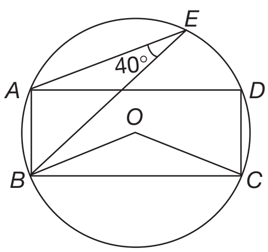
(b) $AB + BC = AE$

(c) $AB + DE < AE$

(d) None of the above



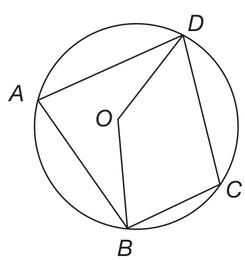
41. In the given figure, (not to scale), rectangle $ABCD$ and triangle ABE are inscribed in the circle with centre O. If $\angle AEB = 40°$, then find $\angle BOC$.



(a) 60° (b) 80°

(c) 100° (d) 120°

42. In the given figure (not to scale), O is the centre of the circle. BC and CD are equal chords. If $\angle OBC = 55°$, then find $\angle BAC$.



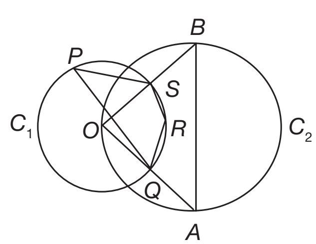
(a) 60°

(b) 70°

(c) 80°

(d) 90°

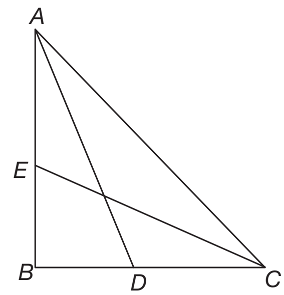
43. In the given figure (not to scale), O is the centre of the circle C_1 and AB is the diameter of the circle C_2. Quadrilateral $PQRS$ is inscribed in the circle with centre O. Find $\angle QRS$.



(a) 105° (b) 115°

(c) 135° (d) 145°

44. In the given figure (not to scale), E and D are the mid-points of AB and BC respectively. Also, $\angle B = 90°$, $AD = \sqrt{292}$ cm and $CM = \sqrt{208}$ cm. Find AC.



(a) 15 (b) 18

(c) 20 (d) 24

45. In ΔABC, P is the mid-point of BC and Q is the mid-point of AP. Find the ratio of the area of ΔABQ and the area of ΔABC. The following are the steps involved in solving the above problem.

(A) We know that a median of a triangle divides a triangle into two triangles of equal area.

(B) $\Rightarrow \text{Ar}(\Delta ABP) = \frac{1}{2}[\text{Ar}(\Delta ABC)]$

(C) $\text{Ar}(\Delta ABQ) = \frac{1}{2}[\text{Ar}(\Delta ABP)] = \frac{1}{4}[\text{Ar}\Delta ABC]$

(D) $\Rightarrow \text{Ar}(\Delta ABQ) : \text{Ar}(\Delta ABC) = 1 : 4$

(a) ACBD (b) ADBC

(c) ABCD (d) ADCB

46. $ABCD$ is a cyclic quadrilateral, ABC is a minor arc and O is the centre of the circle. If $\angle AOC = 160°$, then find $\angle ABC$.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

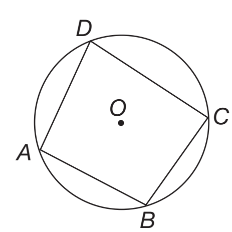
(A) We have, $\angle ABC + \angle ADC = 180°$

(B) $\angle ABC + \frac{1}{2}\angle AOC = 180°$ $\left(\because \angle ADC = \frac{1}{2}\angle AOC\right)$

(C) $\angle ABC = 180° - 80°$

(D) $\angle ABC + \frac{160°}{2} = 180°$

(E) $\therefore \angle ABC = 100°$



(a) ABDEC

(b) ABDCE

(c) BCDAE

(d) BACDE

47. Show that each diagonal of a parallelogram divide it into two congruent triangles.

The following are the steps involved in showing the above result. Arrange them in sequential order.

(A) In ΔABC and ΔCDA, $AB = DC$ and $BC = AD$ ($\because$ opposite angles of parallelogram) $AC = AC$ (common side).

(B) Let $ABCD$ be a parallelogram. Join AC.

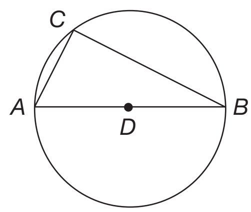
(C) By SSS congruence property, $\Delta ABC \cong \Delta CDA$.

(D) Similarly, BD divides the triangle into two congruent triangles.

(a) BACD (b) BDAC

(c) BADC (d) BDCA

48. Show that any angle in a semi-circle is a right angle.



The following are the steps involved in showing the above result. Arrange them in sequential order.

(A) $\therefore \angle ACB = \dfrac{180°}{2} = 90°$

(B) The angle subtended by an arc at the centre is double of the angle subtended by the same arc at any point on the remaining part of the circle.

(C) Let AB be a diameter of a circle with centre D and C be any point on the circle. Join AC and BC.

(D) $\therefore \angle ADB = 2 \times \angle ACB$

$180° = 2\angle ACB$ ($\because \angle ADB = 180°$)

(a) DBAC (b) DBCA

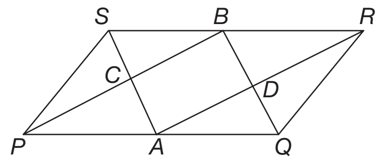
(c) CBAD (d) CBDA

49. A, B, C and D are concyclic. AC bisects BD. If $AB = 9$ cm, $BC = 8$ cm, and $CD = 6$ cm, then find the measure of AD.

(a) 7 cm (b) 10 cm

(c) 12 cm (d) 15 cm

50. In the given figure, $PQRS$ is a parallelogram. A and B are the mid-points of $\overline{PQ}$ and $\overline{SR}$ respectively. If $PS = BR$, then the quadrilateral $ADBC$ is a ______.



(a) rhombus (b) trapezium

(c) square (d) rectangle

51. The sides of a triangle are 2006 cm, 6002 cm and m cm, where m is a positive integer. Find the number of such possible triangles.

(a) 1 (b) 2006

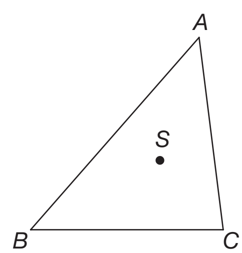
(c) 3996 (d) 4011

52. If a, b and c are the lengths of the sides of a right triangle ABC with $c = 2a$ and $b^2 - 3a^2 = 0$, then $\angle ABC =$ ______.

(a) 60° (b) 30°

(c) 45° (d) 90°

53. In ΔABC, $AC = BC$, S is the circum-centre and $\angle ASB = 150°$. Find $\angle CAB$.

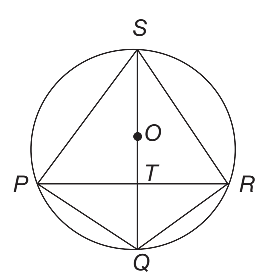


(a) $55\frac{1}{2}°$ (b) $52\frac{1}{2}°$

(c) $62\frac{1}{2}°$ (d) $35\frac{1}{2}°$

54. In the given figure, P, Q, R and S are concyclic points, and O is the mid-point of the diameter QS.

If $\angle QPR = 25°$, then find $\angle SOR$.



(a) 130° (b) 120°

(c) 75° (d) 100°

PRACTICE QUESTIONS

Level 3

55. In ΔABC, $\angle B = 90°$. P, Q and R are the mid-points of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Then which of the following is true?

(a) A, P, Q and R are concyclic points

(b) B, P, R and Q are concyclic points

(c) C, Q, P and R are concyclic points

(d) All of these

56. If p, q and r are the lengths of the sides of a right triangle, PQR, and the hypotenuse $r = \sqrt{2pq}$, then $\angle QPR =$ __________.

(a) 50° (b) 45°

(c) 60° (d) 30°

57. In a triangle PQR, $\overline{PQ} = QR$. A and B are the mid-points of $\overline{QR}$ and $\overline{PR}$ respectively. A circle passes through P, Q, A and B. Then which of the following is necessarily true?

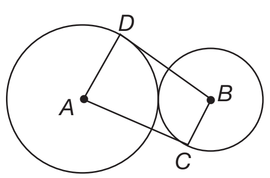
(a) ΔPQR is equilateral

(b) ΔPQR is right isosceles

(c) PQ is a diameter

(d) Both (a) and (c)

58. In the figure given below (not to scale), D is a point on the circle with centre A and C is a point on the circle with centre B. $\overline{AD} \perp \overline{BD}$ and $\overline{BC} \perp \overline{CA}$. Then which of the following is true?



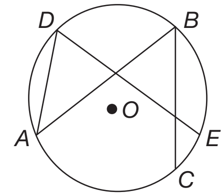
(a) $BD = AC$, when $AD = BC$

(b) $BD = AC$, when $\overline{AD} \,||\, \overline{BC}$

(c) Both (a) and (b)

(d) $BD = AC$ is always true

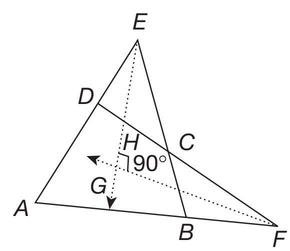
59. In the given figure, the angles $\angle ADE$ and $\angle ABC$ differ by 15°. Find $\angle CAE$.



(a) 10° (b) $7\frac{1}{2}°$

(c) 15° (d) 30°

60. In the given figure, $ABCD$ is a cyclic quadrilateral, $\angle ABC = 70°$, $\overrightarrow{FG}$ bisects $\angle CFA$, $\overrightarrow{EG}$ bisects $\angle DEB$, $\angle DCE = 60°$ and $\angle EGF = 90°$. Find $\angle HEC$.



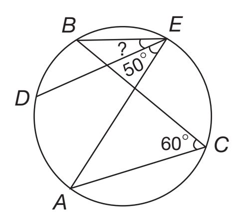
(a) 20°

(b) 40°

(c) 25°

(d) 45°

61. In the given figure, A, D, B, E and C are concyclic. If $\angle ACB = 60°$ and $\angle AED = 50°$, then find $\angle DEB$.



(a) 15°

(b) 10°

(c) 20°

(d) 5°

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Yes
2. No
3. equal
4. Yes
5. Yes
6. No
7. concyclic
8. True
9. No
10. No
11. 6
12. 100°
13. square, rhombus
14. cyclic
15. Four
16. No
17. concurrent
18. $\frac{AC}{2}$
19. less than
20. 72°
21. No
22. 135°
23. Need not be
24. No
25. No
26. outside
27. 44
28. No
29. Yes
30. AB

Shot Answer Type Questions

31. $a = 135$
33. 125°
34. $p = 40$, $q = 60$, $r = 100$, $s = 160$
35. 250
36. $\angle DAB = 46°$, $x = 44°$
37. 40°
38. 15
39. 60°
40. 75°
41. $BC = 5$ cm $= AD$ and $AB = 7$ cm $= CD$
42. 90°
43. $AO + OB = 12$ cm, and side of the square $= 6\sqrt{2}$ cm.
44. $z = 140°$
45. 6 cm

Essay Type Questions

46. $\angle BFD = 45°$
48. $\angle ABD = 40°$
   $\angle DAB = 50°$
   $\angle AED = 140°$
49. Area of parallelogram $ABCF = 60$ cm^2
   Area of $\Delta AEB = 30$ cm^2.
50. $\angle BCD = 60°$

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)
21. (d)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (d)

Level 2

31. (a)	**32.** (b)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (a)
41. (c)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (d)	**49.** (c)	**50.** (d)
51. (d)	**52.** (a)	**53.** (b)	**54.** (a)						

Level 3

55. (b)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (c)	**60.** (c)	**61.** (b)

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Find AB using Pythagoras's theorem and $CM = \frac{1}{2}$ AB.
2. Find the number of sides of the polygon, angle subtended at the centre $= \frac{360°}{n}$.
3. In a rhombus, diagonals bisect at 90°.
4. In a cyclic quadrilateral, opposite angles are supplementary.
5. In ΔABC, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$.
6. (i) Use BPT.

   (ii) By BPT, $\frac{PM}{PQ} = \frac{PN}{PR}$.
7. $\angle CDE = \angle BAE$ ($\because$ angles in the same segment).
8. Equal chords subtend equal angles on the circumference of the circle.
9. The angle bisectors of two adjacent angles in a parallelogram form a right angle at their point of intersection.
10. Recall complement and supplement angles.
11. Angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.
12. $RS = \frac{1}{2} PQ$.
13. In a cyclic quadrilateral, opposite angles are supplementary.
14. Area of trapezium, $FEGI = \frac{1}{2}(FI + EG) \times FD$.
15. Use the concept of similar triangles.
16. Angle subtended by a minor arc is always less than 90°.
17. Number of diagonals of a polygon with n sides $= \frac{n(n-3)}{2}$.
18. $AC^2 = AB^2 + BC^2$. Consider AB as x, BC as $x + k$ and AC as $x + 2k$.
19. ΔMOR and ΔNOR are isosceles triangles.
20. (i) Produce AF and AG to meet CD.

    (ii) Exterior angle of a triangle is equal to the sum of its interior opposite angles.

    (iii) Sum of the angles in a triangle is always 180°.
21. Draw a parallel line to AB through E.
22. Use the concept of similar triangles.
23. Join BD, $ABDE$ is a cyclic quadrilateral.
24. Find the length of the diagonal.
25. $\angle BAC = 90°$ and BC is a diameter.
26. Areas of the parallelograms lying on the same base and between same parallel lines are equal.
27. Use the angle sum property of a triangle.
28. Apply the concept of inequality property of a triangle.
29. Sum of the exterior and interior angles of a polygon = 180°.
30. Find one exterior angle.

Level 2

31. (i) As $BC : CD = 2 : 3$, Area (ΔABC) : Area(ΔACD) $= 2 : 3$.

    Let area of $\Delta ABC = 2x$ and $(\Delta ACD) = 3x$.

    (ii) As $AE : EC = 3 : 4$, Area $(\Delta AEB) = \frac{3}{7}(2x)$ and Area $(\Delta ECD) = \frac{4}{7}(3x)$.
32. (i) As $PB = PC$, $\angle PBO = \angle PCO$.

    (ii) As $\angle BOC = 130°$, $\angle BAC = \angle BDC = 65°$.
33. (i) As the greatest interior angle is 110°, the least exterior angle is (180° – 110°).

    (ii) Sum of all the exterior angles = 360°.

    (iii) All exterior angles are distinct.

34. (i) Join OC and find the relation between OB and BC.

(ii) $OB = BC$ and $OC : OB = \sqrt{2} : 1$.

35. (i) Find the diameter of the smaller circle.

(ii) Join AB, AC and BC.

(iii) $\angle ACB = 90°$ and $AC = BC$.

36. (i) Join O_1, A, O_2 and B and find the angles of rhombus O_1AO_2B.

(ii) Join O_1A, O_2A, O_1B and O_2B.

(iii) $O_1A = O_1B = O_1O_2$.

(iv) $\angle O_1AO_2 = 60°$.

37. (i) $\angle PQR = \frac{1}{2}(\angle POR)$; $\angle NMS = \frac{1}{2}(\angle NOS)$.

(ii) If $\angle OPR = x$, then $\angle ROP = 180° - 2x$.

38. (i) Join DC and use properties of cyclic quadrilateral

(ii) Use $\angle ADC = 90°$ and find $\angle BDC$.

(iii) $BDCP$ is cyclic.

39. (i) Join OD, OE and OF.

(ii) Equal chords subtend equal angles at the centre.

40. (i) In a triangle median is same as altitude drawn on same side, then the remaining two sides are equal.

(ii) As AC is median as well as altitude to BD, $AB = AD$.

(iii) In ΔADE, $AD + DE > AE$.

(iv) As AD is median, $CD = DE$.

41. (i) Join OA and use properties of arcs.

(ii) $\angle AOB = 2\angle AEB$.

(iii) AOC is the diameter, i.e., $\angle AOB + \angle BOC = 180°$.

42. (i) Join OC, and apply the concept of equal chords.

(ii) $\angle OBC = \angle OCB$.

(iii) As $BC = CD$, $\angle BOC = \angle COD$.

(iv) $\angle BAC = \frac{1}{2}\angle BOC$.

43. (i) As AB is the diameter, $\angle AOB = 90°$.

(ii) $\angle QPS = \frac{1}{2}(\angle QOS)$.

(iii) $PQRS$ is a cyclic quadrilateral.

44. (i) Use Pythagoras's theorem.

(ii) Let $AE = BE = x$ and $BD = CD = y$.

(iii) Use Pythagoras's theorem in the right triangles ABD and BEC and obtain the equations in terms of x and y.

(iv) Add the above equations and obtain AC, i.e., $4x^2 + 4y^2$.

45. ABCD is the required sequential order.

46. ABDCE is the required sequential order.

47. BACD is the required sequential order.

48. CBDA is the required sequential order.

49. Let 'O' be the intersection point of AC and BD

By AA similarity,

$\Delta AOB \sim \Delta DOC$

$$\frac{AB}{CD} = \frac{OB}{OC}$$

By AA similarity,

$\Delta AOD \sim \Delta BOC$

$$\frac{AD}{BC} = \frac{OD}{OC}$$

$$\frac{AD}{BC} = \frac{OB}{OC}$$

$(AB)(BC) = (CD)(AD)$

$AD = 12$ cm.

50. Since B is the mid-point of $\overline{SR}$ and $PS = BR$, $PS = BS = PA = AB$.

$\therefore$ $PABS$ is a rhombus, and similarly, $AQRB$ is a rhombus.

$\therefore \angle BCA = \angle ADB = 90°$.

As PB and QB are bisectors of $\angle P$ and $\angle Q$, $\angle PBQ = 90°$.

$\therefore CADB$ is a rectangle.

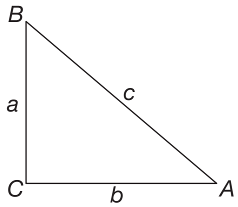
51. From the inequality of triangles, we have:

$6002 + 2006 > m$ and

$6002 - 2006 < m$

$\Rightarrow 3996 < m < 8008$

Since m is an integer, it can take $(8008 - 3996 - 1)$ or 4011 values.

52. 

Given that, $c = 2a$ and $b^2 - 3a^2 = 0. \Rightarrow b = \sqrt{3}a.$

$a : b : c = a : \sqrt{3}a : 2a = 1 : \sqrt{3} : 2.$

$\therefore \angle A = 30°, \angle B = 60°$ and $\angle C = 90°.$

So, $\angle ABC = 60°.$

53. Since, $SA = SB = SC$, S is the circum-centre

$\angle ASB = 150°$ (given) $\Rightarrow \angle ACB = 75°.$

Since, $AC = BC, \angle CAB = \angle CBA = \left(\dfrac{180° - 75°}{2}\right)$

$= 52\dfrac{1}{2}°.$

54. Given, $\angle QPR = 25°.$

Since, QS is the diameter, $\angle QPS = 90°.$

$\Rightarrow \angle SPR = 90 - 25° = 65°$

$\therefore \angle SOR = 2 \times 65° = 130°.$

Level 3

55. (i) Draw the figure and mark the points P, Q and R on sides AB, BC and AC respectively.

(ii) Angle $B = 90°$ and $\angle PRQ$ is formed by lines PR and RQ, which are parallel to lines BC and AB (mid-point theorem).

(iii) Since BC and AB are perpendicular, PR and RQ are also perpendicular, i.e., $\angle PRQ = 90°.$

(iv) Quadrilateral joining $BPRQ$ is a cyclic quadrilateral.

56. (i) Draw the figure and mark the points on it.

(ii) Use Pythagoras's theorem.

(iii) We will get $p^2 + q^2 = 2pq$ which gives $p = q$.

57. (i) Draw the figure and mark the points on it.

(ii) Join BQ, now BQ is the median to PR. Since $PQ = QR$, BQ is also the altitude to PR.

(iii) PBQ is a right triangle, right angled at B. Thus, PQ is the diameter.

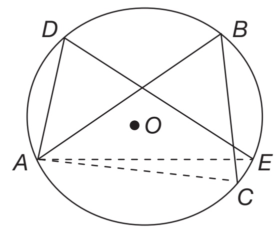
(iv) Joining PA, we get another right triangle PAQ which is right angled at A. PBQ and PAQ are congruent angles.

(v) $PR = QR$, so ΔPQR is an equilateral triangle.

58. (i) Join AB. Now using Pythagoras's theorem, $AB^2 = AD^2 + DB^2$ and $AB^2 = AC^2 + CB^2$.

(ii) If $BD = AC$, then $AD = BC$.

(iii) If $\overline{AD} \,||\, \overline{BD}$, then $\overline{AC} \,||\, BD$, so $ABCD$ is a rectangle.

59. 

From the given figure,

$AE > AC$

$\Rightarrow \angle ADE > \angle ABC$

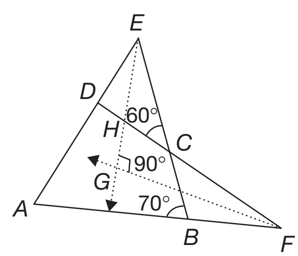
$\Rightarrow (\angle ADE - \angle ABC) = 15°$

$\Rightarrow \dfrac{\angle AOE}{2} - \dfrac{\angle AOC}{2} = 15°$

$\Rightarrow \angle AOE - \angle AOC = 30°$

$\Rightarrow \angle COE = 30°.$

$\therefore \angle COE = \dfrac{1}{2}\angle COE = 15°.$

60. 

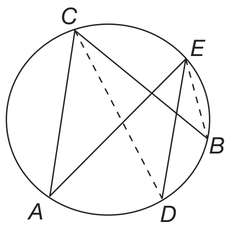
$\angle DAB = \angle DCE = 60°$ and $\angle EDC = \angle CBA = 70°$. (Exterior angle of a cyclic quadrilateral.)

In ΔABE, $\angle DEC = 180 - (60° + 70°) = 50°.$

Since, $\overline{FG}$ bisects $\angle F$

$\therefore \angle HEC = \frac{\angle DEC}{2} = \frac{50°}{2} = 25°.$

61.



Join CD and EB

$\angle ACD = \angle AED = 50°$ (angles in the same segment and $\angle AED = 50°$)

$\angle ACB = 60°$ (given)

$\Rightarrow \angle DCB = \angle ACB - \angle ACD = 60° - 50° = 10°.$

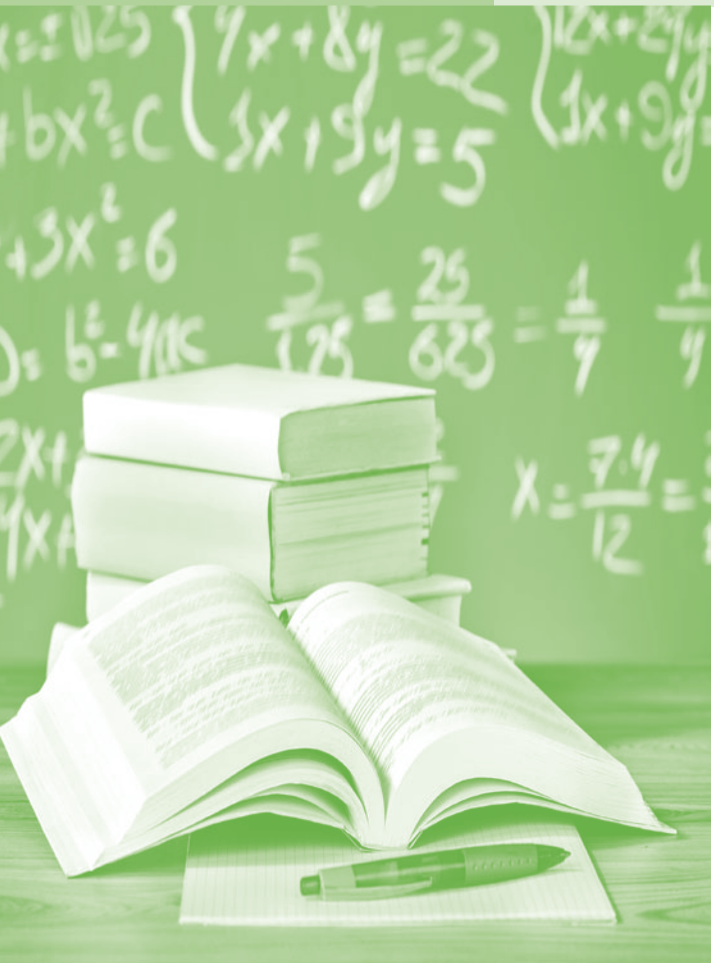
$\angle DEB = \angle DCB = 10°$. (angles in the same segment)

HINTS AND EXPLANATION

Chapter

13

Mensuration



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the concepts of plane figures
- Calculate the areas of plane figures
- Obtain volume of simple solid figures

KEY IDEAS

After completing this chapter, you should be able to:

- Know about polygons and calculate the area of different types of polygons
- Learn about circles, circumference of a circle, sector of a circle and area of a ring
- Understand different solids and find their volumes
- Find lateral surface areas, total surface areas and volumes of prism, cube, cuboid, cylinder, pyramid, cone, torus, sphere and polyhedrons (tetrahedron and octahedron)

INTRODUCTION

Mensuration is a branch of mathematics. It deals with computation of geometric magnitudes, such as the length of a line segment, area of a surface and volume of a solid. In this chapter, we will deal with computation of areas and perimeters of plane figures and surface areas and volume of solid figures. To be precise, we will learn to find the perimeters and areas of plane figures including triangles, quadrilaterals, etc. Further, we will study prisms, cubes, cuboids and cylinder. Then we will learn how to compute area and volume of pyramids, cones, cone frustums, tetrahedrons and octahedrons. Finally, we will discuss spheres, hemispheres, hollow-spheres, hollow-hemispheres and torus.

PLANE FIGURES

A figure lying in a plane is called a **plane (or planar) figure**. Triangles, rectangles, circles are some examples of plane figures. A plane figure is a closed figure if it has no free end. It is called a simple closed figure if it does not cross itself.

A plane figure is made up of line segments or curves or both. A plane figure is called a **rectilinear figure**, if it is made up of only line segments. Triangles, squares, pentagons, etc. are some examples of rectilinear figures. A circle is not a rectilinear figure. The part of the plane which is enclosed by a simple closed figure is called a **plane region**. The magnitude of a plane region is called its **area**.

A line segment has one dimension, i.e., length. Hence, its size is measured in terms of its length.

A planar region has two dimensions, i.e., length and breadth. Hence, its size is measured in terms of its area.

The perimeter of a closed planar figure is the total length of the lines bounding the figure.

In Fig. 13.1, ABC is a triangle and $PQRS$ is a quadrilateral.

The perimeter of the ΔABC is $AB + BC + CA$.

The perimeter of the quadrilateral $PQRS$ is $PQ + QR + RS + SP$.

The perimeter of a rectangle of length l and breadth b is $2(l + b)$.

The perimeter of a square of side s is $4s$.

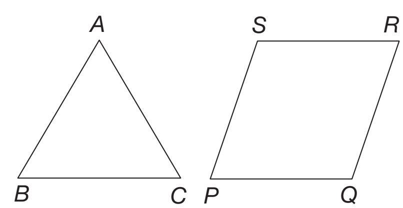


Figure 13.1

Units of Measurement

The length of a line is measured in units, such as centimetres, inches, metres, feet. Area is measured in units, such as square centimetres, square inches, square metres, square feet.

ESTIMATING AREAS

The area of a plane figure can be estimated by finding the number of unit squares that can fit into a whole figure.

Observe the square $ABCD$ (Fig. 13.2), in the square $ABCD$, there are 16 unit squares. Therefore, its area is 16 sq. units.

In Fig. 13.3, *PQR* is a right triangle. All the small units are not squares. Just as the length of a line need not be a whole number of units, the area of a plane figure also need not be a whole number of units.

In the figure, the triangle *PQR* does not cover 8 unit squares. But, its area is equal to that of 8 unit squares. This is because, the partial units, when added, are equal to 2 unit squares. Besides this, there are 6 unit squares. Hence, its area is equal to 8 unit squares.

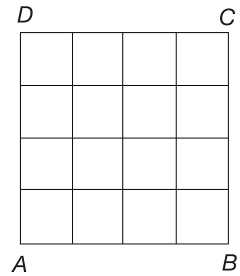


Figure 13.2

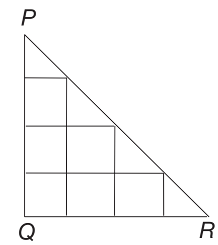


Figure 13.3

Area of a Rectangle

A rectangle is a four-sided closed figure in which both pairs of opposite sides are equal and each angle measures 90°. In Fig. 13.4, *ABCD* is a rectangle.

In rectangle *ABCD*, $AB = 5$ cm and $BC = 4$ cm. Each of the small squares measures 1 cm by 1 cm and the area occupied by each small square is 1 cm^2. There are 4 rows, each row consist of 5 unit squares.

$\therefore$ The area occupied by the rectangle = $(4)(5) = 20$ cm^2.

The number of unit squares in the rectangle *ABCD* is equal to the product of the number of unit squares along the length and the number of unit squares along the breadth. Thus, we observe that the area of a rectangle = (length)(breadth).

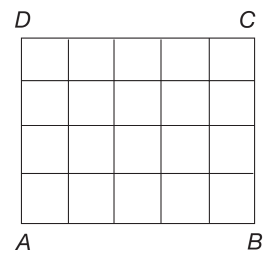


Figure 13.4

Area of a Parallelogram

Consider parallelogram *ABCD* as provided in Fig. 13.5:

Draw a perpendicular $\overline{DE}$ from *D* to $\overline{AB}$.

Draw a perpendicular $\overline{CF}$ from *C* to $\overline{AX}$ (*AB* extended).

By the ASA congruency property, the area of ΔAED is equal to the area of ΔBFC.

$\therefore$ The area of parallelogram *ABCD* = The area of rectangle *DEFC*.

$\Rightarrow (EF)(FC) = (AB)(DE)$

$\therefore$ Area of parallelogram = (Base)(Corresponding height)

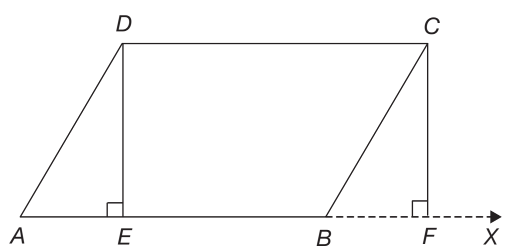


Figure 13.5

Area of a Triangle

Consider ΔABC as provided in Fig. 13.6:

Draw a line *AD* parallel to and equal to *BC*.

Join *CD*.

ABCD is a parallelogram.

By ASA axiom, the area of triangle *ABC* is equal to the area of triangle *ADC*.

$\therefore$ The area of triangle $ABC = \left(\frac{1}{2}\right)$ (Area of a parallelogram)

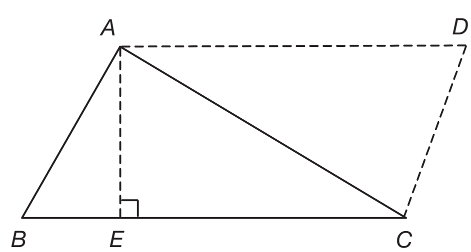


Figure 13.6

(Where AE is the height of the triangle or parallelogram.)

$\therefore$ The area of a triangle = $\left(\frac{1}{2}\right)$ (Base)(Height)

The area of a triangle is also given by $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b and c are the lengths of the sides of the triangle and s is the semi-perimeter, i.e., $s = \frac{(a+b+c)}{2}$. This relation is known as Heron's formula.

Area of a Trapezium

Consider trapezium $ABCD$ (Fig. 13.7), in which AD and BC are the parallel sides and AB and DC are the oblique sides (not parallel).

Join BD.

DE is the perpendicular distance between AD and BC.

The area of the trapezium is equal to the sum of the areas of the ΔABD and ΔBCD.

$$= \left(\frac{1}{2}\right)(AD)(DE) + \left(\frac{1}{2}\right)(BC)(DE)$$

$$= \left(\frac{1}{2}\right)(AD + BC)(DE)$$

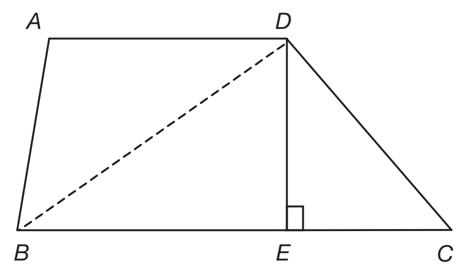


Figure 13.7

$\therefore$ Area of a trapezium $= \left(\frac{1}{2}\right)$(Sum of the parallel sides)(Distance between them).

Area of a Rhombus

A rhombus is a parallelogram in which all sides are equal. Consider rhombus $PQRS$ (Fig. 13.8), in which PR and SQ are the diagonals of the rhombus.

In a rhombus, the diagonals bisect each other at right angles at the point O.

$\therefore$ $\angle POS = 90°$ and $\angle POQ = 90°$.

Area of the rhombus $PQRS$ = Area of ΔPRS + Area of ΔPQR

$$= \left(\frac{1}{2}\right)(PR)(OS) + \left(\frac{1}{2}\right)(PR)(OQ)$$

$$= \left(\frac{1}{2}\right)(PR)(OS + OQ) = \left(\frac{1}{2}\right)(PR)(SQ)$$

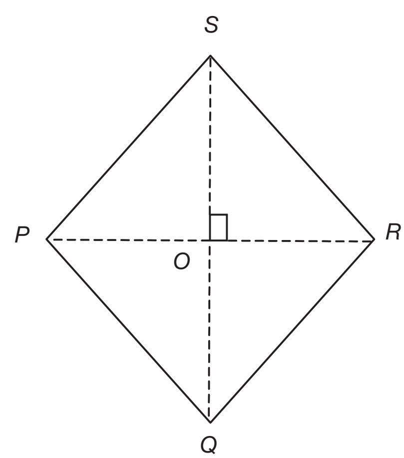


Figure 13.8

$\therefore$ Area of rhombus $= \left(\frac{1}{2}\right)$ (product of the diagonals).

Area of a Square

A square is a rectangle in which all the sides are equal.

Area of a rectangle = (Length)(Breadth)

In a square, Length = Breadth

$\therefore$ Area of a square = (Side)(Side) = $(\text{Side})^2$

Perimeter of a square = 4(Side)

Diagonal of a square $= \sqrt{2}$ (Side)

$$\therefore \text{Area of a square} = \frac{(\text{Diagonal})^2}{2}.$$

POLYGON

A polygon is a closed rectilinear figure that has three or more sides.

The following is a list of different types of polygons:

Name	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Septagon	7
Octogon	8
Nonagon	9
Decagon	10

In a polygon, if all the sides are equal, and all the angles are equal, then it is a regular polygon.

Area of a Polygon

In Fig. 13.9, *ABCDE* is a polygon.

This polygon may be divided into triangles by joining points *B* and *E*, *E* and *C*. Then the areas of the triangles can be found. The sum of the areas of these triangles is the area of the polygon.

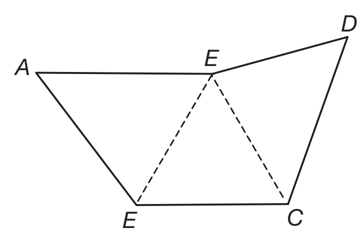


Figure 13.9

Area of a Regular Polygon

The regular polygon of n sides may be divided into n triangles of equal areas.

Consider the regular octagon in Fig. 13.10. The octagon can be divided into 8 equal triangles. Let OP be the perpendicular distance from centre O to side AH. OP is the height of the triangle OHA.

$$\text{Area of triangle } OHA = \left(\frac{1}{2}\right)(AH)(OP)$$

Since the octagon consists of 8 equal triangles, the area of the octagon $= (8)\left(\frac{1}{2}\right)(AH)(OP)$.

But, the perimeter of the octagon = 8(*AH*).

$\therefore$ The area of a regular polygon $= \left(\frac{1}{2}\right)$(Perimeter of the polygon) (Perpendicular distance from the centre to any side.)

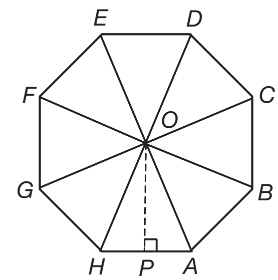


Figure 13.10

Area of an Equilateral Triangle

In Fig. 13.11, *ABC* is an equilateral triangle.

Let $AB = BC = CA = a$

Draw perpendicular $\overline{AD}$ from *A* to $\overline{BC}$.
AD is the height of the triangle.

ADC is a right triangle, where $DC = \frac{a}{2}$ and $AC = a$.

$$AD = \sqrt{(a)^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2}a.$$

$\therefore$ The height of the triangle $\frac{\sqrt{3}}{2}a$.

The area of the triangle $= \frac{1}{2}(a)\left(\frac{\sqrt{3}}{2}a\right) = \left(\frac{\sqrt{3}}{4}a^2\right)$.

$\therefore$ The area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$ sq. units.

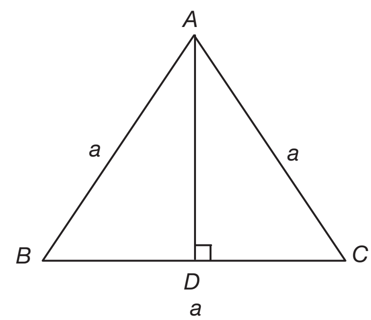


Figure 13.11

Area of an Isosceles Triangle

An isosceles triangle has two equal sides. Generally, the base is considered to be the unequal side. Consider ΔABC (Fig. 13.12), in which $AB = AC$.

Let $AB = AC = a$ and $BC = b$.

Draw perpendicular *AD* from *A* to *D* on *BC*. In an isosceles triangle, *AD* is the altitude as well as the median.

As *D* is the mid-point of *BC*, $DC = \frac{b}{2}$.

ADC is a right triangle,

$$AD = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{\sqrt{4a^2 - b^2}}{2}.$$

The area of triangle $ABC = \left(\frac{1}{2}\right)(BC)(AD)$

$$= \left(\frac{1}{2}\right)(b)\left(\frac{\sqrt{4a^2 - b^2}}{2}\right) = \frac{b}{4}\sqrt{4a^2 - b^2} \text{ sq. units.}$$

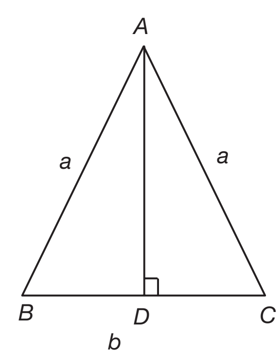


Figure 13.12

Area of Right Triangle

In Fig. 13.13, ABC is a right triangle and right angled at B.

Since AB is perpendicular to the base BC of the triangle, AB is the altitude of the triangle.

$\therefore$ The area of right $\Delta ABC = \left(\frac{1}{2}\right)(b)(a) = \frac{ab}{2}$ sq. units.

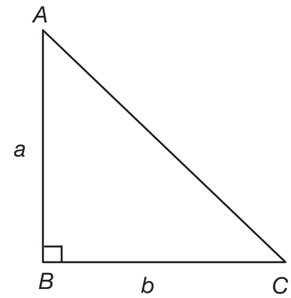


Figure 13.13

Isosceles Right Triangle

A right triangle in which the perpendicular sides are equal is an isosceles right triangle.

In Fig. 13.14, ABC is an isosceles right triangle, where $AB = BC = a$ units.

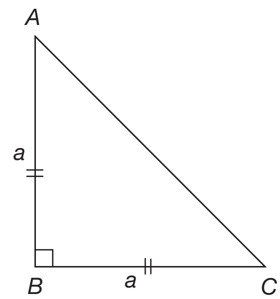


Figure 13.14

The area of the triangle $ABC = \frac{1}{2}(a)(a) = \frac{a^2}{2}$ sq. units.

From Pythagorean theorem, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow \quad a^2 + a^2 = AC^2 \quad \Rightarrow \quad AC = \sqrt{2}a$ units.

$\therefore AB : BC : CA = a : a : \sqrt{2}a = 1 : 1 : \sqrt{2}$.

The hypotenuse is the longest side of a right triangle. The hypotenuse is $\sqrt{2}$ times of each of the equal sides.

Notes

1. In-radius (r) of a triangle is given by the formula, $r = \frac{\Delta}{s}$.
2. Circum-radius (R) of a triangle is given by the formula, $R = \frac{abc}{4\Delta}$.

Where Δ is area of the triangle with sides a, b and c, and s is the semi-perimeter.

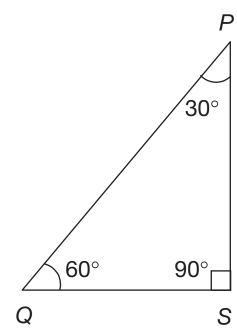


Figure 13.15

Special Right Triangle with Angles 30°, 60° and 90° In the right triangle (Fig. 13.15), $\angle PSQ = 90°$.

We have to determine the ratio of the sides PQ, QS and PS.

Construction: Produce $\overline{QS}$ to the point R, such that $SR = SQ$ (see Fig. 13.16).

$\therefore \Delta PSQ \cong \Delta PSR$ (SAS Congruence property)

Let $PQ = QR = PR = a$. Since PS is also the median to QR, $QS = \frac{a}{2}$.

In an equilateral triangle, altitude is $\frac{\sqrt{3}}{2}$ times its side.

$\therefore PS = \frac{\sqrt{3}}{2}a.$

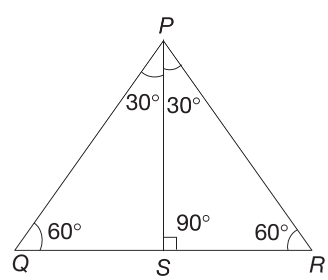


Figure 13.16

In triangle PSQ,

$$SQ : SP : PQ = \frac{a}{2} : \frac{\sqrt{3}}{2}a : a$$

$= 1 : \sqrt{3} : 2.$

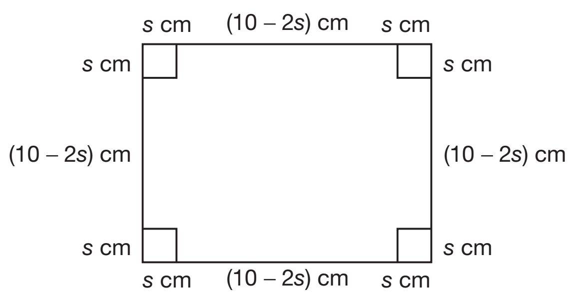
$\therefore$ In a right triangle with angles 30°, 60° and 90°, the ratio of the sides opposite to these angles is $1 : \sqrt{3} : 2$.

EXAMPLE 13.1

From each corner of a square sheet of side 10 cm, a square of side s cm is cut, where s is an integer. The remaining sheet is folded into a cuboid of volume C cubic cm. Which of the following cannot be a value of C?

(a) 64 **(b)** 72 **(c)** 48 **(d)** 30

SOLUTION



The dimensions of cuboid are $l = b = (10 - 2s)$ cm and $h = s$ cm.

Volume of cuboid $= (10 - 2s)(10 - 2s)s$

i.e., $(10 - 2s)^2 s$ cubic cm.

$10 - 2s > 0$, i.e., $s < 5$ and s is an integer.

$\therefore s = 1, 2, 3$ or 4.

When $s = 1$, $C = 64$

When $s = 2$, $C = 72$

When $s = 3$, $C = 48$

When $s = 4$, $C = 16$

$\therefore$ Only option (d) cannot be a possible value of C.

CIRCLES

Circumference of a Circle

Since, a circle is a closed arc, its perimeter is known as circumference. The circumference of a circle is π (read as pie) times its diameter. π is an irrational number. For numerical work, we use the approximate value of π as 3.14 or $\frac{22}{7}$.

The circumference of a circle $= \pi d = 2\pi r$.

The area of a circle $= \pi r^2$.

Area of a Ring

The region enclosed between two concentric circles is known as a ring.

In Fig. 13.17, the shaded region represents a ring. Let the radius of the smaller circle, OA, be r and the radius of the bigger circle, OB, be R.

The area of the ring = (Area of the bigger circle) − (Area of the smaller circle)

$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$.

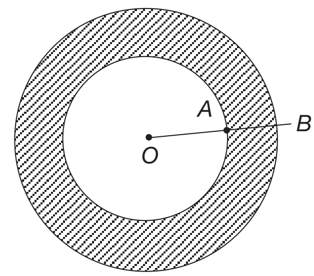


Figure 13.17

Sector of a Circle

A sector is formed when the endpoints of an arc of a circle are joined with two radii of the circle.

In the circle with centre O, AB is an arc of the circle (see Fig. 13.18). The endpoints of the arc, A and B are joined with two radii OA and OB of the circle. AOB is a sector of the circle. The arc makes an angle θ at the centre of the circle. This is also known as **central angle** or **sector angle**.

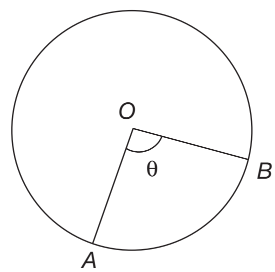


Figure 13.18

The length of an arc of the sector of a circle is proportional to the central angle of the sector of the circle.

$\therefore$ The length of an arc of the sector of a circle $(l) = \dfrac{\theta}{360°} \times 2\pi r$, where r is the radius of the circle.

The perimeter of the sector of the circle $= l + 2r$.

The area of the sector of a circle is proportional to the central angle of the sector of the circle.

$\therefore$ The area of the sector of a circle $= \dfrac{\theta}{360°}(\pi r^2)$ or $\dfrac{lr}{2}$.

EXAMPLE 13.2

A circle is placed in a rectangle such that it touches both the lengths of the rectangle. If the radius of the circle is one-fourth of the length of the rectangle, then find the ratio of the area of the region in the rectangle that is not covered by the circle to the area of the circle $\left(\text{take } \pi = \dfrac{22}{7}\right)$.

(a) 21 : 17 **(b)** 14 : 19 **(c)** 17 : 11 **(d)** 15 : 11

SOLUTION

Let l be the length of the rectangle.

Radius (r) of the circle $= \left(\dfrac{1}{4}\right)$(the length of the rectangle)

$= \dfrac{l}{4}$

therefore, breadth $(b) = 2 \times$ radius of the circle

$= 2 \times \left(\dfrac{l}{4}\right) = \left(\dfrac{l}{2}\right)$

required ratio = Area of the region in the rectangle that is not covered by the circle : Area of the circle

$$= (l)\left(\frac{l}{2}\right) - \pi r^2 : \pi r^2$$

$$= \left(\frac{l^2}{2}\right) - \left(\frac{22}{7}\right)\left(\frac{l}{4}\right)^2 : \left(\frac{22}{7}\right)\left(\frac{l}{4}\right)^2$$

$$= 17 : 11.$$

EXAMPLE 13.3

If a regular hexagon is inscribed in a circle of radius 4 cm, then find the area of the polygon in cm^2.

(a) $6\sqrt{3}$ **(b)** $24\sqrt{3}$ **(c)** $4\sqrt{3}$ **(d)** $48\sqrt{3}$

HINT

Area of hexagon = 6(Area of equilateral triangle).

SOLIDS

A plane figure may have one dimension or two dimensions. Triangles and quadrilaterals have two dimensions. For two-dimensional figures, the dimensions are length and breadth or width or height. But many objects such as a brick, a match box, a pencil, a marble, a tank, an ice cream, a cone have a third dimension. Thus, a solid is a three-dimensional object. In general, any object occupying space can be called a solid.

Some solids, like prisms, cubes and cuboids have plane or flat surfaces, while some solids, like cone, cylinder have curved surfaces as well as flat surfaces. Spheres have only curved surfaces. Lateral surface area of a solid having a curved surface is referred to as curved surface area.

Volume of a Solid

The amount of space enclosed by the bounding surface or surfaces of a solid is called the volume of the solid. It is measured in cubic units.

Prisms

A prism is a solid in which two congruent and parallel polygons form the top and the bottom faces. The lateral faces are parallelograms.

The line joining the centres of the two parallel polygons is called the axis of the prism and the length of the axis is referred to as the height of the prism.

If two parallel and congruent polygons are regular and, if the axis is perpendicular to the base, then the prism is called a right prism. The lateral surfaces of a right prism are rectangles.

Consider two congruent and parallel triangular planes ABC and PQR. If we join the corresponding vertices of both the planes, i.e., A to P, B to Q and C to R, then the resultant solid formed is a triangular prism. A right prism, whose base is a rectangle, is called a cuboid and the one, the base of which is a pentagon, is called a pentagonal prism. If all the faces of a prism are congruent, it is a cube. In case of a cube or a cuboid, any face may be the base of the prism. A prism whose base and top faces are squares, but the lateral faces are rectangular is called a square prism (see Fig. 13.19).

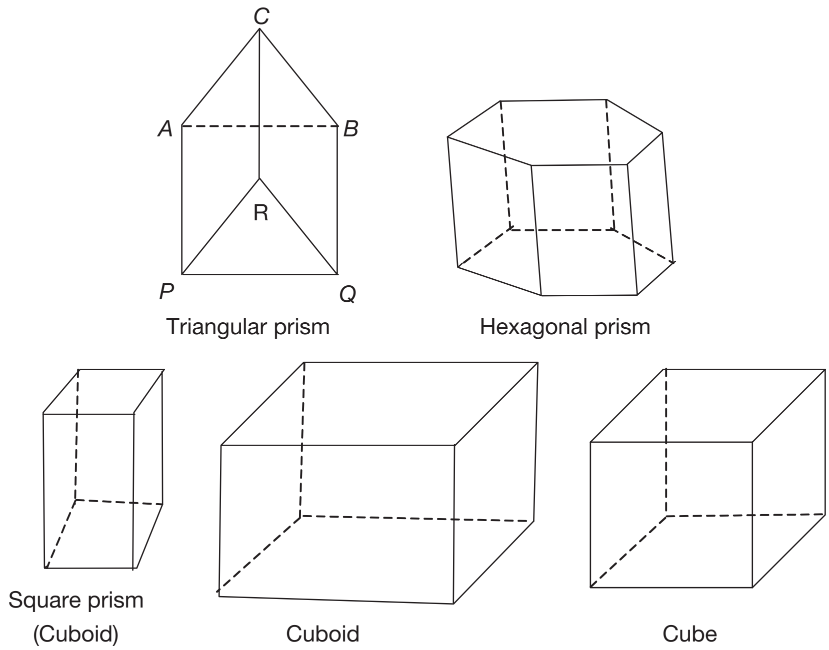


Figure 13.19

Notes The following points hold good for all prisms.

1. The number of lateral faces = The number of sides of the base.
2. The number of edges of a prism = The number of sides of the base × 3.
3. The sum of the lengths of the edges = 2(The perimeter of base) + The number of sides × Height.
4. For a prism, whose base is a polygon, Number of vertices + Number of faces = Number of edges + 2. This is known as Euler's formula, i.e., $V + F = E + 2$.

Lateral Surface Area (LSA) of a Prism

$$\text{LSA} = \text{Perimeter of base} \times \text{Height} = ph$$

Total Surface area (TSA) of a Prism

$$\text{TSA} = \text{LSA} + 2(\text{Area of base})$$

Volume of a Prism

$$\text{Volume} = \text{Area of base} \times \text{Height} = Ah$$

Note The volume of water flowing in a canal in an hour = The cross-section of the canal × The speed of water in an hour.

EXAMPLE 13.4

The base of a right prism is a right-angled triangle. The measure of the base of the right-angled triangle is 3 m and its height 4 m. If the height of the prism is 7 m, then find the following:

1. The number of edges of the prism.
2. The volume of the prism.
3. The total surface area of the prism.

SOLUTION

1. The number of the edges = The number of sides of the base × 3 = 3 × 3 = 9.
2. The volume of the prism = (Area of the base) × (Height of the prism) = $\left(\frac{1}{2}\right)(3 \times 4) \times 7$ $= 42 \text{ m}^3$.
3. TSA = LSA + 2(Area of base)
   $= ph + 2$(Area of base).
   Where, p = perimeter of the base = sum of lengths of the sides of the triangle.
   As hypotenuse of the triangle $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ m.
   ∴ Perimeter of the base = 3 + 4 + 5 = 12 m.
   ⇒ LSA $= ph = 12 \times 7 = 84 \text{ m}^2$.
   TSA = LSA + 2 (Area of base)
   $= 84 + 2\left(\frac{1}{2} \times 3 \times 4\right)$
   $= 84 + 12 = 96 \text{ m}^2$.

Cubes and Cuboids

Cuboid

In a right prism, if the base is a rectangle, then it is called a cuboid (see Fig. 13.20). A match box, a brick, a room, etc., are in the shape of a cuboid.

The three dimensions of the cuboid, i.e., its length (l), breadth (b) and height (h) are, generally, denoted by $l \times b \times h$ and read as l by b by h.

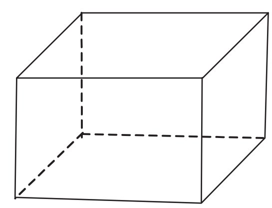


Figure 13.20 A Cuboid

1. The lateral surface area of a cuboid $= ph = 2(l + b)h$ sq. units, where p is the perimeter of the base.
2. The total surface area of a cuboid = LSA + 2(Base area) $= 2(l + b)h + 2lb = 2(lb + bh + lh)$ sq. units.
3. The volume of a cuboid $= Ah = (lb)h = lbh$ cubic units, where A is the area of the base.
4. Diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$ units.

Note If a closed wooden box of thickness t has inner dimensions of l, b and h, then the outer length $= l + 2t$, the outer breadth $= b + 2t$ and the outer height $= h + 2t$.

Cube

In a cuboid, if all the dimensions, i.e., its length, breadth and height are equal, then it is called a cube. All the edges of a cube are equal in length (see Fig. 13.21). Thus, the size of a cube is completely determined by its edge.

If the edge of a cube is a units, then

1. The lateral surface area of the cube $= 4a^2$.
2. The total surface area of the cube = LSA + 2(Area of base) $= 4a^2 + 2a^2 = 6a^2$.

3. The volume of the cube = a^3.
4. The diagonal of the cube $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3}\, a$.

Note If the inner edge of a cubical wooden box of thickness t is a units, then the outer edge of the cube is given by $a + 2t$ units.

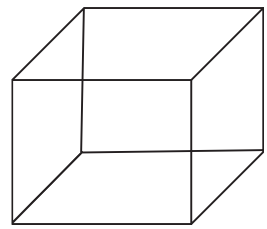


Figure 13.21 A Cube

EXAMPLE 13.5

The dimensions $l \times b \times h$ of a room are 12 m × 7 m × 5 m. Find:

(a) The diagonal of the room.

(b) The cost of flooring at the rate of ₹2 per m^2.

(c) The cost of white-washing of the inside of the room excluding the floor at the rate of ₹3 per m^2.

SOLUTION

(a) The diagonal of the room $= \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 7^2 + 5^2}\sqrt{144 + 49 + 25} = \sqrt{218}$ m.

(b) To find the cost of flooring, we should know the area of the base.

Base area $= lb = 12 \times 7 = 84$ m^2.

$\therefore$ Cost of flooring $= 84 \times 2 =$ ₹168.

(c) Total area to be white-washed

$= \text{LSA} + \text{area of roof} = 2(l + b)h + lb$

$= 2(12 + 7)5 + 12 \times 7 = 2(19)(5) + 84 = 190 + 84 = 274$ m^2.

$\therefore$ Cost of white-washing $= 274 \times 3 =$ ₹822.

EXAMPLE 13.6

A box is in the form of a cube. Its outer edge is 5 m long. Find:

(a) The total length of the edges.

(b) Cost of painting the outside of the box, on all the surfaces, at the rate of ₹5 per m^2.

SOLUTION

(a) Length of edges = Number of edges × 3 × Length of each edge $= 4 \times 3 \times 5 = 60$ m.

(b) To find the cost of painting the box, we need to find the total surface area.

TSA $= 6a^2 = 6 \times 5^2 = 6 \times 25 = 150$ m^2.

$\therefore$ Cost of painting $= 150 \times 5 =$ ₹750.

Right Circular Cylinder

A cylinder consists of two congruent and parallel circular regions which are connected by a curved surface. Each of the circular regions is called a base of the cylinder. A road roller, water pipe, power cables, round pillars are some of the cylinder-shaped objects.

In Fig. 13.22, a right circular cylinder is shown. Let A be the centre of the top face and A^1 be the centre of the base. The line joining the centres (i.e., AA^1) is called the axis of the cylinder. The length AA^1 is called the height of the cylinder. The radius r of the base of the cylinder and the height h, completely describe the cylinder.

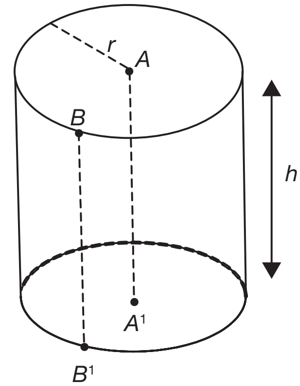


Figure 13.22

Lateral (curved) surface area = Perimeter of base × Height = $2\pi rh$

The total surface area = LSA + 2(Base area)

$= 2\pi rh + 2(\pi r^2) = 2\pi r(h + r)$

Volume = Area of base × Height = $\pi r^2 h$.

Note If a plastic pipe of length l is such that its outer radius is R and the inner radius is r, then the volume of the plastic content of the pipe is $l\pi(R^2 - r^2)$ cubic units.

EXAMPLE 13.7

A closed cylindrical container, the radius of which is 7 cm and height 10 cm is to be made out of a metal sheet. Find:

(a) The area of metal sheet required.

(b) The volume of the cylinder made.

(c) The cost of painting the lateral surface of the cylinder at the rate of ₹4 per cm^2. (Assume that the thickness of the metal sheet is negligible).

SOLUTION

(a) The area of the metal sheet required = The total surface area of the cylinder

$= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7(7+10) = 44(17) = 748 \text{ cm}^2.$$

(b) Volume = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10 = 22 \times 70 = 1540 \text{ cm}^3.$$

(c) To find the cost of painting the lateral surface, we need to find the curved (lateral) surface area.

$$\therefore \text{LSA} = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 10 = 22 \times 70 = 440 \text{ cm}^2.$$

Cost of painting = 440 × 4 = ₹1760.

Pyramid

A **pyramid** is a solid obtained by joining the vertices of a polygon to a point in the space by straight lines. The polygon is the base of the pyramid. Triangles are formed with each side of the base and the point in the space. The fixed point in space where all the triangles (i.e., lateral faces) meet is called the vertex of the pyramid.

In Fig. 13.23, base *ABCD* is a quadrilateral. All the vertices of the base are joined to a fixed point *O* in space by straight lines. The resultant solid obtained is a **pyramid** with a quadrilateral as the base. The straight line joining the vertex of the pyramid and the centre of the base is called the axis of the pyramid. If the axis is not perpendicular to the base, it is an **oblique pyramid**.

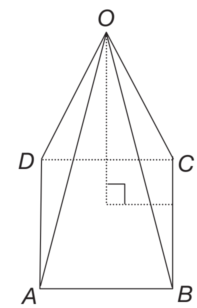


Figure 13.23

Right Pyramid

If the line joining the vertex of the pyramid and the centre of the base is perpendicular to the base, then the pyramid is called a **right pyramid** (see Fig. 13.24).

The length of the line segment joining the vertex of the pyramid and the centre of the base of a right pyramid is called the **height** of the pyramid. It is represented by h. The perpendicular distance between the vertex of the pyramid and the mid-point of any of the sides of the base (i.e., regular polygon) of a right pyramid is called its **slant height**. It is represented by l.

For a right pyramid with perimeter of base = p, height = h and slant height = l:

1. Lateral surface area $= \frac{1}{2}(\text{perimeter of base}) \times (\text{slant height}) = \frac{1}{2}pl$
2. Total surface area = lateral surface area + area of base.
3. Volume of pyramid $= \frac{1}{3} \times \text{area of base} \times \text{height}$.

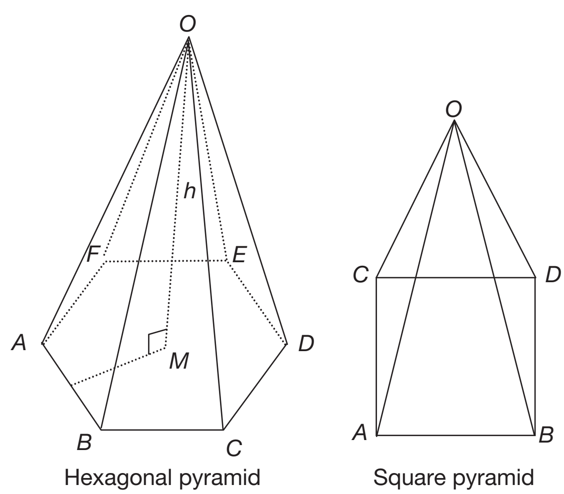


Figure 13.24

EXAMPLE 13.8

A regular hexagonal pyramid is 20 m high. Side of the base is 5 m. Find the volume and the slant height of the pyramid.

SOLUTION

Given, $h = 20$ m

Side of base = $a = 5$ m

$$\therefore \text{Area of base} = \frac{\sqrt{3}}{4} \times a^2 \times 6 = \frac{6\sqrt{3}}{4} \times 5^2$$

$$= \frac{75\sqrt{3}}{2} \text{ m}^2.$$

Volume $= \frac{1}{3} Ah$; where A = area of the base, and h = height.

$$= \frac{1}{3} \times \frac{3\sqrt{3}}{2}(25) \times 20 = \sqrt{3} \times 250 = 250\sqrt{3} \text{ m}^3.$$

To find slant height, refer to the Fig. 13.25. In this figure, O is the vertex of the pyramid and G is the centre of the hexagonal base. OG is the axis of the pyramid. OH is the slant height of the pyramid.

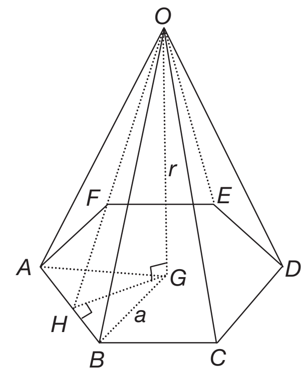


Figure 13.25

ΔOGH is a right-angled triangle.

$\therefore \; OH^2 = GH^2 + OG^2.$

GH = Altitude of ΔAGB

$$= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2} \text{ m}.$$

$$\therefore OH^2 = \left(\frac{5\sqrt{3}}{2}\right)^2 + (20)^2 = \frac{25 \times 3}{4} + 400$$

$$= \frac{75 + 1600}{4} = \frac{1675}{4}$$

$$\Rightarrow OH = \frac{\sqrt{1675}}{2} \text{ m}.$$

$$\therefore \text{Slant height} = \frac{5\sqrt{67}}{2} \text{ m}.$$

Circular Cone

A **circular cone** is a solid pointed figure with a circular base as shown in Fig. 13.26. A circular cone is a kind of pyramid whose base is a circle.

A cone has one vertex, one plane surface (i.e., the base) and a curved surface. The line joining the vertex to the centre of base (i.e., AO) is called the axis of the cone. If the axis is not perpendicular to the base of a cone, then it is an **oblique cone**. An ice cream cone and a conical tent are some of the examples of conical objects.

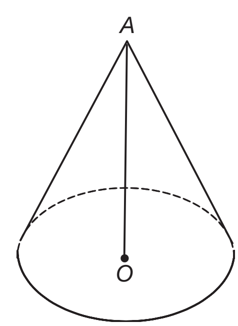


Figure 13.26

Right Circular Cone

In a circular cone, if the line joining the vertex and the centre of the base of the cone is perpendicular to the base, then it is a **right circular cone**. In other words, if the axis of the cone is perpendicular to the base of the cone, then it is a right circular cone. The length of the line segment AO is called the height of the cone. In this chapter, we deal with problems on right circular cones.

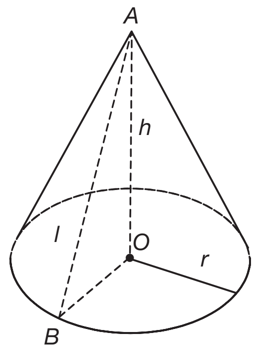


Figure 13.27

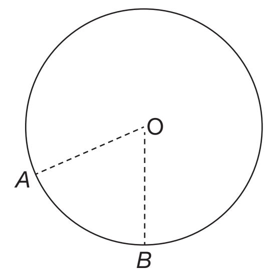


Figure 13.28

We can also define a right circular cone as a solid obtained by the revolution of a right-angled triangle about one of its two perpendicular sides.

If we consider any point B on the periphery of the base of the cone, then the line joining B and the vertex A is called the slant height of the cone and is denoted by l. Since AO is perpendicular to OB, ΔAOB is right-angled (see Fig. 13.27).

$$\therefore l = \sqrt{r^2 + h^2}.$$

Earlier, we have studied about sector. We may recall that sector is a figure bounded by an arc of a circle with its two (see Fig. 13.28).

Now, consider the sector AOB. If we roll the sector up and bring (join) together the radii OA and OB such that they coincide, then the figure formed will be a cone. The radius of the circle becomes the slant height of the cone and the length of the arc of the sector becomes the perimeter of the base of the cone.

For a cone of radius r, height h and slant height l:

1. Curved surface area of a cone = πrl sq. units.
2. Total surface area of a cone

   = Curved surface area + Area of base = $\pi rl + \pi r^2 = \pi r(r + l)$ sq. units.
3. Volume of a cone $= \dfrac{1}{3}\pi r^2 h$ cubic units.

Cone Frustum (or a Conical Bucket)

If a right circular cone is cut by a plane perpendicular to its axis (i.e., a plane parallel to the base), then the solid portion containing the base of the cone is called the **frustum** of the cone.

From Fig. 13.29, we observe that a frustum is in the shape of a bucket.

Let radius of the upper base be R, radius of the lower base be r, height of the frustum be h and slant height of the frustum be l.

1. Curved surface area of a frustum = $\pi l(R + r)$ sq. units.
2. Total surface area of a frustum

   = Curved surface area + Area of upper base + Area of lower base = $\pi l(R + r) + \pi r^2 + \pi R^2$ sq. units.

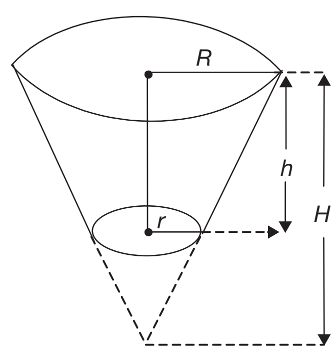


Figure 13.29

3. Volume of a frustum $= \frac{1}{3}\pi h(R^2 + Rr + r^2)$ cubic units.

4. Slant height (l) of a frustum $= \sqrt{(R-r)^2 + h^2}$ units.

EXAMPLE 13.9

A Buffoon's cap is in the form of a cone of radius 7 cm and height 24 cm. Find the area of the paper required to make the cap.

SOLUTION

Area of the paper required = Curved surface area of the cap (or cone) = πrl.

Now, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm.

$\Rightarrow$ Curved surface area $= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$.

$\therefore$ Area of the paper required = 550 cm^2.

EXAMPLE 13.10

The inner diameter of an ice cream cone is 7 cm and its height is 12 cm. Find the volume of ice cream that the cone can contain.

SOLUTION

Volume of ice cream (cone) $= \frac{1}{3}\pi r^2 h$.

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12 = 22 \times 7 = 154 \text{ cm}^3.$$

EXAMPLE 13.11

The diameters of the top and bottom portions of a milk can are 56 cm and 14 cm, respectively. The height of the can is 72 cm. Find the:

(a) Area of metal sheet required to make the can. (without lid)

(b) Volume of milk which the can can hold.

SOLUTION

The milk can is in the shape of a cone frustum with $R = 28$ cm, $r = 7$ cm and $h = 72$ cm.

(a) Area of metal sheet required

= Curved surface area + Area of bottom base $= \pi l(R + r) + \pi r^2$.

$l = \sqrt{(R-r)^2 + h^2} = \sqrt{(28-7)^2 + 72^2}$

$= \sqrt{21^2 + 72^2} = \sqrt{9(7^2 + 24)^2} = 3\sqrt{49 + 576} = 3 \times \sqrt{625}$

$= 3 \times 25 = 75$ cm.

$\therefore$ Area of metal sheet $= \frac{22}{7} \times 75(28 + 7) + \frac{22}{7} \times 7^2$

$= 22 \times 75 \times 5 + 22 \times 7 = 22(375 + 7)$

$= 22(382) = 8404 \text{ cm}^2.$

(b) Amount of milk which the container can hold

$$= \frac{1}{3}\pi h(R^2 + Rr + r^2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 72(28^2 + 7 \times 28 + 7^2)$$

$$= 22 \times 24 \times (112 + 28 + 7)$$

$$= 22 \times 24 \times (147) = 77616 \text{ cm}^3.$$

EXAMPLE 13.12

From a circular canvas of diameter 56 m, a sector of 270° was cut out and a conical tent was formed by joining the straight ends of this piece. Find the radius and the height of the tent.

SOLUTION

As shown in Fig. 13.30, when the free ends of the canvas cut-out are joined to form a cone, the radius of the sector becomes the slant height (l) of the cone.

$$\therefore l = \frac{56}{2} = 28 \text{ m}.$$

The length of the arc of the sector becomes the circumference of the base of the cone. Let the radius of the base of the cone = r.

$$\Rightarrow 2\pi r = \frac{22}{7} \times \frac{56}{2} \times \left(\frac{270}{360}\right)$$

$$\Rightarrow 2\pi r = \frac{22}{7} \times 28 \times \frac{3}{4} \Rightarrow r = 21 \text{ m}.$$

$$\therefore \text{Height, } h = \sqrt{l^2 - r^2}$$

$$= \sqrt{28^2 - 21^2} = \sqrt{7^2(4^2 - 3)^2}$$

$$= 7\sqrt{16 - 9} = 7\sqrt{7} \text{ m}$$

$$\therefore h = 7\sqrt{7} \text{ m and } r = 21 \text{ m}.$$

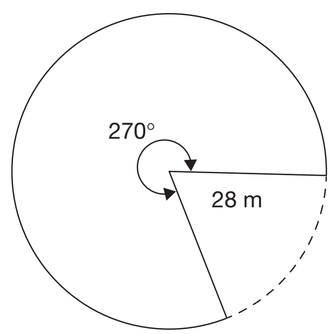


Figure 13.30

Torus (Solid Ring)

A fully-inflated bicycle tube, tennikoit ring, and a life belt are some examples of a torus. A torus is a three-dimensional figure formed by the revolution of a circle about an axis lying in its plane, but not intersecting the circle.

If r is the radius of a circle that rotates and a is the distance of the centre of rotating circle from the axis (see Figs. 13.31 and 13.32), then:

Surface area of torus = $4\pi^2 ra$ sq. units and

Volume of torus = $2\pi^2 r^2 a$ cubic units.

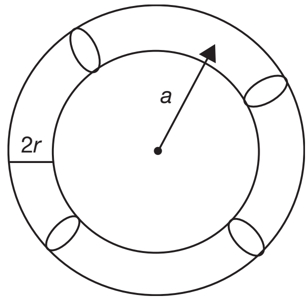


Figure 13.31

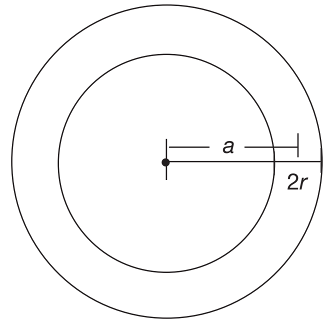


Figure 13.32

EXAMPLE 13.13

The radius of cross-section of a car tube is 7 cm. The outer radius of the tube is 16 cm. Find the surface area and the volume of the tube.

SOLUTION

Here, $r = 7$ cm, hence $a = 16 - r = 16 - 7 = 9$ cm.

$$\text{Surface area} = 4\pi^2 ra = 4 \times \frac{22}{7} \times \frac{22}{7} \times 7 \times 9 = \frac{17424}{7}\ \text{cm}^2.$$

$$\text{Volume} = 2\pi^2 r^2 a$$

$$= 2 \times \frac{22}{7} \times \frac{22}{7} \times 7 \times 7 \times 9 = 8712\ \text{cm}^3.$$

Sphere

Sphere is a set of points in the space which are equidistant from a fixed point. The fixed point is called the centre of the sphere, and the distance is called the radius of the sphere. A lemon, a football, the moon, globe, small lead balls used in cycle bearings are some example of spherical-shaped objects.

A line joining any two points on the surface of sphere and passing through the centre of the sphere is called its diameter. The size of sphere can be completely determined by knowing its radius or diameter.

Solid Sphere

A **solid sphere** is the region in the space bounded by a sphere. The centre of a sphere is also a part of solid sphere, whereas the centre is not a part of hollow sphere. Marbles, lead shots, etc., are examples of solid spheres, while a tennis ball is a hollow sphere.

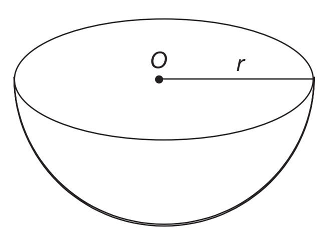


Figure 13.33 Hemisphere

Hemisphere

If a sphere is cut into two halves by a plane passing through the centre of sphere, then each of the halves is called a **hemisphere** (see Fig, 13.33).

Spherical Shell

The region bounded in the space between two concentric solid spheres is called a spherical shell. The thickness of the shell is given by the difference in radii of the two spheres.

Hemispherical Shell A hemispherical shell is shown in Fig. 13.34.

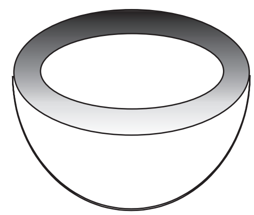


Figure 13.34 Hemispherical shell

Formulas to Memorize

Sphere

1. Surface area of a sphere = $4\pi r^2$
2. Volume of a sphere $= \frac{4}{3}\pi r^3$

Hemisphere

1. Curved surface area of a hemisphere = $2\pi r^2$
2. Total surface area of a hemisphere = $3\pi r^2$
3. Volume of a hemisphere $= \frac{2}{3}\pi r^3$

Spherical Shell

1. Thickness = $R - r$, where R = outer radius and r = inner radius.
2. Volume $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
3. Total surface area of a hemispherical shell

   = (curved surface area of outer hemisphere + curved surface area of inner hemisphere + area of ring)

   $= 3\pi R^2 + \pi r^2$.

EXAMPLE 13.14

The cost of painting a solid sphere at the rate of 50 paise per square metre is ₹1232. Find the volume of steel required to make the sphere.

SOLUTION

Cost of painting = Surface area × Rate of painting

$$\therefore \text{Surface area} = \frac{\text{Cost of painting}}{\text{Rate of painting}}$$

$$\Rightarrow \frac{1232}{0.5} = 2464 \text{ m}^2 \Rightarrow 4\pi r^2 = 2464$$

$$\Rightarrow r^2 = \frac{2464}{4\pi} = \frac{616}{\left(\frac{22}{7}\right)} = \frac{616 \times 7}{22} = 28 \times 7 \Rightarrow r = 7 \times 2 = 14 \text{ m}.$$

$$\therefore \text{Volume of steel required} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = \frac{34496}{3} \text{ m}^3.$$

EXAMPLE 13.15

A hollow hemispherical bowl, as shown in Fig. 13.35, of thickness 1 cm has an inner radius of 6 cm. Find the volume of metal required to make the bowl.

SOLUTION

Inner radius, $r = 6$ cm

Thickness, $t = 1$ cm

$\therefore$ Outer radius, $R = r + t = 6 + 1 = 7$ cm.

$\therefore$ Volume of steel required $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$= \frac{44}{21}(7^3 - 6^3)$

$= \frac{44}{21}(343 - 216) = \frac{44}{21} \times 127 = \frac{5588}{21}$ cm^3.

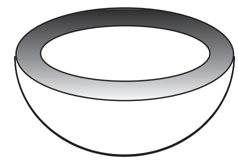


Figure 13.35

EXAMPLE 13.16

A thin hollow hemispherical sailing vessel is made of metal surmounted by a conical canvas tent. The radius of the hemisphere is 14 m and the total height of the vessel (including the height of tent) is 28 m. Find the area of the metal sheet and the canvas required.

SOLUTION

The vessel (with the conical tent) is shown in Fig. 13.36.

Total height, $H = 28$ m

Radius of hemisphere $= r = 14$ m.

$\therefore$ Height of conical tent $= h = H - r = 28 - 14$

$= 14$ m.

We can observe that radius of the base of the cone

$=$ Radius of the hemisphere $= 14$ m.

$\therefore$ Area of canvas required $= \pi rl$.

$= \frac{22}{7} \times 14 \times \sqrt{14^2 + 14^2} = 44 \times 14\sqrt{2} = 616\sqrt{2}$ m^2.

Area of metal sheet required = Surface area of hemisphere

$= 2\pi r^2 = 2 \times \frac{22}{7} \times 14 \times 14 = 1232$ m^2.

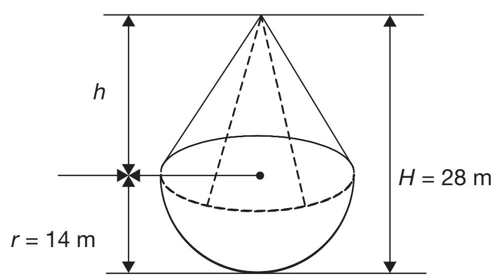


Figure 13.36

Polyhedrons

A solid bounded by plane polygons is a polyhedron. The bounding polygons are known as the faces and the intersection of the faces are edges. The points where three or more edges intersect are called the vertices.

A polyhedron having four faces is a tetrahedron, one having six faces is a hexahedron and one having eight faces is an octahedron.

If all the faces of a polyhedron are regular polygons which are congruent, then it is a regular polyhedron. However, further discussion on regular polyhedrons can be made in higher classes.

Regular Tetrahedron

A regular tetrahedron is a tetrahedron having all of its faces as equilateral triangles (see Fig. 13.37). The line joining the vertex of the tetrahedron and the centre of the base is the vertical height of a regular tetrahedron. The line joining the mid-point of the side of the base and the vertex of a regular tetrahedron is its slant height.

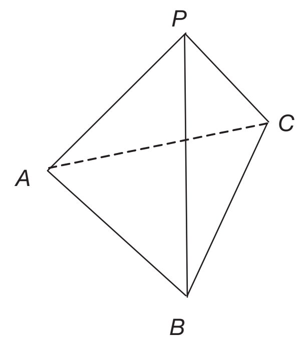


Figure 13.37 Regular Tetrahedron

The lateral surface area of a regular tetrahedron

$$= \frac{1}{2} \times \text{Perimeter of base} \times \text{Slant height}$$

$$= \frac{1}{2} \times 3a \times \frac{\sqrt{3}}{2}a$$

$$= \frac{3\sqrt{3}}{4}a^2 \text{ (where } a \text{ is the edge).}$$

The total surface area of a regular tetrahedron

= LSA + Area of base

$$= \frac{3\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}a^2$$

$$= \sqrt{3}\,a^2.$$

$$\text{Volume of a regular tetrahedron} = \frac{1}{3} \times \text{Area of base} \times \text{Vertical height}$$

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4}a^2 \times \frac{\sqrt{2}}{\sqrt{3}}a = \frac{a^3}{6\sqrt{2}}.$$

Regular Octahedron

A regular octahedron is an octahedron whose faces are all congruent equilateral triangles.

Figure 13.38 is an example of regular octahedron, where $ABCD$ is a square; ABP, BCP, CDP, DAP, ABQ, BCQ, CDQ and DAQ are eight equilateral triangles which are congruent. If each side of an equilateral triangle is a units, then surface area of the octahedron

$$= 8 \times \frac{\sqrt{3}}{4}a^2 = 2\sqrt{3}\,a^2 \text{ sq. units.}$$

Volume of a regular octahedron

$$= 2\left(\frac{1}{3} \times a^2 \times \frac{a}{\sqrt{2}}\right) = \sqrt{\frac{2}{3}}a^3 \text{ cu units.}$$

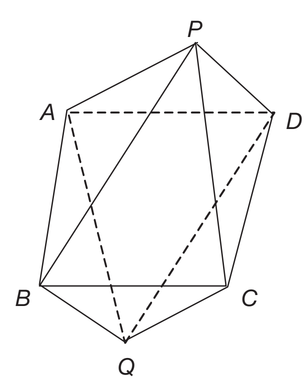


Figure 13.38 Regular Octahedron

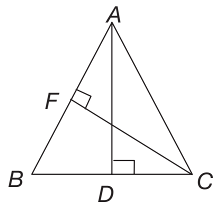
TEST YOUR CONCEPTS

Very Short Answer Type Questions

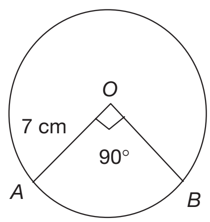
1. The diagonal of a square is 10 cm. What is its area?
2. If the circumference of a circle is numerically equal to its area, then what is the radius of the circle?
3. A solid object has 8 vertices and 12 edges. It has ______ faces.
4. The number of vertices of a pyramid, whose base is a pentagon, is ______.
5. A tetrahedron has ______ edges.
6. What is the number of edges of an octahedron?
7. If $p + q$ and $p - q$ are the sides of a rectangle, then its diagonal is ______ units.
8. Area of an isosceles triangle, one of whose equal sides is 5 units and base 6 units is ______.
9. A triangle has a perimeter of 9 cm. How many combinations of its sides exist, if their lengths in cm are integers?
10. Among an equilateral triangle, an isosceles triangle and a scalene triangle, ______ triangle has the maximum area if the perimeter of each triangle is the same.
11. Ratio of the area of a triangle to the product of its sides is ______ times the reciprocal of its circum-radius.
12. Area of a quadrilateral, which has one of its diagonals as 10 cm and the lengths of the offsets drawn to its 3 cm and 5 cm, is ______.
13. By joining the mid-points of the adjacent sides of a quadrilateral of area 26 cm^2 a ______ is formed and its area, is ______.
14. The longest needle that can be placed in a cylinder of volume $\pi r^2 h$, is ______.
15. The base of a pyramid is an equilateral triangle with side a and the height of the pyramid is h. Then the volume of the pyramid is ______.
16. The heights of two cylinders are equal and their radii are in the ratio of 3 : 2. The ratio of their curved surface areas is ______.
17. What is the volume of a pyramid whose base area is $(2+\sqrt{3})$ cm^2 and its height being $(2-\sqrt{3})$ cm?
18. If the edge of a cube increases by 20%, then find the percentage increase in its volume.
19. If a sphere, cone, cuboid, pyramid and cube have the same surface area, then which of these solid figure have the maximum volume?
20. The volumes of two cones are in the ratio of their respective radii, if they have the same height. [True/False]
21. The radii of two spheres are 2 cm and 3 cm respectively. The ratio of their surface areas is ________.
22. What is the volume of a hemispherical shell with outer radius of x units and thickness of y units?
23. If the volume of a solid hemisphere and its total surface area are numerically equal, then find its radius.
24. The perimeter of the base of a pyramid is $(2x + 2y)$ cm and its slant height is $(x - y)$ cm. What is the lateral surface area of the pyramid? $(x > y)$
25. The sides of a triangle are 5 cm, 7 cm and 8 cm. Find its area.
26. What is the volume of the prism whose base is a hexagon of side 6 cm and height $12\sqrt{3}$ cm?
27. If the dimensions of a cuboid are 5 cm × 4 cm × 3 cm, then find the maximum volume of the cube that can be carved out of it.
28. Find the lateral surface area and the length of the diagonal of a cube of side 8 cm.
29. Find the number of soaps of size 2 cm × 3 cm × 5 cm, that can be arranged in a cuboidal box of dimensions 6 cm × 3 cm × 15 cm.
30. What is the area of a ground that can be levelled by a cylindrical roller of radius 3.5 m and length 4 m by making 10 rounds?

Short Answer Type Questions

31. In the given figure, $AB = 6.4$ cm, $CF = 2.6$ cm, $AD = 3.2$ cm. Find the area of the triangle ABC and length of the side BC.



32. From the given figure, find the length of the major arc AB.



33. Find the area of a circle, if the area of the isosceles right-angle triangle inscribed in it is 18 sq. cm.

34. The perimeter of a rectangle is a two digits number whose units digit and tens digit represent the length and breadth of the rectangle respectively. Find its area.

35. Three equilateral triangles are cut from an equilateral triangle of area 36 cm^2, such that a regular hexagon is left. Find the area of the hexagon.

36. If $a + b$, $a - b$ and $2\sqrt{ab}$ are the sides of a cuboid, then find the longest stick that can be placed in it.

37. A square plate of side 28 cm is made into a cylinder by joining two edges of it. Find the base area of the cylinder formed.

38. From a cylindrical wooden log of length 30 cm and base radius $7\sqrt{2}$ cm, maximum cuboid of square base is made. Find the volume of wood wasted.

39. Find the number of cubes of side 2 m to be dropped in a cylindrical vessel of radius 14 m to increase the water level by 5 m.

40. If eight cubes are stacked to form a big cube, then the percentage decrease in the total surface area is ________.

41. Three cubes, each of side 3.2 cm, are joined end-to-end. Find the total surface area of the resulting cuboid.

42. The base of a right pyramid is an equilateral triangle, each side of which is $6\sqrt{3}$ cm long and height is 4 cm. Find the total surface area of the pyramid in cm^2.

43. What is the volume of a square pyramid whose vertical height is 10 cm and the length of the side of the base is 6 cm?

44. The area of the base of a pyramid, which is an equilateral triangle, is $16\sqrt{3}$ cm^2, and its slant height is 3 cm. What is its lateral surface area?

45. The base radius of a conical tent is 120 cm and its slant height is 750 cm. Find the area of the canvas required to make 10 such tents (in m^2). (Take $\pi = 3.14$).

Essay Type Questions

46. An open metallic conical tank is 6 m deep and its circular top has a diameter of 16 m. Find the cost of tin plating in its inner surface at the rate of ₹0.8 per 100 cm^2. (Take $\pi = 3.14$).

47. A drum in the shape of a frustum of a cone with radii 20 ft and 14 ft and height 5 ft is full of water. The drum is emptied into a cuboidal tank of base 73 ft × 44 ft. Find the rise in the height of the water level in the tank.

48. The radius of a cross-section of a tube of a truck is 28 cm. The outer radius of the tube itself is 63 cm. What is the surface area of the tube?

49. If the thickness of a hemispherical bowl is 12 cm and its outer diameter is 10.24 m, find the inner surface area of the hemisphere. (Take $\pi = 3.14$)

50. The internal surface area of a hollow hemisphere is 77 cm^2 and its external surface area is 308 cm^2. What is the thickness of the hollow hemisphere?

CONCEPT APPLICATION

Level 1

1. The perimeter of a right triangle is 72 cm and its area is 216 cm^2. Find the sum of the lengths of its perpendicular sides. (in cm)
   (a) 36 (b) 32
   (c) 42 (d) 50

2. Find the area of the regular pentagon of side 6 cm and height 8 cm. (in cm^2)
   (a) 40 (b) 60
   (c) $80\sqrt{3}$ (d) 120

3. Find the perimeter of a sector of a circle if the angle and radius of it are 30° and 10.5 cm respectively.
   (a) 26.5 cm (b) 21.5 cm
   (c) 23 cm (d) 8 cm

4. The sides of a pentagonal prism are 10 cm, 12 cm, 15 cm, 8 cm and 6 cm. Its height is 14 cm. Find the total length of its edges.
   (a) 172 cm (b) 162 cm
   (c) 182 cm (d) 152 cm

5. Three cubes of sides 3 cm, 4 cm and 5 cm, respectively, are melted and formed into a larger cube. What is the side of the cube formed?
   (a) 7 cm (b) 6 cm
   (c) 5 cm (d) 4 cm

6. The sum of the length, breadth and the height of a cuboid is 20 cm and the length of its diagonal is 12 cm. Find the total surface area of the cuboid.
   (a) 156 cm^2 (b) 169 cm^2
   (c) 256 cm^2 (d) 269 cm^2

7. The radius of the base of a cone is 7 cm and its slant height is 25 cm. The volume of the cone is _______.
   (a) 3696 cm^3 (b) 1232 cm^3
   (c) 2464 cm^3 (d) 1864 cm^3

8. The side of a square is equal to the diagonal of a cube. The square has an area of 1728 m^2. Calculate the side of the cube.
   (a) 12 m (b) 24 m
   (c) 27 m (d) 36 m

9. The radius of the base and the slant height of a cone is 5 cm and 13 cm respectively. Find the volume of the cone.
   (a) 88π cm^3 (b) 100π cm^3
   (c) 92π cm^3 (d) 106π cm^3

10. If the ratio of the volumes of the spheres is 8 : 27, then the ratio of their surface areas is _______.
    (a) 4 : 9 (b) 9 : 4
    (c) 4 : 3 (d) 2 : 9

11. The volume of a hemisphere is 18π cm^3. What is the total surface area of the hemisphere?
    (a) 18π cm^2 (b) 27π cm^2
    (c) 21π cm^2 (d) 24π cm^2

12. Find the volume of a regular octahedron of each edge $2\sqrt{3}$ cm.
    (a) $4\sqrt{3}$ cm^3 (b) $8\sqrt{3}$ cm^3
    (c) $4\sqrt{6}$ cm^3 (d) $8\sqrt{6}$ cm^3

13. In a polyhedron, if the number of faces is 4 and the number of edges is 6, then the number of vertices of that polyhedron is _______.
    (a) 1 (b) 2
    (c) 3 (d) 4

14. Fifteen identical spheres are made by melting a solid cylinder of 10 cm radius and 5.4 cm height. Find the diameter of each sphere.
    (a) 6 cm (b) 3 cm
    (c) 2 cm (d) 4 cm

15. Find the number of soaps of size 2.1 cm × 3.7 cm × 2.5 cm that can be put in a cuboidal box of size 6.3 cm × 7.4 cm × 5 cm.
    (a) 14 (b) 12
    (c) 13 (d) 11

16. If the radius of a sphere is increased by 25%, then the percentage increase in its volume is _______. (approximately)
    (a) 90% (b) 95%
    (c) 83% (d) 78%

17. Find the volume of a hollow sphere of outer radius 9 cm and inner radius 6 cm.

(a) 342π cm^3 (b) 684π cm^3

(c) 36π cm^3 (d) 128π cm^3

18. A hollow hemispherical bowl of thickness 1 cm has an inner radius of 4.5 cm. Find the curved surface area of the bowl.

(a) 106π cm^2 (b) 108π cm^2

(c) 101π cm^2 (d) 102π cm^2

19. If the length of each edge of a tetrahedron is 18 cm, then the volume of the tetrahedron is _______.

(a) $482\sqrt{2}$ cm^3 (b) $480\sqrt{2}$ cm^3

(c) $484\sqrt{2}$ cm^3 (d) $486\sqrt{2}$ cm^3

20. Find the total surface area of a regular octahedron, each edge of which is 10 cm.

(a) $100\sqrt{3}$ cm^2 (b) $200\sqrt{3}$ cm^2

(c) $300\sqrt{3}$ cm^2 (d) $400\sqrt{3}$ cm^2

21. The number of edges in a pyramid whose base has 20 edges is _______.

(a) 10 (b) 20

(c) 30 (d) 40

22. A copper cable, 32 cm long, having a diameter 6 cm, is melted to form a sphere. Find the radius of the sphere.

(a) 6 cm (b) 8 cm

(c) 10 cm (d) 12 cm

23. The number of diagonals of a regular polygon in which each interior angle is 156°, is _______.

(a) 24 (b) 54

(c) 90 (d) 45

24. *ABCDEF* is a regular hexagon. If *AB* is 6 cm long, then what is the area of triangle *ABD*?

(a) $8\sqrt{3}$ cm^2 (b) $12\sqrt{3}$ cm^2

(c) $15\sqrt{3}$ cm^2 (d) $18\sqrt{3}$ cm^2

25. The area and the radius of a sector are 6.93 sq. cm and 4.2 cm respectively. Find the length of the arc of the sector.

(a) 1.72 cm (b) 0.86 cm

(c) 3.3 cm (d) 6.6 cm

26. The perimeter of the base of a right prism, whose base is an equilateral triangle, is 24 cm. If its total surface area is $(288+32\sqrt{3})$cm^2, then its height is _______.

(a) 10 cm (b) 12 cm

(c) 14 m (d) 14 cm

27. A large sphere of radius 3.5 cm is carved from a cubical solid. Find the difference between their surface areas.

(a) 224 cm^2 (b) 140 cm^2

(c) 176 cm^2 (d) 80.5 cm^2

28. If one of the edges of a regular octahedron is $4\sqrt{3}$ cm, then find its height.

(a) $2\sqrt{3}$ cm (b) $4\sqrt{6}$ cm

(c) $6\sqrt{3}$ cm (d) $2\sqrt{6}$ cm

29. The dimensions of a cuboidal container are 12 cm × 10 cm × 8 cm. How many bottles of syrup can be poured into the container, if each bottle contains 20 cm^3 of syrup?

(a) 46 (b) 54

(c) 48 (d) 58

Level 2

30. A circle of radius 2 cm is inscribed in an equilateral triangle. Find the area of the triangle in cm^2.

(a) $12\sqrt{3}$ (b) $4\sqrt{3}$

(c) $6\sqrt{3}$ (d) $24\sqrt{3}$

31. The circum-radius of a right triangle is 10 cm and one of the two perpendicular sides is 12 cm. Find the area of the triangle in sq. cm.

(a) 96 (b) 128

(c) 48 (d) 64

32. A parallelogram has two of its adjacent sides measuring 13 each. Find the sum of the squares of its diagonals.

(a) 169 (b) 338

(c) 676 (d) 507

PRACTICE QUESTIONS

33. A classroom is 5 m long, 2.5 m broad and 3.6 m high. If each student is given 0.5 m^2 of the floor area, then how many cubic metres of air would each student get?

(a) 1.4 (b) 1.8
(c) 1.2 (d) 1.6

34. If the volume of a right equilateral triangular prism is $8500\sqrt{3}$ dm^3 whose height is 50 cm, then find the side of its base.

(a) $10\sqrt{17}$ cm (b) $10\sqrt{17}$ dm
(c) $20\sqrt{17}$ dm (d) $20\sqrt{17}$ cm

35. How many cubes, each of total surface area 54 sq.dm, can be made from a cube of edge 1.2 metre.

(a) 64 (b) 81
(c) 125 (d) 25

36. Four times the sum of the areas of the two circular faces of a cylinder is equal to the twice its curved surface area. Find the diameter of the cylinder if its height is 8 cm.

(a) 4 cm (b) 8 cm
(c) 2 cm (d) 6 cm

37. If the base of a right pyramid is a square of side 4 cm and its height is 18 cm, then the volume of the pyramid is _______.

(a) 90 cm^3 (b) 104 cm^3
(c) 100 cm^3 (d) 96 cm^3

38. A right circular conical tent is such that the angle at its vertex is 60° and its base radius is 14 m. Find the cost of the canvas required to make the tent at the rate of ₹25 per m^2.

(a) ₹15,400 (b) ₹30,800
(c) ₹16,400 (d) ₹32,800

39. A goat is tied to a corner of a rectangular plot of dimensions 14 m × 7 m with a 21 m long rope. It cannot graze inside the plot, but can graze outside it as far as it is permitted by the rope. Find the area it can graze (in m^2). (Take $\pi = \frac{22}{7}$)

(a) 240 (b) 1560.5
(c) 1543.5 (d) 1232

40. If the breadth of a rectangle is increased by 5 cm, its area increases by 25 cm^2. If its length is increased by 5 cm, its area increases by 20 cm^2. Find the area of the rectangle (in cm^2).

(a) 20 (b) 25
(c) 30 (d) 35

41. A cylinder-shaped tank is surmounted by a cone of equal radius. The height of the cone is 6 m and the total height of the tank is 18 m. Find the volume of the tank if the base radius of the cylinder is 5 m.

(a) 1650 m^3 (b) 1244 m^3
(c) 1100 m^3 (d) 2200 m^3

42. The radius of the cross-section of an inflated cycle tyre is 7 cm. The distance of the centre of the cross-section from the axle is 20 cm. Find the volume of air in the tyre.

(a) 19360 cm^3 (b) 1760 cm^3
(c) 6160 cm^3 (d) 880 cm^3

43. If h is the length of the perpendicular drawn from a vertex of a regular tetrahedron to the opposite face and each edge is of length s, then s^2 is equal to _______.

(a) $\frac{3h^2}{8}$ (b) $\frac{3h^2}{2}$
(c) h^2 (d) $\frac{8h^2}{3}$

44. The radii of the ends of a frustum of a cone are 28 cm and 7 cm. The height of the cone is 40 cm. Find its volume.

(a) 32340 cm^3 (b) 43120 cm^3
(c) 10780 cm^3 (d) None of these

45. The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\therefore$ Volume of the cube = a^3 cm^3 = $(6)^3$ cm^3 = 216 cm^3.

(B) Then, diagonal of the cube = $a\sqrt{3}$ cm.

(C) From the given data $a\sqrt{3} = 6\sqrt{3} \Rightarrow a = 6$ cm.

(D) Let the side of the cube be a cm.

(a) DCBA (b) DBCA
(c) DACB (d) DBAC

PRACTICE QUESTIONS

46. Find the length of the arc of a sector of a circle whose angle at the centre is 120° and area of the sector is 462 cm^2.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) Given, $\theta = 120°$, area of the sector = 462 cm^2.

We know that $A = \frac{\theta}{360°} \times \pi r^2$.

(B) $\Rightarrow 462 = \frac{120°}{360°} \times \frac{22}{7}(r)^2 \Rightarrow r = 21$ cm.

(C) Length of the arc of the sector $= \frac{\theta}{360°} \times 2\pi r$

$= \frac{120}{360°} \times 2 \times \frac{22}{7} \times 21.$

(D) $\therefore$ Length of the arc = 44 cm

(a) ABCD

(b) ACBD

(c) BACD

(d) BCAD

47. The radii of the top and the bottom of a metal can which is cone-shaped frustum, are 20 cm and 8 cm, respectively. The height of the can is 16 cm. Find the area of the metal sheet required to make the can with a lid.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) Area of metal sheet = $\pi l(R + r) + \pi R^2 + \pi r^2 = \pi \times 20(20 + 8) + \pi(20)^2 + \pi(8)^2$.

(B) Given, $R = 20$ cm, $r = 8$ cm and $h = 16$ cm.

(C) $l = \sqrt{(R-r)^2 + h^2} = \sqrt{(20-8)^2 + (16)^2} = 20$ cm.

(D) $\therefore$ Area of the sheet = 1024 π sq. cm.

(a) BCAD

(b) ACBD

(c) BDAC

(d) BACD

48. The diagonal of a cube is $8\sqrt{3}$ cm. Find its total surface area.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) Then the diagonal of cube = $a\sqrt{3}$ cm.

(B) $\therefore$ Total surface area = $6a^2 = 6(8)^2 = 384$ cm^2.

(C) Let the side of cube be a cm.

(D) From the given data, $a\sqrt{3} = 8\sqrt{3} \Rightarrow a = 8$ cm.

(a) ACBD

(b) CADB

(c) ABCD

(d) CBAD

49. The length of the diagonals of a rhombus are 10 cm and 24 cm. Find its side. (in cm)

(a) 10 (b) 13

(d) 12 (d) 11

50. Area of a right-angled triangle is 6 cm^2 and its perimeter is 12 cm. Find its hypotenuse. (in cm)

(a) 5 (b) 6

(c) 7 (d) 8

51. A largest possible right-circular cylinder is cut out from a wooden cube of edge 7 cm. Find the volume of the wood left over after cutting the cylinder. (in cu cm)

(a) 73.5 (b) 82.5

(c) 76 (d) 92

52. A solid sphere of radius 4 cm is melted and recast into 'n' solid hemispheres of radius 2 cm each. Find n.

(a) 32 (b) 16

(c) 8 (d) 4

53. Three small metallic cubes whose edges are in the ratio 3 : 4 : 5 are melted to form a big cube. If the diagonal of the cube so formed is 18 cm, then find the total surface area of the smallest cube. (in cm^2)

(a) 154 (b) 184

(c) 216 (d) 162

54. A solid hemisphere of radius 8 cm is melted and recast into x spheres of radius 2 cm each. Find x.

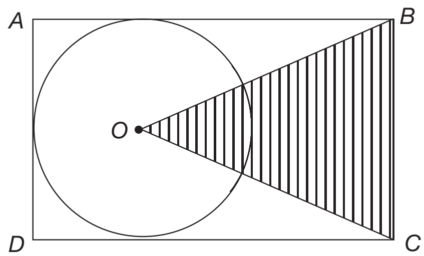
(a) 4 (b) 8

(c) 16 (d) 32

PRACTICE QUESTIONS

Level 3

55. In the following figure, $ABCD$ is a rectangle with $AB = 9$ cm and $BC = 6$ cm. O is the centre of the circle. Find the area of the shaded region. (in cm^2)



(a) 18 (b) 24
(c) 27 (d) 15

56. A circus tent is cylindrical upto a height of 8 m and conical above it. If its base diameter is 70 m and the slant height of the conical part is 50 m, the area of the canvas required to make the tent is ______ m^2.

(a) 6380 (b) 7260
(c) 6850 (d) 7460

57. A water tank of dimensions 11 m × 6 m × 5 m is full of water. The tank is emptied through a pipe of cross-section 33 cm^2 in 20 hours. Find the rate of flow of water. (in kmph)

(a) 2 (b) 5
(c) 6 (d) 10

58. P and Q are two cylinders having equal total surface areas. The radius of each cylinder is equal to the height of the other. The sum of the volumes of both the cylinders is 250π cm^3. Find the sum of their curved surface areas. (in cm^2)

(a) 80π (b) 100π
(c) 60π (d) 120π

59. In a triangle, the average of any two sides is 6 cm more than half of the third side. Find area of the triangle. (in sq. cm)

(a) $64\sqrt{3}$ (b) $48\sqrt{3}$
(c) $72\sqrt{3}$ (d) $36\sqrt{3}$

60. The lengths of the diagonals of a rhombus are 9 cm and 12 cm. Find the distance between any two parallel sides of the rhombus.

(a) 7.2 cm (b) 8 cm
(c) 7.5 cm (d) 6.9 cm

61. In a triangle, the sum of any two sides exceed the third side by 6 cm. Find its area (in sq. cm).

(a) $12\sqrt{3}$ (b) $9\sqrt{3}$
(c) $15\sqrt{3}$ (d) $18\sqrt{3}$

62. From each corner of a square sheet of side 8 cm, a square of side y cm is cut. The remaining sheet is folded into a cuboid. The minimum possible volume of the cuboid formed is M cubic cm. If y is an integer, then find M.

(a) 32 (b) 18
(c) 36 (d) 12

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $50\ cm^2$
2. 2 units
3. 6
4. 6
5. 6
6. 12
7. $\sqrt{2(p^2+q^2)}$
8. 12 sq. units
9. 3
10. equilateral
11. $\frac{1}{4}$
12. $40\ cm^2$
13. parallelogram, $13\ cm^2$
14. $\sqrt{4r^2+h^2}$
15. $\frac{\sqrt{3}}{12}a^2 \times h$
16. 3 : 2
17. $\frac{1}{3}$c.c
18. 72.8%
19. Sphere
20. False
21. 4 : 9
22. $\frac{2}{3}\pi x^3 - \frac{2}{3}\pi(x-y)^3$
23. 4.5 units
24. $(x^2 - y^2)\ cm^2$
25. $10\sqrt{3}\ cm^2$
26. $1944\ cm^3$
27. $3 \times 3 \times 3 = 27\ cm^3$
28. (i) $256\ cm^2$
    (ii) $8\sqrt{3}$ cm
29. 9
30. $880\ m^2$

Short Answer Type Questions

31. 5.2 cm
32. 33 cm
33. $18\pi\ m^2$
34. $8\ cm^2$
35. $24\ m^2$
36. $\sqrt{2}\,(a+b)$ units
37. $\frac{686}{11}cm^2$
38. $3360\ cm^3$
39. 385
40. 50%
41. $143.36\ cm^2$
42. $72\sqrt{3}\ cm^2$
43. $120\ cm^3$
44. $36\ cm^2$
45. $282.6\ m^2$

Essay Type Questions

46. ₹20,096
47. $1\frac{3}{7}$ ft
48. $38720\ cm^2$
49. $157\ m^2$
50. 3.5 cm

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (a)
11. (b)	**12.** (d)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (b)
21. (d)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (c)	

Level 2

30. (a)	**31.** (a)	**32.** (c)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (d)	**38.** (b)	**39.** (d)
40. (a)	**41.** (c)	**42.** (a)	**43.** (b)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (b)	**49.** (b)
50. (a)	**51.** (a)	**52.** (b)	**53.** (d)	**54.** (d)					

Level 3

55. (a)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (a)	**61.** (b)	**62.** (d)

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Use Pythagoras theorem.
2. Area of regular polygon $= \frac{1}{2} \times$ Perimeter of polygon $\times$ Height.
3. Length of the arc $= \frac{\theta}{360} \times 2\pi r$.
4. Sum of the lengths of the edges $= nh + 2s$, where h = height, s = perimeter of the base and n = number of sides of the base.
5. Volume of the larger cube = Sum of the volumes of smaller cubes.
6. Length of the diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$.
7. Use the formula to find volume of the cone.
8. Find the side of the square.
9. Find height of the cone.
10. Find $\frac{r_1}{r_2}$.
11. Find r by using the formula of volume of a sphere.
12. Volume of the regular octahedron = 2 × Volume of the pyramid.
13. Use Euler's formula.
14. Volume of the cylinder = Volume of 15 spheres.
15. Number of soaps $= \frac{\text{Volume of box}}{\text{Volume of each soap}}$.
16. Percentage increase in volume $= \frac{\text{Increase in volume}}{\text{Original volume}} \times 100$.
17. Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$.
18. CSA of hollow hemisphere $= 2\pi(R^2 + r^2)$.
19. Volume of tetrahedron $= \frac{1}{3}$(Area of base) × Height, where height of the tetrahedron $= \sqrt{\frac{2}{3}} \times$ Length of the edge.
20. TSA of the octahedron = 8 × Area of each face.
21. The number of the side of the base = The number of lateral edges.
22. Volume of the sphere = Volume of the cylinder.
23. (i) Find the number of sides n of the polygon.

    (ii) Number of diagonals $= \frac{n(n-3)}{2}$.
24. (i) ABD is a right triangle and its area is $\frac{1}{2} AB \cdot BD$.

    (ii) BCD is an isosceles triangle.

    (iii) $\angle ABD = 90°$. Find BD by using the basics of isosceles triangle (i.e., ΔBDC).
25. (i) $l = \frac{2A}{r}$.

    (ii) Use the formula, $A = \frac{(lr)}{2}$.
26. (i) LSA = 288 cm^2.

    (ii) Take perimeter of the base as $3a$ units and find a side.

    (iii) LSA of prism is half the product of base, perimeter and slant height.

    (iv) TSA = LSA + 2(Area of the base).

    (v) Equate the given value to the above formula and find h.
27. (i) Edge of the cube = Diameter of the sphere.

    (ii) Radius of sphere is 3.5 cm, edge of the cube is 2(3.5 cm), i.e., 7 cm.

    (iii) Difference between their surface areas $= 4\pi r^2 - 6s^2$.
28. (i) Height of regular octahedron = 2(Height of regular tetrahedron).

    (ii) Height of the octahedron $(h) = \sqrt{2}\, a$.
29. Number of bottles $= \frac{\text{Volume of container}}{\text{Volume of each bottle}}$.

Level 2

30. (i) Radius of the circle is one-third of the median of the triangle.

(ii) Inradius is 2 cm.

(iii) Height of the equilateral triangle = 3(2 cm) = 6 cm.

(iv) Area of an equilateral triangle $= \dfrac{h^2}{\sqrt{3}}$.

31. (i) Circum-radius is half of the hypotenuse. Use Pythagoras's theorem.

(ii) Circum-radius is 10 cm, so hypotenuse is 2(10 cm).

(iii) Use Pythagoras's theorem and get the length of the other perpendicular side and find the area.

32. (i) Given parallelogram is a rhombus.

(ii) As adjacent sides are equal, parallelogram becomes rhombus.

(iii) Sum of the squares of diagonals is equal to four times the square of the side.

33. (i) Find the number of students.

(ii) Number of students $= \dfrac{\text{Area of the floor}}{0.5}$.

(iii) The amount of air each student would get $= \dfrac{\text{Volume of the room}}{\text{Number of students}}$.

34. (i) Volume of prism = (Area of the base)(Height).

(ii) Area of equilateral triangle $= \dfrac{\sqrt{3}a^2}{4}$.

(iii) 1 dm = 10 cm.

35. (i) Edge (a) of small cube is to be found by using $6a^2 = 54$.

(ii) Number of small cubes $= \dfrac{\text{Volume of the big cube}}{\text{Volume of each small cube}}$.

36. (i) Use relevant formula.

(ii) Given condition is $8\pi r^2 = 4\pi rh$.

(iii) Substitute $h = 8$ cm in the above equation and solve for r.

37. (i) Volume $= \dfrac{1}{3}$(Area of base)(Height).

(ii) Area of square $= s^2$.

(iii) Volume of the pyramid $= \dfrac{1}{3}$ (Area of the base) (height).

38. (i) Base radius, vertical height and slant height form a right triangle with angles 30°, 60° and 90°.

(ii) Base radius (14 cm) is opposite to 30°.

(iii) Cost of canvas = LSA × Cost/m^2.

39. (i) Draw the figure and proceed.

(ii) The required area is equal to the sum of the areas of a sector of an angle 270° and radius 21 m, a sector of an angle 90° and radius 7 m and a sector of an angle 90° and radius 14 m.

(iii) Area of sector $= \dfrac{\theta}{360°}(\pi r^2)$.

40. $l(b + 5) = lb + 25$ and $b(l + 5) = lb + 20$.

41. Required volume is the sum of the volumes of conical part and cylindrical part.

42. (i) Volume of torus $= 27\pi r^2 a$.

(ii) Given, $r = 7$ cm and $a = 20$ cm.

(iii) Volume of the tyre $= 2\pi r^2 a$.

43. (i) Height of the regular tetrahedron, $h = \sqrt{\dfrac{2}{3}}s$.

(ii) Square the above equation and get s^2.

44. (i) Volume of frustum $= \dfrac{1}{3}\pi h(R^2 + r^2 + Rr)$.

(ii) Volume of cone frustum $= \dfrac{\pi h}{3}(R^2 + Rr + r^2)$.

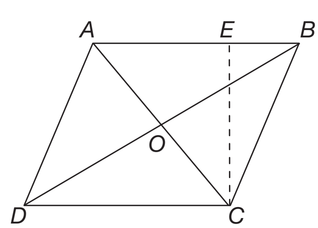
45. DBCA is the required sequential order.

46. ABCD is the required sequential order.

47. BCAD is the required sequential order.

48. CADB is the required sequential order.

49.



Let $ABCD$ be a rhombus.

Let CE be the height of the rhombus.

Given $AC = 10$ cm and $BD = 24$ cm.

$BC = \sqrt{OC^2 + OB^2} = \sqrt{5^2 + 12^2} = 13$ cm.

50. Let the sides of the triangle (in cm) be a, b and c, where $a < b < c$.

$\therefore$ c is the hypotenuse and a and b are the perpendicular sides.

Given, $a + b + c = 12$ (1)

$\frac{1}{2}(a)\ (b) = 6$ (2)

From Eq. (1) $\Rightarrow a + b = 12 - c$

Squaring both sides,

$a^2 + b^2 + 2ab = 12^2 - 24c + c^2$

$= a^2 + b^2 + 4\left(\frac{1}{2}ab\right) = 12^2 - 24c + c^2$

$\Rightarrow c^2 + 4(6) = 12^2 - 24c + c^2$

$c = 5$.

51. Diameter of the cylinder = 7 cm.

$\therefore$ Radius of the cylinder = 3.5 cm.

Height of the cylinder = 7 cm.

Volume of cube = $7^3 = 343$ cm^3.

Volume of the cylinder $\Rightarrow \pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5$

$\times\ 7 = 269.5$.

$\therefore$ The required volume $= 343 - 269.5 = 73.5$ cm^3.

52. Given that, $n \times \frac{2}{3}\pi(2)^3 = \frac{4}{3}\pi(4)^3 \Rightarrow n = 16$.

53. Let the edges of the cubes be $3x$, $4x$ and $5x$ respectively. Volume of the cube formed = Sum of the volumes of the small cubes = $(3x)^3 + (4x)^3 + (5x)^3 = 216x^3$.

$\therefore$ Edges of the cube formed = $6x$.

Diagonal of the cube formed $= \sqrt{3}(6x) \Rightarrow 6\sqrt{3}x = 18 \Rightarrow x = \sqrt{3}$.

$\therefore$ TSA of smallest cube $= 6a^2 = 6(3\sqrt{3})^2$

$= 162$ cm^2.

54. Given that, $x \times \frac{4}{3}\pi(2)^3 = \frac{2}{3}\pi(8)^3 \Rightarrow x = 32$.

Level 3

59. Let the sides of the triangle be a, b and l.

Given, $\frac{a+b}{2} = 6 + \frac{c}{2}, \frac{a+c}{2} = 6 + \frac{b}{2}$ and $\frac{b+c}{2}$

$= 6 + \frac{a}{2}$.

$\Rightarrow a + b = 12 + c$ (1)

$b + c = 12 + a$ (2)

$c + a = 12 + b$ (3)

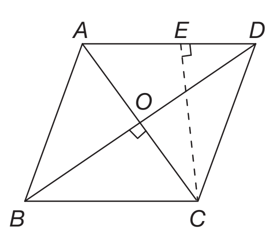
By solving, we get

$a = b = c = 12$

$\Rightarrow$ Area of $\Delta ABC = \frac{\sqrt{3}}{2} \times 12^2$

$= 36\sqrt{3}$ sq. cm.

60.



In ΔBOC, $BO^2 + OC^2 = BC^2$

$\Rightarrow 6^2 + (4.5)^2 = BC^2 \Rightarrow BC = 7.5$ cm.

Let CE be the required distance.

Area of rhombus $= \frac{12 \times 9}{2} = 54$ sq. cm

$\Rightarrow \frac{CE}{2}(AB + CD) = 54$ (Area of parallelogram)

$\Rightarrow CE = \frac{108}{15} = 7.2$ cm.

61. Let the sides of the triangle be a cm, b cm and c cm.

$a + b - c = a + c - b = b + c - a = 6$

$\Rightarrow a + b - c + a + c - b + b + c - a = 18$

$a + b + c = 18$.

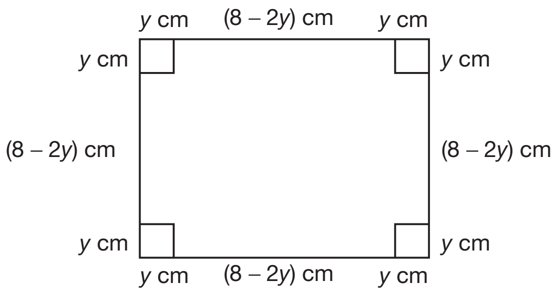
Area of the triangle =

$$\sqrt{\left(\frac{a+b+c}{2}\right)\left(\frac{a+b+c}{2} - a\right)\left(\frac{a+b+c}{2} - b\right)\left(\frac{a+b+c}{2} - c\right)}$$

$$= \sqrt{\left(\frac{a+b+c}{2}\right)\left(\frac{b+c-a}{2}\right)\left(\frac{a+c-b}{2}\right)\left(\frac{a+b-c}{2}\right)}$$

$$= \frac{1}{4}\sqrt{(18)(6)(6)(6)} = 9\sqrt{3}\text{ cm}^2.$$

62.



Length = Breadth = $(8 - 2y)$ cm and height = y cm.

Its volume $= (8 - 2y)(8 - 2y)y$

$= (8 - 2y)^2\, y$ cubic cm.

$8 - 2y > 0$, i.e., $y < 4$ and y is an integer.

$\therefore$ $y = 1$ or 2 or 3.

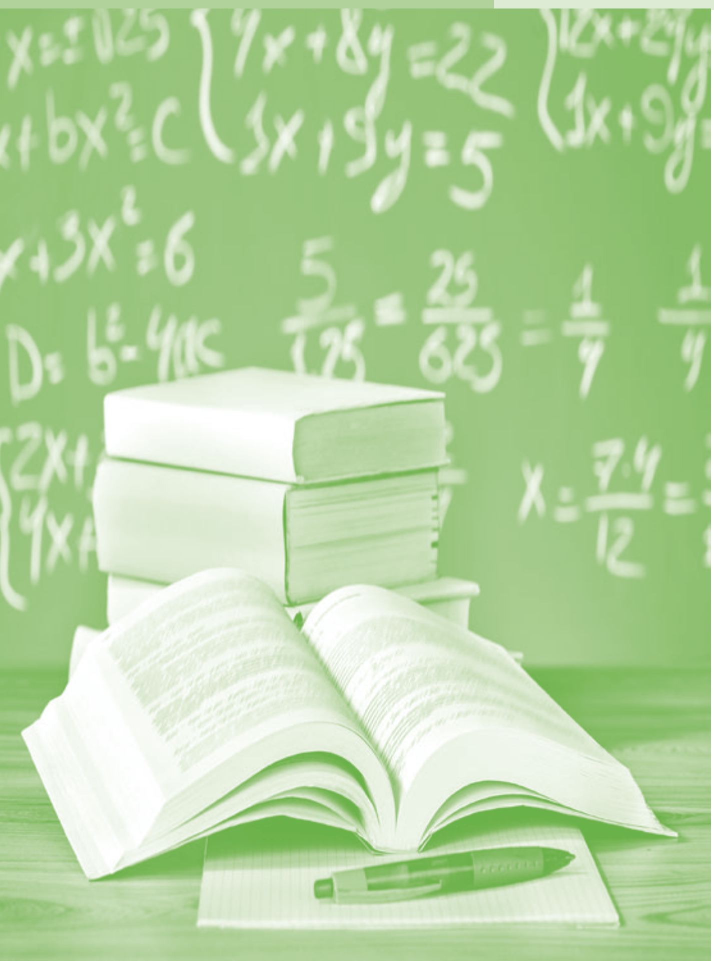
Among these values of y, volume is minimum when $y = 3$. When $y = 3$, volume = 12 cm^3.

$\therefore$ $M = 12$.

Chapter

14

Coordinate Geometry



REMEMBER

Before beginning this chapter, you should be able to:

- Know planes, lines and angles
- Remember different types of triangles and polygons

KEY IDEAS

After completing this chapter, you should be able to:

- Find the coordinates of a point and conversion of signs
- Study about points on a plane and the distance between points
- Know the applications of distance formula, mid-point of a line segment and centroid of a triangle
- Learn about equation of some standard lines

INTRODUCTION

Let $X'OX$ and YOY' be two mutually perpendicular lines intersecting at point O in a plane.

These two lines are called reference lines or coordinate axes. The horizontal reference line $X'OX$ is called X-axis and the vertical reference line YOY' is called Y-axis.

The point of intersection of these two axes, i.e., O is called the origin. The plane containing the coordinate axes is called coordinate plane or XY-plane.

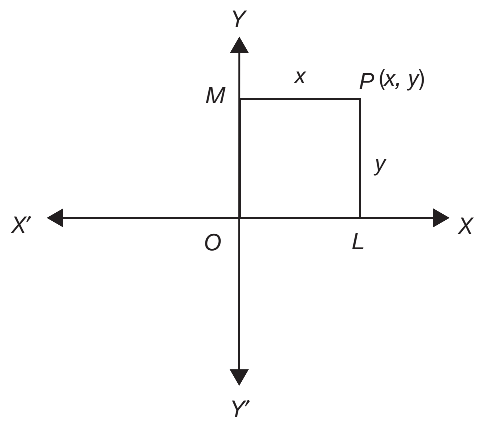


Figure 14.1

COORDINATES OF A POINT

Let P be a point in the XY-plane. Draw perpendiculars PL and PM to X-axis and Y-axis respectively (see Fig. 14.1).

Let $PL = y$ and $PM = x$. The point P is taken as (x, y). Here, x and y are called the rectangular Cartesian coordinates or coordinates of the point P. x is called x-coordinate or abscissa and y is called y-coordinate or ordinate of the point P.

Convention of Signs

1. Towards the right side of the Y-axis, x-coordinate of any point on the graph paper is taken positive and towards the left side of the Y-axis, x-coordinate is taken negative.
2. Above the X-axis, the y-coordinate of any point on the graph paper is taken positive and below the X-axis, y-coordinate is taken negative.

If (x, y) is a point in the plane and Q_1, Q_2, Q_3 and Q_4 are the four quadrants of rectangular coordinate system, then:

1. If $x > 0$ and $y > 0$, then $(x, y) \in Q_1$.
2. If $x < 0$ and $y > 0$, then $(x, y) \in Q_2$.
3. If $x < 0$ and $y < 0$, then $(x, y) \in Q_3$.
4. If $x > 0$ and $y < 0$, then $(x, y) \in Q_4$.

EXAMPLE 14.1

If $x > 0$ and $y < 0$, then $(x, -y)$ lies in which quadrant?

SOLUTION

$y < 0$

$\Rightarrow -y > 0$

$\therefore$ The point $(x, -y)$ lies in first quadrant, i.e., Q_1.

EXAMPLE 14.2

If $(x, -y) \in Q_2$, then (x, y) belongs to which quadrant?

SOLUTION

Given, $(x, -y) \in Q_2 \Rightarrow x < 0,\ y < 0$.

$\therefore$ (x, y) belongs to third quadrant, i.e., Q_3.

EXAMPLE 14.3

Plot the points $A(2, 3)$, $B(-1, 2)$, $C(-3, -2)$ and $D(4, -2)$ in the XY-plane.

SOLUTION

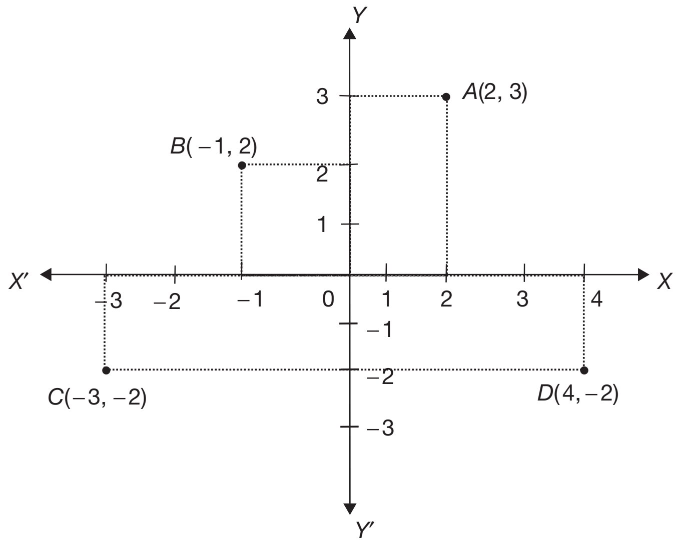


Figure 14.2

POINTS ON THE PLANE

Point on X-axis and Y-axis

Let P be a point on X-axis, so that its distance from X-axis is zero. Hence, point P can be taken as $(x, 0)$.

Let P' be a point on Y-axis, so that its distance from Y-axis is zero. Hence, point P' can be taken as $(0, y)$ (see Fig. 14.3).

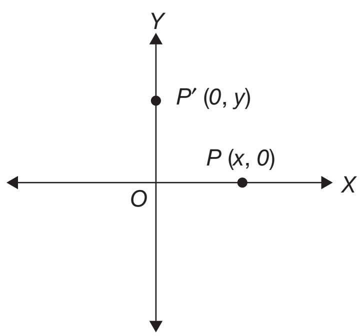


Figure 14.3

Distance Between Two Points

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$. Draw perpendiculars AL and BM from A and B to X-axis. AN is the perpendicular drawn from A on to BM (see Fig. 14.4).

From right triangle ABN, $AB = \sqrt{AN^2 + BN^2}$, (we have $AB^2 = AN^2 + BN^2$).
Here, $AN = x_2 - x_1$ and $BN = y_2 - y_1$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Hence, the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ units.}$$

Note The distance of a point $A(x_1, y_1)$ from origin $O(0, 0)$ is $OA = \sqrt{x_1^2 + y_1^2}$.

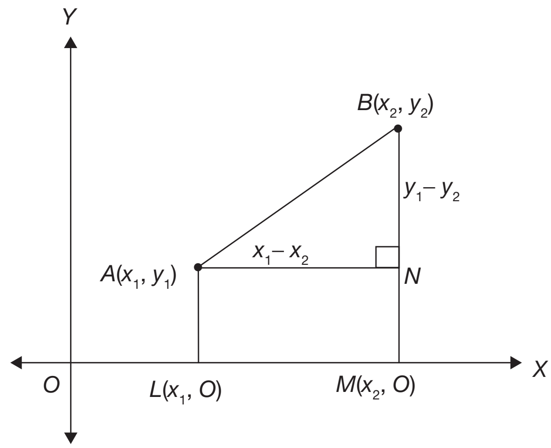


Figure 14.4

EXAMPLE 14.4

Find the distance between points (–4, 5) and (2, –3).

SOLUTION

Let the given points be $A(-4, 5)$ and $B(2, -3)$

$$AB = \sqrt{(2-(-4))^2 + (-3-5)^2}$$

$$= \sqrt{36+64} = 10 \text{ units.}$$

EXAMPLE 14.5

Find a, if the distance between points $A(8, -7)$ and $B(-4, a)$ is 13 units.

SOLUTION

Given, $AB = 13$

$$\Rightarrow \sqrt{(-4-8)^2 + (a+7)^2} = 13$$

Taking squares on both sides, we get

$$(a+7)^2 = 169 - 144 = 25$$

$$a + 7 = \sqrt{25}$$

$$a + 7 = \pm 5$$

$$\therefore\ a = -2 \text{ or } -12.$$

EXAMPLE 14.6

Find the coordinates of a point on Y-axis which is equidistant from points (13, 2) and (12, –3).

SOLUTION

Let $P(0, y)$ be the required point and the given points be $A(12, -3)$ and $B(13, 2)$.

Then, $PA = PB$ (given)

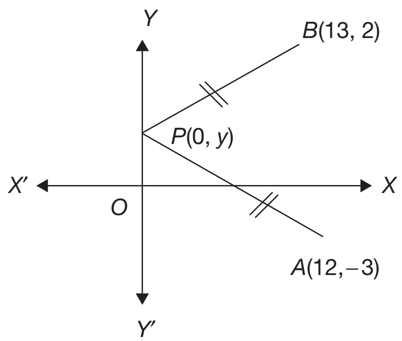


Figure 14.5

$$\sqrt{(12-0)^2+(-3-y)^2}=\sqrt{(13-0)^2+(2-y)^2}$$
$$\Rightarrow \sqrt{144+(y+3)^2}=\sqrt{169+(2-y)^2}$$

Taking squares on both sides, we get

$$169+4+y^2-4y=144+9+y^2+6y$$
$$\Rightarrow 10y=20 \Rightarrow y=2$$

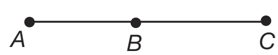
$\therefore$ The required point on Y-axis is (0, 2).

Collinearity of Three Points

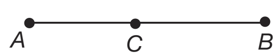
Let A, B and C be three given points. The distances AB, BC and CA can be calculated using distance formula. If the sum of any two of these distances is found to be equal to the third distance, then points A, B and C are said to be collinear.

Notes

1. If $AB + BC = AC$, then points A, B and C are collinear.



2. If $AC + CB = AB$, then points A, C and B are collinear.



3. $BA + AC = BC$, then points B, A and C are collinear.

By Notes (1), (2) and (3), we can find the position of the points in collinearity.

Applications of Distance Formula

EXAMPLE 14.7

Show that points $P(5, 6)$, $Q(4, 5)$ and $R(3, 4)$ are collinear.

SOLUTION

Given, $P = (5, 6)$, $Q = (4, 5)$ and $R = (3, 4)$.

$PQ=\sqrt{(4-5)^2+(5-6)^2}=\sqrt{2}$ units.

$QR = \sqrt{(3-4)^2 + (4-5)^2} = \sqrt{2}$ units.

$PR = \sqrt{(3-5)^2 + (4-6)^2} = \sqrt{8} = 2\sqrt{2}$ units.

Now, $PQ + QR = \sqrt{2} + \sqrt{2} = 2\sqrt{2} = PR$.

That is, $PQ + QR = PR$.

Hence, points P, Q and R are collinear.

EXAMPLE 14.8

Show that points $A(3, -1)$, $B(-1, 2)$ and $C(6, 3)$ form an isosceles right-angled triangle when joined.

SOLUTION

Given, $A = (3, -1)$, $B = (-1, 2)$ and $C = (6, 3)$.

$AB = \sqrt{(-1-3)^2 + (2+1)^2} = 5$ units

$BC = \sqrt{(6-(-1))^2 + (3-2)^2} = \sqrt{50}$ units

$AC = \sqrt{(6-3)^2 + (3-(-1))^2} = 5$ units

Clearly,

$BC^2 = AB^2 + AC^2$.

Also, $AB = AC$.

Hence, the given points form the vertices of a right-angled isosceles triangle.

EXAMPLE 14.9

Show that points $(2-\sqrt{3}, \sqrt{3}+1)$, $(1, 0)$ and $(3, 2)$ form an equilateral triangle.

SOLUTION

Let $A(2-\sqrt{3}, \sqrt{3}+1)$, $B(1, 0)$ and $C(3, 2)$ be the given points.

$$AB = \sqrt{(1-2+\sqrt{3})^2 + \left(0-(\sqrt{3}+1)\right)^2}$$

$$= \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}.$$

$AB = \sqrt{8}$ units.

$$BC = \sqrt{(3-1)^2 + (2-0)^2} = \sqrt{8} \text{ units}$$

$$AC = \sqrt{\left(3-(2-\sqrt{3})\right)^2 + \left(2-(\sqrt{3}+1)\right)^2}$$

$$= \sqrt{(1+\sqrt{3})^2 + (1-\sqrt{3})^2} = \sqrt{8} \text{ units.}$$

$\therefore AB = BC = AC = \sqrt{8}$ units.

Hence, the given points form an equilateral triangle.

EXAMPLE 14.10

Show that points $A(-1, 0)$, $B(-2, 1)$, $C(1, 3)$ and $D(2, 2)$ form a parallelogram.

SOLUTION

Given, $A(-1, 0)$, $B(-2, 1)$, $C(1, 3)$ and $D(2, 2)$.

$AB = \sqrt{(-2+1)^2 + (1-0)^2} = \sqrt{2}$ units

$BC = \sqrt{(1-(-2))^2 + (3-1)^2} = \sqrt{13}$ units

$CD = \sqrt{(2-1)^2 + (2-3)^2} = \sqrt{2}$ units

$DA = \sqrt{(2-(-1))^2 + (2-0)^2} = \sqrt{13}$ units

$AC = \sqrt{(1-(-1))^2 + (3-0)^2} = \sqrt{13}$ units

$BD = \sqrt{(2-(-2))^2 + (2-1)^2} = \sqrt{17}$ units

Clearly,

$AB = CD$, $BC = DA$ and $AC \neq BD$.

That is, the opposite sides of the quadrilateral are equal and diagonals are not equal.

Hence, the given points form a parallelogram.

EXAMPLE 14.11

Find the circum-centre and the circum-radius of a triangle ABC formed by the vertices $A(2, -2)$, $B(-1, 1)$ and $C(3, 1)$.

SOLUTION

Let $S(x, y)$ be the circum-centre of ΔABC.

$\therefore SA^2 = SB^2 = SC^2$

Consider, $SA^2 = SB^2$

$$\begin{aligned} \Rightarrow (x-2)^2 + (y+2)^2 &= (x+1)^2 + (y-1)^2 \\ x^2 - 4x + 4 + y^2 + 4y + 4 &= x^2 + 2x + 1 + y^2 - 2y + 1 \\ -4x + 4y + 8 &= 2x - 2y + 2 \\ 6x - 6y - 6 &= 0 \\ x - y - 1 &= 0 \qquad (1) \end{aligned}$$

$SB^2 = SC^2$

$$\begin{aligned} \Rightarrow (x+1)^2 + (y-1)^2 &= (x-3)^2 + (y-1)^2 \\ x^2 + 2x + 1 + y^2 - 2y + 1 &= x^2 - 6x + 9 + y^2 - 2y + 1 \\ 2x - 2y + 2 &= -6x - 2y + 10 \\ 8x - 8 &= 0 \\ \Rightarrow x &= 1. \end{aligned}$$

Substituting $x = 1$ in Eq. (1), we get $y = 0$.

$\therefore$ The required circum-centre of ΔABC is $(1, 0)$.

Circum-radius, $SA = \sqrt{(2-1)^2 + (-2-0)^2} = \sqrt{5}$ units.

EXAMPLE 14.12

Find the area of the circle whose centre is (–1, –2), and (3, 4) is a point on the circle.

SOLUTION

Let the centre of the circle be $A(-1, -2)$, and the point on the circumference be $B(3, 4)$.

Radius of circle = AB

$$= \sqrt{(3-(-1))^2 + (4-(-2))^2} = \sqrt{52} \text{ units.}$$

$\therefore$ The area of the circle $= \pi r^2$

$$= \pi(\sqrt{52})^2 = 52\pi \text{ sq. units.}$$

EXAMPLE 14.13

Find the area of the square whose one pair of opposite vertices are (2, –3) and (4, 5).

SOLUTION

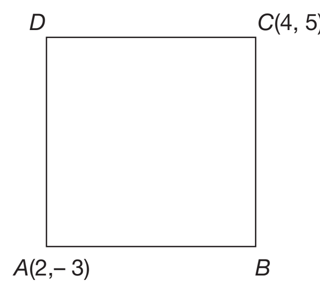


Figure 14.6

Let the given vertices be $A(2, -3)$ and $C(4, 5)$.

Length of $AC = \sqrt{(4-2)^2 + (5+3)^2}$

$$= \sqrt{68} \text{ units.}$$

$\therefore$ Area of the square $= \dfrac{AC^2}{2} = \dfrac{(\sqrt{68})^2}{2} = 34$ sq. units.

STRAIGHT LINES

Inclination of a Line

The angle made by a straight line with positive direction of X-axis in the anti-clockwise direction is called its inclination.

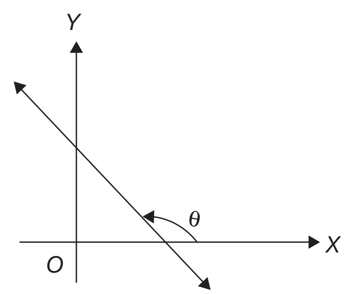


Figure 14.7

Slope or Gradient of a Line

If θ is the inclination of a line L, then its slope is denoted by m and is given by $m = \tan\theta$ (see Fig. 14.7).

Example: The inclination of the line l in adjacent Fig. 14.8 is 45°.

$\therefore$ The slope of the line is $m = \tan 45° = 1$.

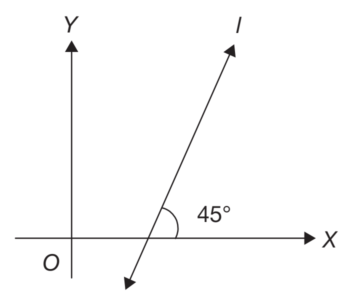


Figure 14.8

Example: The line L in Fig. 14.9 makes an angle of 45° in clockwise direction with X-axis. So, the inclination of the line L is 180° – 45° = 135°.

$\therefore$ The slope of the line L is $m = \tan 135° = -1$.

Some Results on the Slope of a Line

1. The slope of a horizontal line is zero. Hence,

- **(i)** Slope of X-axis is zero.
- **(ii)** Slope of any line parallel to X-axis is also zero.

2. The slope of a vertical line is not defined. Hence,
   (i) Slope of Y-axis is undefined.
   (ii) Slope of any line parallel to Y-axis is also undefined.

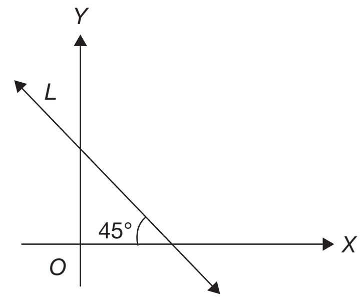


Figure 14.9

Theorem 1 Two non-vertical lines are parallel, if and only if, their slopes are equal.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2 respectively. If θ_1 and θ_2 are the inclinations of the lines, L_1 and L_2 respectively, then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$, since $L_1 \parallel L_2$ .Then, $\theta_1 = \theta_2$ (see Fig. 14.10).

($\because$ They form a pair of corresponding angles)

$$\Rightarrow \tan\theta_1 = \tan\theta_2$$
$$\Rightarrow m_1 = m_2$$

Conversely: Let $m_1 = m_2$

$$\Rightarrow \tan\theta_1 = \tan\theta_2$$
$$\Rightarrow \theta_1 = \theta_2$$
$$\Rightarrow L_1 \parallel L_2$$

($\because$ θ_1 and θ_2 form a pair of corresponding angles.)

Hence, two non-vertical lines are parallel, if and only if, their slopes are equal.

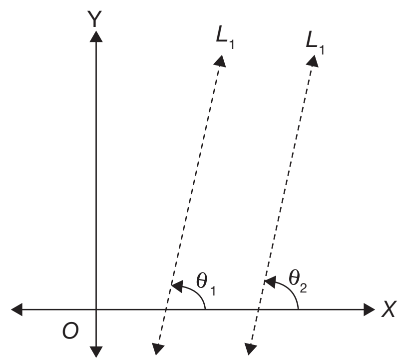


Figure 14.10

Theorem 2 Two non-vertical lines are perpendicular to each other, if and only if, the product of their slopes is -1.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2. If θ_1 and θ_2 are the inclinations of the lines L_1 and L_2 respectively, then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$ (see Fig. 14.11).

If $L_1 \perp L_2$, then

$$\theta_2 = 90° + \theta_1$$

($\because$ The exterior angle of a triangle is equal to the sum of two opposite interior angles.)

$$\Rightarrow \tan\theta_2 = \tan(90° + \theta_1)$$
$$\Rightarrow \tan\theta_2 = -\cot\theta_1$$
$$\Rightarrow \tan\theta_2 = \frac{-1}{\tan\theta_1} \quad [\because \theta_1 \neq 0]$$
$$\Rightarrow \tan\theta_1 \times \tan\theta_2 = -1$$
$$\therefore\ m_1 m_2 = -1.$$

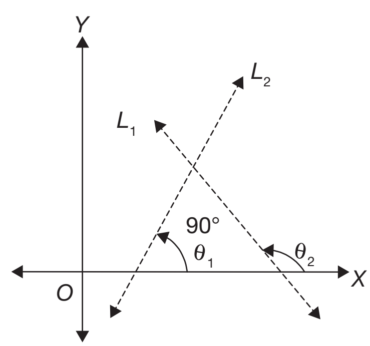


Figure 14.11

Conversely: Let $m_1 m_2 = -1$

$$\Rightarrow \tan\theta_1 \tan\theta_2 = -1$$
$$\Rightarrow \tan\theta_2 = \frac{-1}{\tan\theta_1} \quad [\because \theta_1 \neq 0]$$
$$\Rightarrow \tan\theta_2 = -\cot\theta_1$$
$$\Rightarrow \tan\theta_2 = \tan(90° + \theta_1)$$
$$\Rightarrow \theta_2 = 90° + \theta_1$$
$$\Rightarrow L_1 \perp L_2$$

Hence, two non-vertical lines are perpendicular to each other, if and only if, the product of their slopes is -1.

The Slope of a Line Passing through Points (x_1, y_1) and (x_2, y_2)

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points.

Let AB be the straight line passing through points A and B.

Let θ be the inclination of line $\overleftrightarrow{AB}$.

Draw perpendiculars AL and BM on to X-axis from A and B respectively. Also, draw $AN \perp BM$ (see Fig. 14.12).

Then, let $\angle NAB = \theta$.

Here, $BN = BM - MN = BM - AL = y_2 - y_1$

$AN = LM = OM - OL = x_2 - x_1$

$\therefore$ The slope of the line L is

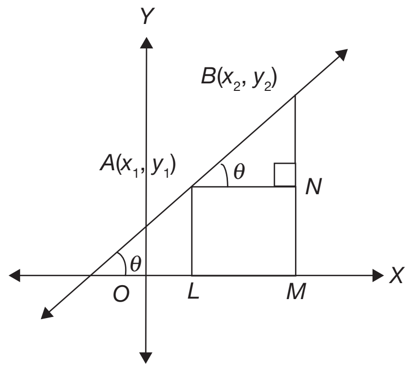


Figure 14.12

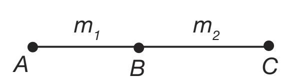
$$m = \tan\theta = \frac{BN}{AN} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Hence, the slope of a line passing through points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

The following table gives inclination (θ) of the line and its corresponding slope (m) for some particular values of θ.

θ	0°	30°	45°	60°	90°	120°	135°	150°
$m = \tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$

Note If points A, B and C are collinear, then the slope (m_1) of AB = the slope (m_2) of BC.



EXAMPLE 14.14

Find the slope of the line joining points (3, 8) and (–9, 6).

SOLUTION

Let $A(3, 8)$ and $B(-9, 6)$ be the given points.

Then, the slope of $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{6-8}{-9-3} = \frac{1}{6}.$$

EXAMPLE 14.15

Find the value of p if the slope of the line joining points $(5, -p)$ and $(2, -3)$ is $\frac{-1}{3}$.

SOLUTION

Let the given points be $A(5, -p)$ and $B(2, -3)$.

Given, the slope of $\overleftrightarrow{AB} = \frac{-1}{3}$.

That is, $\frac{-3-(-p)}{2-5} = \frac{-1}{3}$

$$\Rightarrow \frac{p-3}{-3} = \frac{-1}{3}$$

$$\Rightarrow p-3 = 1$$

$$\Rightarrow p = 4.$$

EXAMPLE 14.16

Find the value of k, if lines AB and CD are perpendicular, where $A = (4, 5)$, $B = (k + 2, -3)$, $C = (-3, 2)$ and $D = (2, 4)$.

SOLUTION

The slope of $\overleftrightarrow{AB}$ $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-3-5}{(k+2)-4} = \frac{-8}{k-2}.$$

Slope of $\overleftrightarrow{CD}$ $(m_2) = \frac{4-2}{2-(-3)} = \frac{2}{5}.$

Since, $AB \perp PQ \Rightarrow m_1 m_2 = -1$

That is, $\frac{-8}{k-2} \times \left(\frac{2}{5}\right) = -1$

$$\Rightarrow \frac{16}{5k-10} = 1$$

$$\Rightarrow 16 = -10 + 5k$$

$$\Rightarrow k = \frac{26}{5}.$$

EXAMPLE 14.17

Find the value of k, if points $(-2, -4)$, $(k, -2)$ and $(3, 4)$ are collinear.

SOLUTION

Let the given points be $A(-2, -4)$, $B(k, -2)$ and $C(3, 4)$.

The slope of $AB = \frac{-2+4}{k+2} = \frac{2}{k+2}.$

The slope of $BC = \frac{4+2}{3-k} = \frac{6}{3-k}$.

Since the points A, B and C are collinear,

The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$$\Rightarrow \frac{2}{k+2} = \frac{6}{3-k}$$
$$\Rightarrow 2(3-k) = 6(k+2)$$
$$\Rightarrow 3-k = 3k+6$$
$$\Rightarrow 4k = -3$$
$$\Rightarrow k = \frac{-3}{4}.$$

EXAMPLE 14.18

Find the ortho-centre of the ΔABC formed by vertices $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$.

SOLUTION

The given vertices of ΔABC are $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$.

Slope of $AB = \frac{2-6}{5-1} = \frac{-4}{4} = -1$

Slope of $BC = \frac{9-2}{12-5} = \frac{7}{7} = 1$

Slope of $AC = \frac{9-6}{12-1} = \frac{3}{11}$

Slope of $AB \times$ Slope of $BC = -1$

$\therefore AB \perp BC$

Hence, ABC is a right triangle, right angle at B.

Hence, ortho-centre is the vertex containing right angle, i.e., $B(5, 2)$.

Intercepts of a Straight Line

Say a straight line L meets X-axis in A and Y-axis in B.

Then, OA is called the x-intercept and OB is called the y-intercept (see Fig. 14.13).

Note OA and OB are taken as positive or negative, based on whether the line meets positive or negative axes.

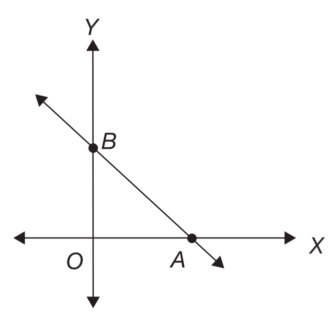


Figure 14.13

EXAMPLE 14.19

The line l in Fig. 14.14 meets X-axis at $A(-5, 0)$ and Y-axis at $B(0, -3)$.

SOLUTION

Hence, x-intercept $= -5$ and y-intercept $= -3$.

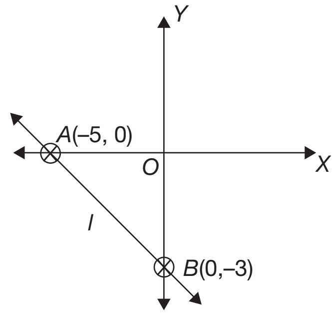


Figure 14.14

Equation of a Line in General Form

An equation of the form, $ax + by + c = 0$ (where $|a| + |b| \neq 0$, i.e., a and b are not simultaneously equal to zero), which is satisfied by every point on a line is called the equation of a line.

Equations of Some Standard Lines

Equation of X-axis

We know that the y-coordinate of every point on X-axis is zero. So, if $P(x, y)$ is any point on X-axis, then $y = 0$.

Hence, the equation of X-axis is $y = 0$.

Equation of Y-axis

We know that the x-coordinate of every point on Y-axis is zero. So, if $P(x, y)$ is any point on Y-axis, then $x = 0$.

Hence, the equation of Y-axis is $x = 0$.

Equation of a Line Parallel to X-axis

Let L be a line parallel to X-axis and at a distance of k units away from X-axis.

Then the y-coordinate of every point on the line L is k.

So, if $P(x, y)$ is any point on the line L, then $y = k$.

Hence, the equation of a line parallel to X-axis at a distance of k units from it, is $y = k$ (see Fig. 14.15).

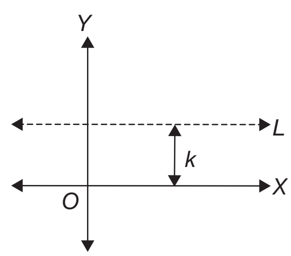


Figure 14.15

Note For the lines lying below X-axis, k is taken as negative.

Equation of a Line Parallel to Y-axis

Let L' be a line parallel to Y-axis and at a distance of k units away from it. Then the x-coordinate of every point on the line L' is k.

So, if $P(x, y)$ is any point on the line L', then $x = k$.

Hence, the equation of a line parallel to Y-axis and at a distance of k units from it, is $x = k$ (see Fig. 14.16).

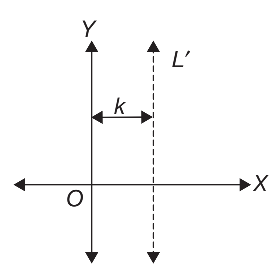


Figure 14.16

Note For the lines lying on the left side of Y-axis, k is taken as negative.

Oblique Line

A straight line which is neither parallel to X-axis nor parallel to Y-axis is called an oblique line or an inclined line.

Different Forms of Equations of Oblique Lines

Gradient Form (or) Slope Form The equation of a straight line with slope m and passing through the origin is given by $y = mx$.

Point–Slope Form The equation of a straight line passing through point (x_1, y_1) and with slope m is given by $y - y_1 = m(x - x_1)$.

Slope–intercept Form The equation of a straight line with slope m and having y-intercept as c is given by $y = mx + c$.

Note Area of triangle formed by the line $y = mx + c$ is $\frac{1}{2}\left|\frac{c^2}{m}\right|$ sq. units.

Two-point Form The equation of a straight line passing through points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad \text{or} \qquad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}.$$

Intercept Form The equation of a straight line with x-intercept as a and y-intercept as b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

Note Area of triangle formed by line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}|ab|$ sq. units.

EXAMPLE 14.20

Find the equation of the line parallel to Y-axis and passing though point $(5, -7)$.

SOLUTION

The equation of a line parallel to Y-axis is $x = k$.

Given, the line passes through point $(5, -7)$

$\Rightarrow k = 5$.

Hence, the equation of the required line is $x = 5$.

That is, $x - 5 = 0$.

EXAMPLE 14.21

Find the equation of the line passing through $(3, 4)$ and having a slope $\frac{4}{5}$.

SOLUTION

The equation of the line passing through (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$.

Hence, the equation of the required line is

$$y - 4 = \frac{4}{5}(x - 3)$$
$$5y - 20 = 4x - 12$$
$$4x - 5y + 8 = 0.$$

EXAMPLE 14.22

Find the equation of a line making intercepts 4 and 5 on the coordinate axes.

SOLUTION

Given, x-intercept $(a) = 4$ and y-intercept $(b) = 5$.

$\therefore$ The equation of the required line is $\frac{x}{a}+\frac{y}{b}=1$.

That is, $\frac{x}{4}+\frac{y}{5}=1$

$$\Rightarrow 5x + 4y - 20 = 0.$$

Equation of a Line Parallel or Perpendicular to the Given Line

Let $ax + by + c = 0$ be the equation of a straight line, then:

1. The equation of a line passing through point (x_1, y_1) and parallel to the given line:

   The slope of the required line (m) = The slope of $ax + by + c = 0$

   $$= \frac{-a}{b} \quad \text{(Since the lines are parallel)}$$

   $\therefore$ The required line is

   $$y - y_1 = m(x - x_1)$$
   $$\Rightarrow y - y_1 = \frac{-a}{b}(x - x_1)$$
   $$\Rightarrow b(y - y_1) = -a(x - x_1)$$
   $$\Rightarrow a(x - x_1) + b(y - y_1) = 0.$$

2. The equation of a line passing through point (x_1, y_1) and perpendicular to the given line:

   The slope of $ax + by + c = 0$ is $\frac{-a}{b}$.

   $\therefore$ The slope of the required line is $\frac{b}{a}$. (Since the line are perpendicular.)

   $\therefore$ The required line is

   $$(y - y_1) = m(x - x_1)$$
   $$\Rightarrow (y - y_1) = \frac{b}{a}(x - x_1)$$
   $$\Rightarrow b(x - x_1) - a(y - y_1) = 0.$$

EXAMPLE 14.23

Find the equation of a line passing through the point $P(-3, 2)$ and parallel to line $4x - 3y - 7 = 0$.

SOLUTION

Here, $(x_1, y_1) = (-3, 2)$, $a = 4$ and $b = 2$.

$\therefore$ Equation of the line passing through $P(-3, 2)$ and parallel to $4x - 3y - 7 = 0$.

$$a(x - x_1) + b(y - y_1) = 0$$
$$\Rightarrow 4(x + 3) - 3(y - 2) = 0$$
$$\Rightarrow 4x - 3y + 18 = 0.$$

Hence, the equation of the required line is $4x - 3y + 18 = 0$.

EXAMPLE 14.24

Find the equation of a line passing through point (–2, 3) and perpendicular to $7x + 2y + 3 = 0$.

SOLUTION

Here, $(x_1, y_1) = (-2, 3)$, $a = 7$ and $b = 2$.

$\therefore$ Equation of the line perpendicular to $7x + 2y + 3 = 0$ and passing through (–2, 3) is $b(x - x_1) - a(y - y_1) = 0$.

That is, $2(x + 2) - 7(y - 3) = 0$

$$\Rightarrow 2x - 7y + 25 = 0.$$

Hence, the required equation of the line is $2x - 7y + 25 = 0$.

EXAMPLE 14.25

The line $(8x + 3y - 15) + \lambda(3x - 8y + 2) = 0$ is parallel to X-axis. Find λ.

SOLUTION

The given line is $(8x + 3y - 15) + \lambda(3x - 8y + 2) = 0$.

That is, $x(8 + 3\lambda) + y(3 - 8\lambda) + (2\lambda - 15) = 0$.

Since the given line is parallel to X-axis, its slope = 0.

$$\frac{-(8+3\lambda)}{3-8\lambda} = 0$$
$$\Rightarrow 8 + 3\lambda = 0$$

Hence, $\lambda = \frac{-8}{3}$.

EXAMPLE 14.26

The equation of the line passing through the point of intersection of lines $2x - y + 3 = 0$ and $3x + y + 7 = 0$ and perpendicular to $2x - 3y + 4 = 0$, is _________.

(a) $3x + 2y - 7 = 0$ **(b)** $3x + 2y + 8 = 0$ **(c)** $3x + 2y - 8 = 0$ **(d)** $3x - 2y + 1 = 0$

HINTS

(i) Find m and the intersection point. Then use slope-point form.

(ii) Find the common point (x_1, y_1) of first two equations.

(iii) Find the slope (m) of third line.

(iv) Find the equation of the line passing through (x_1, y_1) and having slope $\left(-\frac{1}{m}\right)$.

EXAMPLE 14.27

The area of the figure formed by $|x| + |y| = 2$ is______. (in sq. units)

(a) 2 **(b)** 4 **(c)** 6 **(d)** 8

HINTS

(i) Plot the figure.

(ii) Find the intercepts made by given line.

(iii) If the intercepts are a and b, then the area of the triangle is $\frac{|ab|}{2}$.

EXAMPLE 14.28

The sum of the reciprocals of the intercepts of a line is $\frac{1}{2}$, then the line passes through the point is______.

(a) (1, 1) **(b)** (2, 1) **(c)** $\left(\frac{1}{4}, \frac{1}{4}\right)$ **(d)** (2, 2)

HINTS

(i) Use $\frac{x}{a} + \frac{y}{b} = 1$.

(ii) Solve, $\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ and get the relation between a and b.

(iii) Use the formula $\frac{x}{a} + \frac{y}{b} = 1$.

Mid-point

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points and M be the mid-point of AB.

Then, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Hence, the coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

EXAMPLE 14.29

Find the mid-point of the line segment joining the points (2, −6) and (6, −4).

SOLUTION

Let $A(2, -6)$ and $B(6, -4)$ be the given points and M be the mid-point of AB.

Then,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2 + 6}{2}, \frac{-6 + (-4)}{2}\right) = (4, -5).$$

Hence, the mid-point of AB is (4, −5).

Centroid

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC, and G be its centroid. Then, the coordinates of G are given by, $G = \left(\dfrac{x_1 + x_2 + x_3}{2}, \dfrac{y_1 + y_2 + y_3}{2}\right)$.

EXAMPLE 14.30

Find the centroid of ΔABC whose vertices are $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$.

SOLUTION

Given, $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$.

So, centroid of ΔABC

$$\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right) = \left(\frac{2 + 4 - 3}{3}, \frac{-3 + 2 - 2}{3}\right) = (1, -1).$$

Hence, $(1, -1)$ is the centroid of ΔABC.

EXAMPLE 14.31

Find the third vertex of ΔABC, if two of its vertices are $A(-2, 3)$, $B(4, 5)$ and its centroid is $G(1, 2)$.

SOLUTION

Let $C(x, y)$ be the third vertex.

Given, centroid of $\Delta ABC = (1, 2)$

$$\Rightarrow \left(\frac{-2 + 4 + x}{3}, \frac{3 + 5 + y}{3}\right) = (1, 2)$$

$$\Rightarrow \left(\frac{x + 2}{3}, \frac{y + 8}{3}\right) = (1, 2)$$

$$\Rightarrow \frac{x + 2}{3} = 1, \frac{y + 8}{3} = 2$$

$$\Rightarrow x = 1, \; y = -2.$$

$\therefore$ The third vertex is $(1, -2)$.

Notes

1. If the mid-points of the sides BC, AC and AB of ΔABC, respectively, are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$, then its vertices are $A(-x_1 + x_2 + x_3, -y_1 + y_2 + y_3)$, $B(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$ and $C(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$.
2. The fourth vertex of a parallelogram whose three consecutive vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) when taken in order is $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.

EXAMPLE 14.32

Find the fourth vertex of the parallelogram whose three consecutive vertices are (8, 8), (6, 1) and (–1, 1).

SOLUTION

Let the three vertices of the parallelogram be $A(8, 8)$, $B(6, 1)$ and $C(-1, 1)$, then fourth vertex $D(x, y)$ is given by

$$\begin{aligned} D(x, y) &= (x_1 - x_2 + x_3, y_1 - y_2 + y_3) \\ &= (8 - 6 - 1, 8 - 1 + 1) \\ &= (1, 8). \end{aligned}$$

Hence, the fourth vertex is $D(1, 8)$.

EXAMPLE 14.33

If the centroid of a triangle is (6, 6) and its ortho-centre is (0, 0), then find its circum-centre.

(a) (3, 3) **(b)** (6, 6) **(c)** (9, 9) **(d)** (12, 12)

SOLUTION

Ortho-centre, centroid and circum-centre are collinear.

We know that centroid divides the line segment joining the ortho-centre, centroid and circum-centre ($\overline{OGS}$) in the ratio 2 : 1 from the ortho-centre (O).

Let $S(x, y)$, $G(6, 6)$ and $O(0, 0)$

$$(6, 6) = \left(\frac{2x \times 1 \times 0}{2+1}, \frac{2 \times y \times 0}{2+1}\right)$$

$$(6, 6) = \left(\frac{2x}{3}, \frac{2y}{3}\right)$$

$$\Rightarrow \frac{2x}{3} = 6, \frac{2y}{3} = 6$$

$$\Rightarrow x = 6 \text{ and } y = 9.$$

∴ The circum-centre is (9, 9).

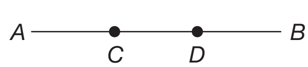
EXAMPLE 14.34

$C(3, 0)$ and $D(3, 1)$ are the points of trisection of a line segment AB. Find the respective coordinates of A and B.

(a) (3, 2), (3, 0) **(b)** (3, –1), (3, 2) **(c)** (–3, 1), (3, 2) **(d)** None of these

SOLUTION

Let A and B be (a_1, b_1) and (a_2, b_2). Given, $C(3, 0)$ and $D(3, 1)$ are the points of trisection of AB.



$\Rightarrow$ C is the mid-points of AD and D is the mid-points of CB.

$\Rightarrow (3, 0) = \left(\frac{a_1 + 3}{2}, \frac{b_1 + 1}{2}\right)$

$\Rightarrow a_1 = 3$ and $b_2 = -1$.

Also, $(3, 1) = \left(\frac{3 + a_2}{2}, \frac{0 + b_2}{2}\right)$

$\Rightarrow a_2 = 3$ and $b_2 = 2$.

$\therefore$ The coordinates of A and B are $(3, -1)$ and $(3, 2)$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If $x > 0$ and $y < 0$, then the point $(x, -y)$ lies in _______ quadrant.
2. Which point among (2, 3), (–3, –4) and (1, –7) is nearest to the origin?
3. The lines $2y + 3 = 0$ and $x = 3$ intersect at _______.
4. The points (0, 0), (0, 4) and (4, 0) form a/an _______ triangle.
5. A linear equation in two variables is always a _______.
6. The slope of the line $ax + by + c = 0$ is _______.
7. If (x, y) represents a point and $|x| > 0$ and $y < 0$, then in which quadrant(s) can the point lie?
8. The equation of a line parallel to Y-axis and passing through (–3, –4) is _______.
9. The slope of line perpendicular to the line joining points (2, 3) and (–2, 5) is _______.
10. The slope of altitude from A to BC of triangle $A(2, 3)$, $B(-3, 2)$ and $C(3, 5)$ is _______.
11. If the line $\frac{x}{a} + \frac{y}{b} = m$ passes through origin, then the value of m is _______.
12. If (x, y) represents a point and $xy > 0$, then the point may lie in _______ or _______quadrant.
13. The slope-intercept form of the line $2x + 3y + 5 = 0$ is _______.
14. The lines $3x + 2y + 7 = 0$ and $6x + 4y + 9 = 0$ are _______to each other.
15. The points $(p, q + r)$, $(q, r + p)$ and $(r, q + p)$ are _______.
16. The area of triangle formed by the line $y = mx + c$ with the coordinate axes is _______.
17. The points (2, 3), (–1, 5) and $(x, -2)$ form a straight line, then x is _______.
18. If the point (x, y) lies in the second quadrant, then x is _______ and y is _______.
19. The angle between lines $x = 5$ and $x = 7$ is _______.
20. The point of intersection of X-axis and $3x + 2y - 5 = 0$ is _______.
21. If $a = 0$, then the line $ax + by + c = 0$ is parallel to _______.
22. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other, then _______.
23. The point of intersection of X-axis and Y-axis is _______.
24. The line $y = k$ is parallel to _______ axis.
25. A, B and C are three points such that $AB = AC + CB$, then A, B and C are _______.
26. The line $ax + by + c = 0$ meets Y-axis at _______ point.
27. If slope of a line (l) is $\tan\theta$, then slope of a line perpendicular to (l) is _______.
28. The lines $x = 2$ and $y = -3$ intersect in _______ quadrant.
29. The slope of a line which is parallel to the line making an inclination of 45° with positive X-axis is _______.
30. If the slope of two lines are equal, then the lines are _______.

Short Answer Type Questions

31. Find the equation of a line passing through points $A(-2, 3)$ and $B(4, 7)$.
32. Find the area of the circle passing through (–2, 3) with centre (5, 2).
33. If $(2x + 3y + 1) + \lambda(x - 2y - 3) = 0$ represents the equation of a horizontal line, then find the value of λ.
34. Let $A(-3, 2)$, $B(4, 1)$ and $C(-2, k)$ be three points such that $AC = BC$. Find the value of k.
35. Find the distance between points (2, –3) and (4, 6).
36. If the line $2x - ky + 6 = 0$ passes through the point (2, –8), then find the value of k.

PRACTICE QUESTIONS

37. Find the area of square, whose diagonally opposite vertices are (–2, 3) and (4, 5).

38. If $A(a + b, a - b)$ and $B(-a + b, -a - b)$, then find the distance AB.

39. Find the intercepts made by the line $3x - 2y - 6 = 0$ on the coordinate axes.

40. Find the inclination of the line $\sqrt{3}x - 3y + 6 = 0$.

41. Find the circum-centre of the triangle whose vertices are $A(-3, -1)$, $B(1, 2)$ and $C(0, -4)$.

42. Find the equation of a line having inclination 60° and making an intercept of $\frac{-1}{3}$ on Y-axis.

43. Find the point on X-axis, which is equidistant from $A(6, 3)$ and $B(-1, 4)$.

44. Show that the points (–1, –1), (6, 1), (8, 8) and (1, 6), when joined in the given order form a rhombus.

45. Find the equation of a line, whose y-intercept is –5 and passes through point $A(-3, 2)$.

Essay Type Questions

46. Find the equations of the lines whose intercepts are the roots of the equation $4x^2 - 3x - 1 = 0$.

47. The equation of one of the diagonals of a rhombus is $3x + 4y - 7 = 0$. Find the equation of the other diagonal passing through (–1, –2).

48. Find the equation of the line passing through (–5, 11) and making equal intercepts, but opposite in magnitude on the coordinate axes.

49. Find the equations of a line which forms area 5 sq. units with the coordinate axes and having sum of intercepts is 7.

50. If points $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$ are three consecutive vertices of a parallelogram, then find the equation of the diagonal BD.

CONCEPT APPLICATION

Level 1

1. If (1, –3), (–2, –3) and (–2, 2) are the three vertices of a parallelogram taken in that order, then the fourth vertex is ______.

(a) (–1, –2) (b) (1, 2)
(c) (–1, 2) (d) (1, –2)

2. Find the equation of the line that passes through point (5, –3) and makes an intercept 4 on the X-axis.

(a) $3x - y + 12 = 0$ (b) $3x + y + 12 = 0$
(c) $3x - y - 12 = 0$ (d) $3x + y - 12 = 0$

3. The inclination of line $x - \sqrt{3}y + 1 = 0$ with the positive X-axis is ______.

(a) 60° (b) 30°
(c) 45° (d) 90°

4. The equation of the line perpendicular to Y-axis and passing through point (–5, 7) is ______.

(a) $y = -5$ (b) $x = 7$
(c) $x = -5$ (d) $y = 7$

5. If (2, 0) and (–2, 0) are the two vertices of an equilateral triangle, then the third vertex can be ______.

(a) (0, 0) (b) (2, –2)
(c) $(0, 2\sqrt{3})$ (d) $(\sqrt{3}, \sqrt{3})$

6. The points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$

(a) are collinear.
(b) form a scalene triangle.
(c) form an equilateral triangle.
(d) None of the above.

7. The equation of the line making equal intercepts and passing through the point (–1, 4) is ______.

(a) $x - y = 3$ (b) $x + y + 3 = 0$
(c) $x + y = 3$ (d) $x - y + 3 = 0$

PRACTICE QUESTIONS

8. The endpoints of the longest chord of a circle are (–4, 2) and (–6, –8). Find its centre.

(a) $\left(-\frac{10}{3}, -2\right)$ (b) (–5, –2)

(c) (–5, –4) (d) (–5, –3)

9. The equation of the line passing through point (–3, –7) and making an intercept of 10 units on X-axis can be ______.

(a) $4x + 3y = -9$ (b) $8x - 3y = 80$

(c) $7x - 13y - 70 = 0$ (d) $7x + 3y - 70 = 0$

10. The points on the Y-axis which are at a distance of 5 units from (4, –1) are ______.

(a) (0, –2), (0, 4) (b) (0, 2), (0, –4)

(c) (0, 2) (0, 4) (d) (0, –2) (0, –4)

11. If the slope and the y-intercept of a line are the roots of the equation $x^2 - 7x - 18 = 0$, then the equation of the line can be ______.

(a) $2x + y - 9 = 0$ (b) $2x - y + 9 = 0$

(c) $9x + y + 2 = 0$ (d) $9x + 2y - 2 = 0$

12. If the points $(k, k - 1)$, $(k + 2, k + 1)$ and $(k, k + 3)$ are three consecutive vertices of a square, then its area (in square units) is ______.

(a) 2 (b) 4

(c) 8 (d) 6

13. The equation of the line making intercepts of equal magnitude and opposite signs, and passing through the point (–3, –5) is _______.

(a) $x - y = 2$ (b) $2x + y = -4$

(c) $3x + 3y = 6$ (d) $x - y = -10$

14. If the endpoints of the diameter of a circle are (–2, 3) and (6, –3), then the area of the circle (in square units) is ______.

(a) $\frac{550}{3}$ (b) $\frac{540}{7}$

(c) $\frac{560}{7}$ (d) $\frac{550}{7}$

15. The inclination of the line $\sqrt{3}x - y + 3 = 0$ with the positive X-axis is _______.

(a) 30° (b) 45°

(c) 60° (d) 90°

16. The two lines $3x + 4y - 6 = 0$ and $6x + ky - 7 = 0$ are such that any line which is perpendicular to the first line is also perpendicular to the second line. Then, $k =$ ____.

(a) –8 (b) –6

(c) 6 (d) 8

17. The line $x = my$, where $m < 0$, lies in the quadrants.

(a) 1st, 2nd (b) 2nd, 4th

(c) 3rd, 4th (d) 3rd, 1st

18. Find the area in square units, of the rhombus with vertices (2, 1), (–5, 2), (–4, –5) and (3, –6), taken in that order.

(a) 24 (b) 48

(c) 36 (d) 50

19. The radius of a circle with centre (–2, 3) is 5 units, then the point (2, 5) lies ______.

(a) on the circle

(b) inside the circle

(c) outside the circle

(d) None of the above

20. One end of the diameter of a circle with the centre as origin is (–2, 10). Find the other end of the diameter.

(a) (–2, –10) (b) (0, 0)

(c) (2, –10) (d) (2, 10)

21. If the roots of the quadratic equation $x^2 - 7x + 12 = 0$ are intercepts of a line, then the equation of the line can be ______.

(a) $2x + 3y = 6$ (b) $4x + 3y = 12$

(c) $4x + 3y = 6$ (d) $3x + 4y = 6$

22. Find the value of λ, if the line $x - 3y + 4 + \lambda(8x - 3y + 2) = 0$ is parallel to the X-axis.

(a) $\frac{1}{5}$ (b) $\frac{5}{8}$

(c) $-\frac{3}{8}$ (d) $-\frac{1}{8}$

23. The slope of the line joining the points $(2, k - 3)$ and (4, –7) is 3. Find k.

(a) –10 (b) –6

(c) –2 (d) 10

PRACTICE QUESTIONS

24. The angle between the lines $x = 10$ and $y = 10$ is ______.
(a) $0°$ (b) $90°$
(c) $180°$ (d) None of these

25. The two lines $5x + 3y + 7 = 0$ and $kx - 4y + 3 = 0$ are perpendicular to the same line. Find the value of k.
(a) $-\frac{20}{7}$ (b) $-\frac{20}{3}$
(c) $\frac{20}{9}$ (d) $\frac{12}{5}$

26. The lines $x - 2y + 3 = 0$, $3x - y = 1$ and $kx - y + 1 = 0$ are concurrent. Find k.
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

27. Find the quadrant in which the lines $2x + 3y - 1 = 0$ and $3x + y - 5 = 0$ intersect each other.
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

28. The circum-centre of the triangle formed by points $O(0, 0)$, $A(6, 0)$ and $B(0, 6)$ is ______.
(a) (3, 3) (b) (2, 2)
(c) (1, 1) (d) (0, 0)

29. The lines $3x - y + 2 = 0$ and $x + 3y + 4 = 0$ intersect each other in the ______.
(a) 1st quadrant (b) 4th quadrant
(c) 3rd quadrant (d) 2nd quadrant

30. Centre of the circle is (a, b). If (0, 3) and (2, 0) are two points on a circle, then find the relation between a and b.
(a) $4a - 6b - 5 = 0$
(b) $4a + 6b - 5 = 0$
(c) $-4a + 5 = 0$
(d) $4a - 6b + 5 = 0$

Level 2

31. The equation of a line passing through $P(3, 4)$, such that P bisects the part of it intercepted between the coordinate axes is ______.
(a) $3x + 4y = 25$ (b) $4x + 3y = 24$
(c) $x - y = -1$ (d) $x + y = 7$

32. The line $7x + 4y = 28$ cuts the coordinate axes at A and B. If O is the origin, then the ortho-centre of ΔOAB is ______.
(a) (4, 0) (b) (0, 7)
(c) (0, 0) (d) None of these

33. If the roots of the quadratic equation $x^2 - 5x + 6 = 0$ are the intercepts of a line, then the equation of the line can be ______.
(a) $2x + 3y = 6$
(b) $3x + 2y = 6$
(c) Either (a) or (b)
(d) None of these

34. The equation of the line whose x-intercept is 5, and which is parallel to the line joining the points (3, 2) and (−4, −1) is ______.
(a) $4x + 7y - 20 = 0$
(b) $3x - 7y + 3 = 0$
(c) $3x + 2y + 15 = 0$
(d) $3x - 7y - 15 = 0$

35. Find the area of the triangle formed by the line $3x - 4y + 12 = 0$ with the coordinate axes.
(a) 6 units^2 (b) 12 units^2
(c) 1 units^2 (d) 36 units^2

36. The line joining the points $(2m + 2, 2m)$ and $(2m + 1, 3)$ passes through $(m + 1, 1)$, if the values of m are ______.
(a) $5, -\frac{1}{5}$ (b) 1, −1
(c) $2, -\frac{1}{2}$ (d) $3, -\frac{1}{3}$

37. The length (in units) of the line joining the points (4, 3) and (−4, 9) intercepted between the coordinate axes is ______.
(a) 10 (b) 8
(c) 6 (d) 4

PRACTICE QUESTIONS

38. The equation of a line parallel to $8x - 3y + 15 = 0$ and passing through the point $(-1, 4)$ is ______.

(a) $8x - 3y - 4 = 0$
(b) $8x - 3y - 20 = 0$
(c) $8x - 3y + 4 = 0$
(d) $8x - 3y + 20 = 0$

39. $(0, 0), (3, \sqrt{3})$ and $(0, 2\sqrt{3})$ are the three vertices of a triangle. The distance between the ortho-centre and the cirum-centre of the triangle is _______. (in units)

(a) $\sqrt{3}$ (b) $\sqrt{5}$
(c) $\sqrt{6}$ (d) 0

40. In a parallelogram $PQRS$, $P(15, 9)$, $Q(7, 10)$, $R(-5, -4)$, then the fourth vertex S is _______.

(a) $(3, -2)$ (b) $(3, -4)$
(c) $(9, -5)$ (d) $(3, -5)$

41. If the roots of the quadratic equation $3x^2 - 2x - 1 = 0$ are the intercepts of a line, then the line can be ______.

(a) $x - 3y - 1 = 0$
(b) $3x - y + 1 = 0$
(c) Either (a) or (b)
(d) None of these

42. The length (in units) of a line segment intercepted between the coordinate axes by the line joining the points (1, 2) and (3, 4) is ______.

(a) 4 (b) 6
(c) 8 (d) $\sqrt{2}$

43. If $A = (3, -4)$, $B = (7, 0)$ and $C = (14, -7)$ are the three consecutive vertices of a parallelogram $ABCD$, then find the slope of the diagonal BD. The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\left(\dfrac{x+7}{2}, \dfrac{y+0}{2}\right) = \left(\dfrac{3+14}{2}, \dfrac{-4-7}{2}\right)$.

(B) The slope of $BD = \dfrac{-11-0}{10-7} = \dfrac{-11}{3}$.

(C) $\dfrac{x+7}{2} = \dfrac{17}{2}$ and $\dfrac{y+0}{2} = \dfrac{-11}{2}$

$\Rightarrow x = 10, y = -11$

$\therefore D = (10, -11)$.

(D) Let the fourth vertex be $D(x, y)$. We know that the diagonals of a parallelogram bisect each other.

(a) ADCB (b) DCAB
(c) DACB (d) CDAB

44. If $A = (1, -6)$, $B = (5, -2)$ and $C = (12, -9)$ are the three consecutive vertices of a parallelogram, then find the fourth vertex. The following are the steps involved in solving the above problem. Arrange them in sequential order from beginning to end.

(A) $\dfrac{5+x}{2} = \dfrac{13}{2}, \dfrac{-2+y}{2} = \dfrac{-15}{2} \Rightarrow x = 8$, and $y = -13$. Therefore, $D = (8, -13)$.

(B) $\therefore \left(\dfrac{5+x}{2}, \dfrac{-2+y}{2}\right) = \left(\dfrac{1+12}{2}, \dfrac{-6-9}{2}\right)$.

(C) Let the fourth vertex be $D = (x, y)$.

(D) We know that diagonals of a parallelogram bisect each other.

(a) ACBD (b) ABDC
(c) CBDA (d) CDBA

45. Find the product of intercepts made by the line $7x - 2y - 14 = 0$ with coordinate axes.

(a) -7 (b) 2
(c) 14 (d) -14

46. Find the value of k, if points $(-2, 5)$, $(-5, -10)$ and $(k, -13)$ are collinear.

(a) $\dfrac{5}{28}$ (b) $\dfrac{-28}{5}$
(c) 28 (d) 5

47. The inclination of the line $\sqrt{3}y - x + 24 = 0$, is ______.

(a) 60° (b) 30°
(c) 45° (d) 135°

48. Find the product of intercepts of the line $3x + 8y - 24 = 0$.

(a) 8 (b) 24
(c) 3 (d) 12

49. Find the value of k, if points $(10, 14)$, $(-3, 3)$ and $(k, -8)$ are collinear.

(a) 16 (b) 18
(c) -18 (d) -16

50. The inclination of the line $y - x + 11 = 0$, is _______.

(a) 30°
(b) 60°
(c) 0°
(d) 45°

51. The equation of a line whose x-intercept is -3 and which is parallel to $5x + 8y - 7 = 0$ is _______.

(a) $5x + 8y + 15 = 0$
(b) $5x + 8y - 15 = 0$
(c) $5x + 8y - 17 = 0$
(d) $5x - 8y - 18 = 0$

Level 3

52. The area of a square with one of its vertices as $(5, -2)$ and the mid-point of the diagonals as $(3, 2)$, is _____. (in sq. units)

(a) 40 (b) 20
(c) 60 (d) 70

53. The equation of the line perpendicular to the line inclined equally to the coordinate axes and passing through $(2, -3)$ is _______.

(a) $x + y + 1 = 0$
(b) $x - y - 2 = 0$
(c) $x + y + 2 = 0$
(d) $2x + y - 1 = 0$

54. A triangle is formed by points (6, 0), (0, 0) and (0, 6). How many points with the integer coordinates are in the interior of the triangle?

(a) 7 (b) 6
(c) 8 (d) 10

55. The equation of one of the diagonals of a square is $3x - 8y + 4 = 0$. Find the equation of the other diagonal passing through the vertex $(4, -6)$.

(a) $8x + 3y - 15 = 0$
(b) $3x - 8y - 11 = 0$
(c) $8x + 3y - 14 = 0$
(d) $8x + 3y + 15 = 0$

56. The lines $2x + 3y - 6 = 0$ and $2x + 3y - 12 = 0$ are represented on the graph. The difference between the areas of triangles formed by the lines with the coordinate axes is ______. (in sq. units)

(a) 12 (b) 9
(c) 6 (d) 3

57. The equation of a line whose x-intercept is 11 and perpendicular to $3x - 8y + 4 = 0$, is _______.

(a) $7x + 3y - 77 = 0$
(b) $8x + 3y - 88 = 0$
(c) $5x + 3y - 55 = 0$
(d) $3x + 8y - 88 = 0$

58. $A(-11, 7)$ and $B(-10, 6)$ are the points of trisection of a line segment PQ. Find the coordinates of P and Q.

(a) $(-12, 8)$; $(-9, 5)$
(b) $(-12, -8)$; $(-9, 5)$
(c) $(12, 0)$; $(9, -5)$
(d) $(12, -8)$; $(9, -5)$

59. If one of the diagonals of a rhombus is $3x - 4y + 10 = 0$, then find the equation of the other diagonal which passes through point $(-2, -3)$.

(a) $4x + 3y + 17 = 0$
(b) $3x - 4y + 15 = 0$
(c) $4x + 3y - 15 = 0$
(d) $3x - 4y - 11 = 0$

60. The equation of the diagonal AC of a square $ABCD$ is $3x + 4y + 12 = 0$. Find the equation of BD, where D is $(2, -3)$.

(a) $4x - 3y - 8 = 0$
(b) $4x - 3y - 17 = 0$
(c) $4x - 3y + 17 = 0$
(d) $4x + 3y - 17 = 0$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. first
2. (2, 3)
3. $\left[3, -\frac{3}{2}\right]$
4. right-angled isosceles triangle
5. straight line
6. $\frac{-a}{b}$
7. The point may lie in Q_3 or Q_4.
8. $x + 3 = 0$
9. 2
10. –2
11. 0
12. the first or third quadrant
13. $y = \frac{-2}{3}x - \frac{5}{3}$
14. parallel
15. collinear
16. $\frac{1}{2}\left|\frac{c^2}{m}\right|$ sq. units
17. $x = \frac{19}{2}$
18. (–ve, +ve)
19. 0°
20. $\left(\frac{5}{3}, 0\right)$
21. X-axis
22. $a_1a_2 + b_1b_2 = 0$
23. origin (or) (0, 0)
24. X
25. collinear
26. $\left(0, \frac{-c}{b}\right)$
27. $-\cot\theta$
28. fourth
29. 1
30. parallel

Short Answer Type Questions

31. $2x - 3y + 13 = 0$
32. 50π sq. units
33. $\lambda = -2$
34. $k = -16$
35. $\sqrt{85}$ units
36. $k = \frac{-5}{4}$
37. 20 sq. units
38. $2\sqrt{2a}$ units
39. x-intercept $(a) = 2$
    y-intercept $(b) = -3$
40. 30°
41. $\left(\frac{1}{14}, \frac{-13}{14}\right)$
42. $3\sqrt{3}\,x - 3y - 1 = 0$
43. (2, 0)
45. $7x + 3y + 15 = 0$

Essay Type Questions

46. $\frac{x}{1} + \frac{y}{-\frac{1}{4}} = 1$ or $\frac{x}{-\frac{1}{4}} + \frac{y}{1} = 1$
47. $4x - 3y - 2 = 0$
48. $x - y + 16 = 0$
49. $2x + 5y = 10$ or $5x + 2y = 10$
50. $x + 3y - 8 = 0$

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (d)

Level 2

31. (b)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (d)
41. (c)	**42.** (d)	**43.** (c)	**44.** (d)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (d)	**50.** (d)
51. (a)									

Level 3

52. (a)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (b)

CONCEPT APPLICATION

Level 1

1. Fourth vertex of a parallelogram is $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$.
2. If x-intercept is a, then $(a, 0)$ is a point on the line. Now, use two point form.
3. Use slope $m = \tan\theta$, where θ is the inclination.
4. Equation of a line perpendicular to Y-axis is of the form x = constant.
5. (i) All the three vertices of an equilateral triangle are not rational
   (ii) Let the third vertex be (x, y).
   (iii) Sides of an equilateral triangle are equal.
6. Find slopes of AB and BC, if they are equal then the given points are collinear.
7. Equation of the line making equal intercepts is of the form, $x + y = a$.
8. The centre of a circle is the mid-point of the diameter.
9. If x-intercept is 10, then the line passing through (10, 0). Now, use the two point form.
10. Take the point on Y-axis as $(0, y)$.
11. Find the roots, and then use slope-intercept form of line.
12. Area of square = $(\text{side})^2$.
13. Use $\frac{x}{a} + \frac{y}{b} = 1$ and $a = -b$.
14. (i) Find the diameter of the circle then find its area.
    (ii) The distance between (–2, 3) and (6, –3) is the length of the diameter of the circle.
    (iii) Area of circle $= \frac{\pi d^2}{4}$, where d is the length of the diameter.
15. Use $m = \tan\theta$.
16. If two lines are perpendicular to the same line, then they are parallel to each other.
17. Identify the sign of y for each sign of x.
18. Find lengths of the diagonals, then area of rhombus $= \frac{1}{2} \times d_1 \times d_2$.
19. Find the distance between the given two points and compare that distance with the radius given.
20. The mid-point of the diameter is the centre of a circle.
21. Find the roots of the given equation, then use intercepts form of line.
22. (i) Equation of a line parallel to X-axis is of the form y = constant.
    (ii) Slope of a line parallel to X-axis is zero.
    (iii) Slope of a line $= \frac{-(x\text{-coefficient})}{(y\text{-coeffcient})}$.
23. The slope of the line joining two points is $\frac{y_2 - y_1}{x_2 - x_1}$.
24. The first line is parallel to the Y-axis. The second line is parallel to the X-axis.
25. Two lines, which are perpendicular to the same line, must be parallel to each other.
26. Solve the first two equations and substitute (x, y) in the third equation and evaluate k.
27. Find the point of intersection of the given lines, then decide.
28. The circum-centre of a right-angled triangle is the mid-point of its hypotenuse.
29. Find the coordinates of the point of intersection.
30. The distance from centre of the circle to any point on the circle is its radius.

Level 2

31. (i) P is the mid-point of the line joining the intercepts. Find the intercepts using the mid-point formula.
    (ii) Let the line cut coordinate axes at $A(a, 0)$ and $B(0, b)$.
    (iii) Using the above data find a and b, then the equation of line, i.e., $\frac{x}{a} + \frac{y}{b} = 1$.
32. (i) Identify the type of ΔOAB.

(ii) Ortho-centre of right triangle is the vertex containing right angle.

33. (i) Find the roots and take the equation as $\frac{x}{a}+\frac{y}{b}=1$.

(ii) Roots of $x^2 - 5x + 6 = 0$ are 2 and 3.

(iii) x-intercept is either 2 or 3. y-intercept is either 3 or 2.

34. (i) Find m, then use slope–point form.

(ii) Find the equation of the line passing through the given points.

(iii) Any line parallel to $ax + by + c_1 = 0$ is $ax + by + c_2 = 0$.

(iv) The required line $ax + by + c_2 = 0$ passes through (5, 0).

35. (i) Area $=\frac{1}{2}\left|\frac{c^2}{ab}\right|$, when the equation of the line is $ax + by + c = 0$.

(ii) If a and b are x- and y-intercepts, then the area of the triangle formed by the line with coordinate axes is $\left|\frac{ab}{2}\right|$.

36. (i) The three points are collinear.

(ii) Given points A, B and C are collinear.

(iii) Use, slope of AB = slope of AC and find m.

37. (i) Find the intercepts, then find the distance between them.

(ii) Find the equation of the line joining the given points.

(iii) Find the intercepts (a and b) made by the above line with coordinate axes.

(iv) The distance between $(a, 0)$ and $(0, b)$.

38. (i) Use the formula $a(x - x_1) + b(y - y_1) = 0$.

(ii) Slopes of parallel lines are equal.

(iii) Use, point-slope form, i.e., $(y - y_1) = m(x - x_1)$ and find the equation of the line.

39. (i) The given vertices form an equilateral triangle.

(ii) Given points are the vertices of an equilateral triangle.

(iii) In any equilateral triangle, geometric centres (except ex-centre) coincide.

40. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the successive vertices of a parallelogram, then the fourth vertex $= (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.

41. (i) Find the roots, then use $\frac{x}{a}+\frac{y}{b}=1$.

(ii) Roots of $3x^2 - 2x - 1 = 0$ are $\frac{-1}{3}$ and 1.

(iii) x-intercept is either $\frac{-1}{3}$ or 1.

y-intercept is either 1 or $\frac{-1}{3}$.

(iv) The required equation of the line is $\frac{x}{a}+\frac{y}{b}=1$.

42. (i) Find the intercepts, then find the distance between the intercepted points.

(ii) Find the equation of the line joining points (1, 2) and (3, 4).

(iii) Find the intercepts (a and b) of the line by putting $x = 0$ and $y = 0$.

(iv) Length of the required line is $\sqrt{a^2+b^2}$.

43. DACB is the required sequential order.

44. CDBA is the required sequential order.

45. $7x - 2y - 14 = 0 \Rightarrow 7x - 2y = 14$

$$\Rightarrow \frac{x}{2}-\frac{y}{7}=1 \Rightarrow \frac{x}{2}+\frac{y}{(-7)}=1.$$

∴ Intercepts are 2 and −7, and their product $= 2 \times (-7) = -14$.

46. (−2, 5), (−5, −10) and (k, −13) are collinear

$$\Rightarrow \frac{-10-5}{-5+2}=\frac{-13+10}{k+5}$$

$$\Rightarrow \frac{-15}{-3}=\frac{-3}{k+5}$$

$$\Rightarrow 5(k+5) = -3$$

$$\Rightarrow k+5=\frac{-3}{5}$$

$$\Rightarrow k=\frac{-3}{5}-5=\frac{-28}{5}$$

$$\therefore k=\frac{-28}{5}.$$

HINTS AND EXPLANATION

47. $\sqrt{3}y - x + 24 = 0 \Rightarrow m = \frac{1}{\sqrt{3}}$.

$\therefore m = \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°. \quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$

48. $3x + 8y = 24$

$\Rightarrow \frac{x}{8} + \frac{y}{3} = 1$

$\Rightarrow$ Intercepts are 8 and 3 and their product is $8 \times 3 = 24$.

49. Let the points be $A = (10, 14)$, $B = (-3, 3)$ and $C = (k, -8)$.

Given, they are collinear $\Rightarrow$ The slopes are same.

$\Rightarrow$ Slope of AB = Slope of BC

$\Rightarrow \frac{3-14}{-3-10} = \frac{-8-3}{k+3}$

$\Rightarrow \frac{-11}{-13} = \frac{-11}{k+3}$

$\Rightarrow k + 3 = -13$

$\Rightarrow k = -16$.

51. $y - x + 11 = 0 \Rightarrow y = x - 11$

$\Rightarrow m = \tan\theta = 1$

$\Rightarrow \theta = 45°$.

52. Slope of the line parallel to $5x + 8y - 7 = 0$ is $\frac{-5}{8}$.

Given that the x-intercept of the required line is -3.

$\therefore$ It passes through $(-3, 0)$. Hence, the required line is

$-\frac{5}{8} = \frac{y-0}{x-(-3)}$

$y - 0 = \frac{-5}{8}(x + 3)$

$\Rightarrow 8y = -5x - 15 \Rightarrow 5x + 8y + 15 = 0$.

Level 3

53. (i) Required line is perpendicular to $x = y$ and passes through $(2, -3)$.

(ii) Find m, then use the slope–point form.

54. Draw the triangle and list the possible points.

55. (i) In a square, the diagonals are perpendicular to each other.

(ii) Find the slope of the second diagonal and use the slope–point form.

56. (i) Find the intercepts made by the lines on the coordinate axes by writing the equations in the intercept form.

(ii) If the intercepts made are a and b, then the area of the triangle formed is $\frac{ab}{2}$.

57. Slope of the line perpendicular to $3x - 8y + 4 = 0$ is $\frac{-1}{\text{Slope of } 3x - 8y + 4 = 0} = \frac{-1}{\frac{3}{8}} = -\frac{8}{3}$.

Given that, x-intercept of the required line is 11.

$\therefore$ It passes through $(11, 0)$.

Hence, the required line is $-\frac{8}{3} = \frac{y-0}{x-11}$.

That is, $y - 0 = -\frac{8}{3}(x - 11)$

$\Rightarrow 8x + 3y - 88 = 0$.

58. Let $P = (a_1, b_1)$, and $Q = (a_2, b_2)$. Given, $A = (-11, 7)$ and $B = (-10, 6)$.

$\Rightarrow A$ is the mid-point of the line segment PB.

$\therefore A = \left(\frac{a_1 - 10}{2}, \frac{b_1 + 6}{2}\right)$

$(-11, 7) = \left(\frac{a_1 - 10}{2}, \frac{b_1 + 6}{2}\right)$

$\Rightarrow a_1 = -12$ and $b_1 = 8$

$\Rightarrow P = (-12, 8)$. Similarly, $Q = (-9, 5)$.

59. One diagonal of the rhombus is $3x - 4y + 10 = 0$.

Slope of $3x - 4y + 10 = 0$ is $\frac{3}{4}$.

In a rhombus, diagonals bisect each other perpendicularly.

$\therefore$ The slope of the other diagonal

$= \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$.

The required equation is $-\frac{4}{3} = \frac{y-(-3)}{x-(-2)}$

$\Rightarrow y + 3 = -\frac{4}{3}\,(x + 2)$

$\Rightarrow 4x + 3y + 17 = 0$.

60. The equation of diagonal AC is $3x + 4y + 12 = 0$.

Its slope $= -\frac{3}{4}$

Slope of the other diagonal BD $\frac{1}{\frac{-3}{4}} = \frac{4}{3}$.

BD passes through $(2, -3)$

$\therefore \frac{4}{3} = \frac{y-(-3)}{x-2}$

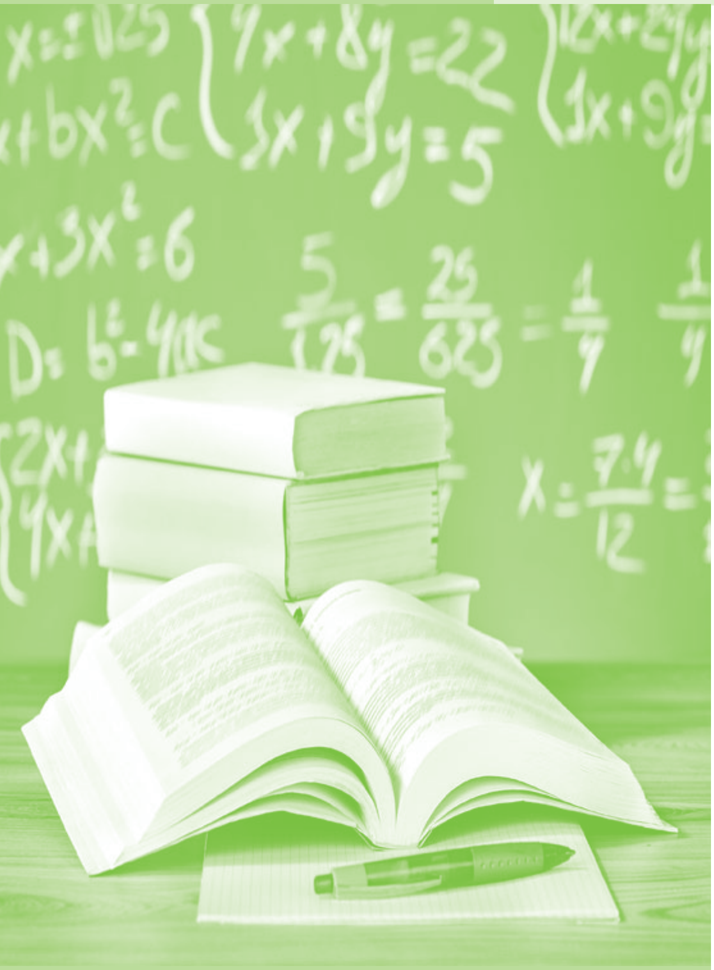
$\therefore y + 3 = \frac{4}{3}(x - 2)$

$4x - 3y - 17 = 0$.

Chapter

15

Locus



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the basic concepts of geometrical shapes and patterns
- Know different types of angles and their values
- Remember theorems related to triangles and angles

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the concept of locus and point of locus
- Calculate the equation of a locus
- Know the concept of congruency and learn the terms related to geometric centres of a triangle

INTRODUCTION

Mark a fixed point O on a sheet of paper. Now, start marking points P_1, P_2, P_3, P_4, … on the sheet of paper such that $OP_1 = OP_2 = OP_3 = \ldots = 4$ cm. What do we observe on joining these points by a smooth curve? We observe a pattern, which is circular in shape and every point is at a distance of 4 cm from point O.

It can be said that whenever points satisfying a certain condition are plotted, a pattern is formed. This pattern formed by all possible points satisfying the given condition is called the locus of points satisfying the given conditions. In the above example, we have the locus of points which are equidistant (4 cm) from the given point O.

The collection (set) of all points which satisfy certain given geometrical conditions is called the **locus of a point** satisfying the given conditions.

Alternatively, a locus can be defined as the path or curve traced by a point in a plane when subjected to some geometrical conditions.

Consider the following examples:

1. The locus of the point in a plane which is at a constant distance r from a fixed point O is a circle with centre O and radius r units (see Fig. 15.1).
2. The locus of the point in a plane which is at a constant distance from a fixed straight line is a pair of lines, parallel to the fixed line. Let the fixed line be l. The lines, m and n, form the set of all points which are at a constant distance from l (see Fig. 15.2).

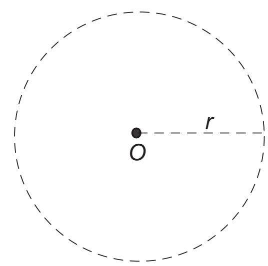


Figure 15.1

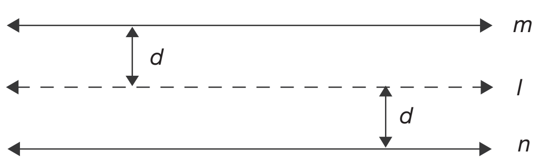


Figure 15.2

3. The locus of a point in a plane, which is equidistant from a given pair of parallel lines is a straight line, parallel to the two given lines and lying midway between them.

   In Fig. 15.3, m and n are the given lines and line l is the locus.

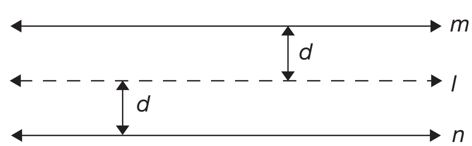


Figure 15.3

Before proving that a given path or curve is the desired locus, it is necessary to prove the following:

1. Every point lying on the path satisfies the given geometrical conditions.
2. Every point that satisfies the given conditions lies on the path.

EXAMPLE 15.1

Show that the locus of a point, equidistant from the endpoints of a line segment, is the perpendicular bisector of the segment.

SOLUTION

The proof will be taken up in two steps.

Step 1: We initially prove that any point equidistant from the endpoints of a line segment lies on the perpendicular bisector of the line segment.

Given: M and N are two points on a plane. A is a point in the same plane such that $AM = AN$ (see Fig. 15.4).

To prove: A lies on the perpendicular bisector of MN.

Proof: Let L be the mid-point of $\overline{MN}$.

If A coincides with L, then A lies on the bisector of MN.

Suppose A is different from L.

Then, in ΔMLA and ΔNLA,

$ML = NL$, $AM = AN$ and AL is a common side.

$\therefore$ By SSS congruence property, $\Delta MLA \cong \Delta NLA$.

$\Rightarrow \angle MLA = \angle NLA$ ($\because$ The corresponding elements of congruent triangles are equal.) (1)

But, $\angle MLA + \angle NLA = 180°$ ($\because$ Linear pair)

$\Rightarrow 2\angle MLA = 180°$ (From Eq. (1))

$\therefore \angle MLA = \angle NLA = 90°$.

So, $\overline{AL} \perp \overline{MN}$. Hence, $\overline{AL}$ is the perpendicular bisector of $\overline{MN}$.

$\therefore$ A lies on the perpendicular bisector of $\overline{MN}$.

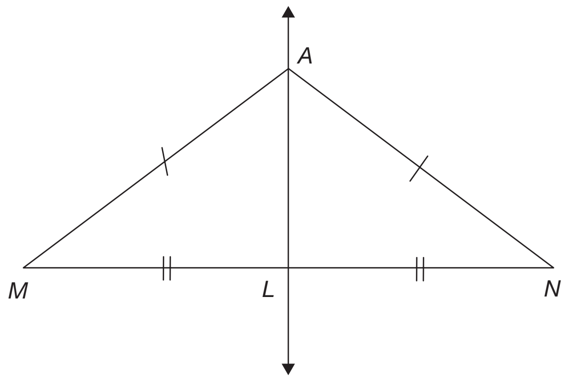


Figure 15.4

Step 2: Now, we prove that any point on the perpendicular bisector of the line segment is equidistant from the end points of the line segment.

Given: MN is a line segment and P is a point on the perpendicular bisector. L is the mid-point of MN (see Fig. 15.5).

To prove: $MP = NP$.

Proof: If P coincides with L, then $MP = NP$.

Suppose P is different from L. Then, in ΔMLP and ΔNLP, $ML = LN$.

LP is the common side and $\Delta MLP = \Delta NLP = 90°$.

$\therefore$ By the SAS congruence property, $\Delta MLP \cong \Delta NLP$.

So, $MP = PN$ ($\because$ The corresponding elements of congruent triangles are equal.)

$\therefore$ Any point on the perpendicular bisector of $\overline{MN}$ is equidistant from points M and N.

Hence, from Step I and Step II of the proof, it can be said that the locus of point equidistant from two fixed points is the perpendicular bisector of the line segment joining the two points.

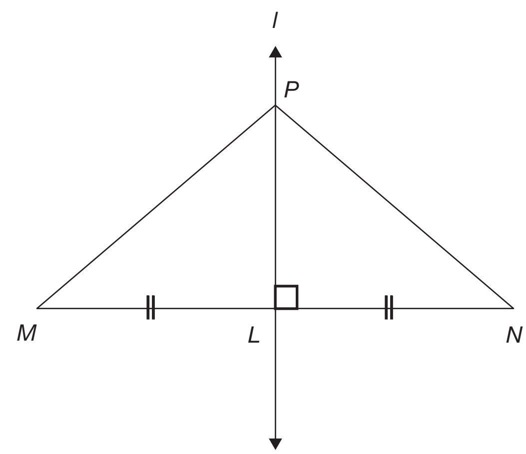


Figure 15.5

EXAMPLE 15.2

Show that the locus of a point, equidistant from two intersecting lines in the plane, is a pair of lines bisecting the angles formed by the given lines.

SOLUTION

Step 1: We initially prove that any point, equidistant from two given intersecting lines, lies on one of the lines bisecting the angles formed by the given lines.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two lines intersecting at O. P is the point on the plane such that $PM = PN$. Line l is the bisector of $\angle BOD$ and $\angle AOC$.

Line m is the bisector of $\angle BOC$ and $\angle AOD$ (see Fig. 15.6).

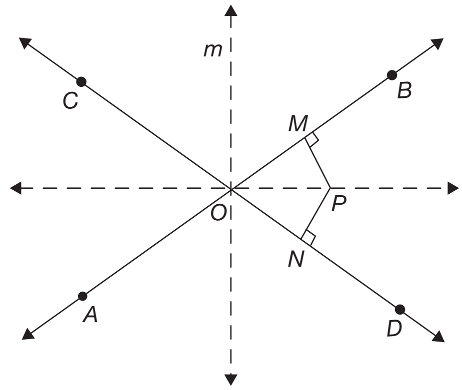


Figure 15.6

To prove: P lies either on the line l or on the line m.

Proof: In ΔPOM and ΔPON, $PM = PN$,

OP is a common side and $\angle PMO = \angle PNO = 90°$.

$\therefore$ By RHS congruence property, $\Delta POM \cong \Delta PON$.

So, $\angle POM = \angle PON$, i.e., P lies on the angle bisector of $\angle BOD$.

As l is the bisector of $\angle BOD$ and $\angle AOC$, P lies on the line l.

Similarly, if P lies in any of the regions of $\angle BOC$, $\angle AOC$ or $\angle AOD$, such that it is equidistant from $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, then we can conclude that P lies on the angle bisector l or on the angle bisector m.

Step 2: Now, we prove that any point on the bisector of one of the angles formed by two intersecting lines is equidistant from the lines.

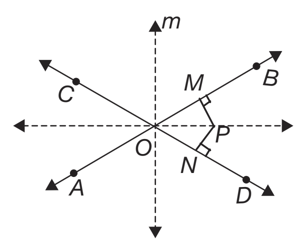


Figure 15.7

Given: Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, intersect at O. Lines l and m are the angle bisectors.

Proof: Let l be the angle bisector of $\angle BOD$ and $\angle AOC$, and m be the angle bisector of $\angle BOC$ and $\angle AOD$.

Let P be a point on the angle bisector l, as shown in Fig. 15.7.

If P coincides with O, then P is equidistant from line $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Suppose P is different from O.

Draw the perpendiculars $\overline{PM}$ and $\overline{PN}$ from the point P onto the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ respectively.

Then in ΔPOM and ΔPON,

$\angle POM = \angle PON$, $\angle PNO = \angle PMO = 90°$ and OP is a common side.

$\therefore$ By the AAS congruence property,

$\Delta POM \cong \Delta PON$.

So, $PN = PM$ ($\because$ The corresponding sides in congruent triangles.)

That is, P is equidistant from lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Hence, from Step I and Step II of the proof, it can be said that the locus of the point, which is equidistant from the two intersecting lines is the pair of the angle bisectors of the two pairs of vertically opposite angles formed by the lines.

EQUATION OF A LOCUS

We know that the locus is the set of points that satisfy a given geometrical condition. When we express the geometrical condition in the form of an algebraic equation, the equation is called the equation of the locus.

Steps to Find the Equation of a Locus

1. Consider any point (x_1, y_1) on the locus
2. Express the given geometrical condition in the form of an equation using x_1 and y_1.
3. Simplify the equation obtained in Step 2.
4. Replace (x_1, y_1) by (x, y) in the simplified equation, which gives the required equation of the locus.

The following formulae will be helpful in finding the equation of a locus:

1. Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. Area of the triangle formed by joining points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

   $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$, where the value of $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.
3. Equation of the circle with centre (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$.
4. The perpendicular distance from a point $P(x_1, y_1)$ to a given line $ax + by + c = 0$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|.$$

EXAMPLE 15.3

Find the equation of the locus of a point which forms a triangle of area 5 square units with the points $A(2, 3)$ and $B(-1, 4)$.

SOLUTION

Let $P(x_1, y_1)$ be point on the locus, $(x_2, y_2) = (2, 3)$ and $(x_3, y_3) = (-1, 4)$.

Given area of $\Delta PAB = 5$ sq. units.

$$\therefore \frac{1}{2}\begin{vmatrix} x_1 - 2 & 2-(-1) \\ y_1 - 3 & 3 - 4 \end{vmatrix} = 5$$

$$\Rightarrow \begin{vmatrix} x_1 - 2 & 3 \\ y_1 - 3 & -1 \end{vmatrix} = 10 \Rightarrow -(x_1 - 2) - 3(y_1 - 3) = \pm 10$$

$$\Rightarrow x_1 + 3y_1 - 11 = \pm 10.$$

The required equation is,

$$x + 3y = 10 + 11 \quad \text{or} \quad x + 3y = -10 + 11$$
$$x + 3y - 21 = 0 \quad \text{or} \quad x + 3y - 1 = 0$$

CONCURRENCY—GEOMETRIC CENTRES OF A TRIANGLE

Let us recall that if three or more lines pass through a fixed point, then the lines are said to be **concurrent** and the fixed point is called the point of **concurrence**. In this context, we recall different concurrent lines and their points of concurrence associated with a triangle.

Geometric Centres of a Triangle

Circum-centre

The locus of the point equidistant from the endpoints of the line segment is the perpendicular bisector of the line segment. The three perpendicular bisectors of the three sides of a triangle

are concurrent. The point of their concurrence is called the **circum-centre** of the triangle. It is usually denoted by S. The circum-centre is equidistant from all the vertices of the triangle. The circum-centre of the triangle is the locus of the point in the plane of the triangle equidistant from the vertices of the triangle (see Fig. 15.8).

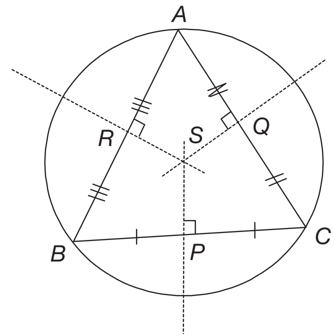


Figure 15.8

In-centre

The angle bisectors of the triangle are concurrent. The point of concurrence is called in-centre. It is usually denoted by I. The point I is equidistant from the sides of the triangle. The **in-centre** of the triangle is the locus of the point, in the plane of the triangle, equidistant from the sides of the triangle (see Fig. 15.9).

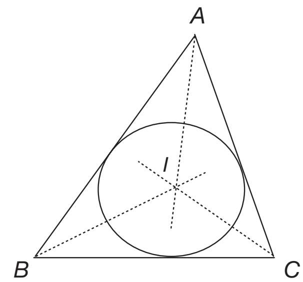


Figure 15.9

Ex-centre

The point of concurrence of the external bisector of two angles of a triangle and the internal bisector of the third angle is called **ex-centre** of the triangle (see Fig. 15.10).

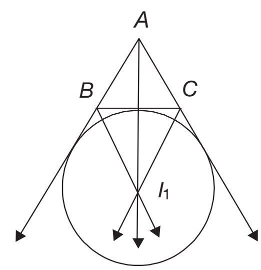


Figure 15.10

Ortho-centre

The altitudes of the triangle are concurrent and the point of concurrence of the altitudes of a triangle is called **ortho-centre**. It is usually denoted by O (see Fig. 15.11).

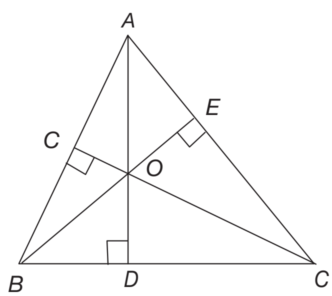


Figure 15.11

Centroid

The medians of a triangle are concurrent and the point of concurrence of the medians of a triangle is called **centroid**. It is usually denoted by G. The centroid divides each of the medians in the ratio 2 : 1, beginning from vertex, i.e., $AG : GD = 2 : 1$, $BG : GE = 2 : 1$ and $CG : GF = 2 : 1$, as shown in Fig. 15.12.

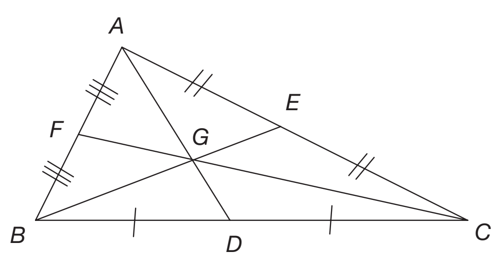


Figure 15.12

Some Important Points

1. In an equilateral triangle, the centroid, the ortho-centre, the circum-centre and the in-centre all coincide.
2. In an isosceles triangle, the centroid, the ortho-centre, the circum-centre and the in-centre all lie on the median to the base.
3. In a right triangle, the length of the median drawn to the hypotenuse is equal to half of the hypotenuse. The median is also equal to the circum-radius. The mid-point of the hypotenuse is the circum-centre.
4. In an obtuse-angled triangle, the circum-centre and ortho-centre lie outside the triangle. For an acute-angled triangle, the circum-centre and the ortho-centre lie inside the triangle.
5. For all triangles, the centroid and the in-centre lie inside the triangle.
6. For all triangles, the ex-centre lies outside the triangle.

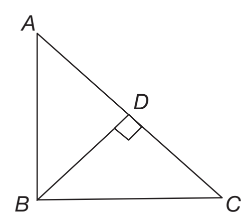
TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Centroid divides the median from the vertex in the ratio ______.
2. Any point on the perpendicular bisector of a line segment joining two points is ______ from the two points.
3. What is the locus of a point in a plane which is at a distance of p units from the circle of radius q units ($p \neq q$)?
4. If A and B are two fixed points, then the locus of a point P, such that $\angle APB = 90°$ is ______.
5. The locus of the point equidistant (in a plane) from the three vertices of a triangle is ______.
6. The locus of the point in a plane which is equidistant from two intersecting lines is ______.
7. The line segment from the vertex of a triangle perpendicular to its opposite side is called ______.
8. The path traced out by a moving point which moves according to some given geometrical conditions is ______.
9. Is the statement 'In ΔABC, a point equidistant from AB and AC lies on the median', true?
10. The path of a freely falling stone is ______.
11. The orthocentre of a right triangle is the ______.
12. The line segment joining the vertex of a triangle and the mid-point of its opposite side is called ______.
13. The locus of the centre of the circles (in a plane) passing through two given points is the ______ of the line segment joining the two points.
14. The circum-centre of a right triangle always lies ______.
15. The locus of the tip of the hour hand is ______.

Short Answer Type Questions

16. Find the locus of the vertex of a triangle with fixed base and having constant area.
17. Find the locus of the centre of a circle passing through two fixed points A and B.
18. Let A and B be two fixed points in a plane. Find the locus of a point P, such that $PA^2 + PB^2 = AB^2$.
19. Find the locus of the point P, such that $TP : MP = 3 : 2$, where T is $(-2, 3)$ and M is $(4, -5)$.
20. Find the locus of the point which is equidistant from sides AB and AD of a rhombus $ABCD$.
21. In the following figure (not to scale), ABC is a right isosceles triangle, right-angled at B and $\overline{BD} \perp \overline{AC}$. If the triangle ABC is rotated about the hypotenuse, then find the locus of the triangle ABC.



22. Show that the locus of a point, equidistant from three distinct given collinear points in a plane does not exist.
23. If ZQ and RU be two lines intersecting at point E, then find the locus of a point moving in the interior of $\angle UEZ$, such that the sum of its distances from the lines ZQ and RU is b units.
24. Find the locus of the point which is equidistant from the sides AB and AC of triangle ABC.
25. ΔAPQ, ΔBPQ and ΔCPQ are three isosceles triangles with the same base PQ. Show that the points A, B and C are collinear.

Essay Type Questions

26. In a square $ABCD$, if A and B are (5, 1) and (7, 1) respectively, then what is the locus of the mid-point of diagonal AC?
27. If $P(x, y)$ and $Q(1, 4)$ are the points on the circle whose centre is $C(5, 7)$, then find the locus of P.

28. If two lines intersect at P at right angles and pass through $A(1, 1)$ and $B(1, 0)$ respectively then what is the locus of P?

29. In ΔPAB, D, E and F are the mid-points of PA, AB and BP, respectively. The area of DEF is 8 sq. units. If A is (2, 5) and B is (3, 4), then what is the locus of P?

30. If P is a point on the circle with AB as a diameter, where A and B are (0, 2) and (2, 4) respectively, then the locus of P is ______.

CONCEPT APPLICATION

Level 1

1. The locus of a point equidistant from three fixed points is a single point. The three points are ______.
   (a) collinear (b) non-collinear
   (c) coincidental (d) None of these

2. The locus of a point moving in a space which is at a constant distance from a fixed point in space is called a ______.
   (a) square (b) sphere
   (c) circle (d) triangle

3. The locus of the centre of the circle that touch a given circle internally is a ______.
   (a) straight line (b) hellix
   (c) circle (d) None of these

4. In ΔABC, $\angle A = \angle B + \angle C$, then the circum-centre is at ______.
   (a) A (b) B
   (c) C (d) the mid-point of BC

5. The locus of a point equidistant from three points does not exist. This implies that the three points are ______.
   (a) collinear (b) non-collinear
   (c) coincidental (d) None of these

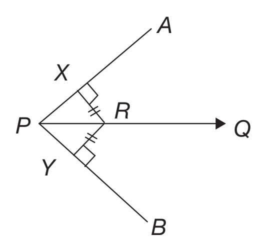
6. The locus of a point which is equidistant from two non-intersecting lines l and m is a ______.
   (a) straight line parallel to the line l
   (b) straight line parallel to the line m
   (c) straight line parallel to the lines l and m and midway between them
   (d) straight line that intersects both the lines l and m

7. The locus of a point which is at a constant distance k from Y-axis is ______.
   (a) $x = \pm k$ (b) $y = \pm k$
   (c) $x = 0$ (d) $y = 0$

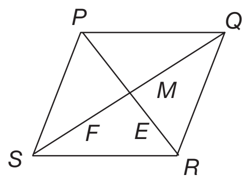
8. The locus of the centre of a wheel rolling on a straight road is a ______.
   (a) concentric circle (b) straight line
   (c) curve path (d) parabola

9. If A and B are two fixed points, then the locus of a point P, such that $PA^2 + PB^2 = AB^2$ is a/an ______.
   (a) circle with AB as the diameter
   (b) right triangle with $\angle P = 90°$
   (c) semi-circle with AB as the diameter
   (d) circle with AB as the diameter, excluding points A and B

10. In the figure, $\overline{RX} \perp \overline{PA}$, $\overline{RY} \perp \overline{PB}$, $RX = RY$ and $\angle APB = 70°$. Find $\angle APQ$.
   (a) 70° (b) 140°
   (c) 35° (d) 50°



11. Consider a point M inside a quadrilateral $PQRS$. If M be the point of intersection of angle bisectors PE and QF, then ______.
   (a) M is nearer to $\overline{PS}$ than to $\overline{QR}$
   (b) M is equidistant from opposite sides $\overline{PS}$ and $\overline{QR}$
   (c) M is nearer to $\overline{QR}$ than to $\overline{PS}$
   (d) M is equidistant from opposite sides $\overline{PQ}$ and $\overline{SR}$



12. The locus of the point which is equidistant from the three lines determined by the sides of a triangle is ______.

(a) the in-centre

(b) the ex-centre

(c) the ortho-centre

(d) either (a) or (b)

13. Find the locus of any fixed point on the circumference of a coin when the coin is rolling on a straight path.

(a) Circle (b) Straight line
(c) Sphere (d) Helix

14. The locus of a point which is equidistant from the coordinate axes can be a ______.

(a) line making a non-zero intercept on X-axis

(b) line making a non-zero intercept on Y-axis

(c) line passing through the origin making an angle of 45° with X-axis

(d) None of the above

15. The locus of a point equidistant from two intersecting lines PQ and RS, and at a distance of 10 cm from their point of intersection O is ______.

(a) four points lying on the angle bisectors at a distance of 5 cm from O

(b) two points lying on the angle bisectors at a distance of 10 cm from O

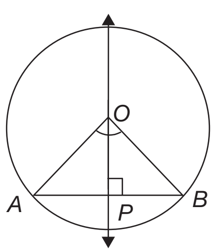
(c) four points lying on the angle bisectors at a distance of 10 cm from O

(d) two points lying on the angle bisectors at a distance of 5 cm from O

Level 2

16. In the figure following figure, O is the centre of the circle and $\overline{AL} \perp \overline{MN}$. If $\angle AOB = 90°$, then find $\angle AOP$.

(a) 60° (b) 20°
(c) 30° (d) 45°

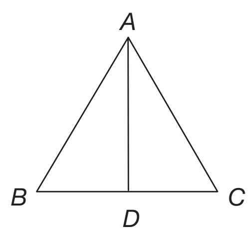


17. The solid formed when a right triangle is rotated about one of the sides containing the right angle is a ______.

(a) prism (b) cylinder
(c) cone (d) sphere

18. In the figure (not to scale), $AB = AC$ and $BD = CD$. Find $\angle ADB$.

(a) 60° (b) 90°
(c) 120° (d) Cannot be determined



19. A part of the locus of a point P, which is equidistant from two intersecting lines $ax + by + c = 0$ and $px + qy + r = 0$, is ______.

(a) $(a - p)x + (b - q)y + (c - x) = 0$

(b) $apx + qby + cy = 0$

(c) $\sqrt{a^2 + b^2}(px + qy + r) = \sqrt{p^2 + q^2}(ax + by + c)$

(d) None of these

20. If PAB is a triangle of area 4 sq. units and A is (2, 5) and B is (3, 4), then part of the locus of P is ______.

(a) $x - y + 15 = 0$

(b) $x - y - 15 = 0$

(c) $x + y - 15 = 0$

(d) $x + y + 15 = 0$

21. Given, triangle PBC and parallelogram $ABCD$ lie between the same parallel lines. On the same base, BC and the area of parallelogram is 2 sq. units.

PRACTICE QUESTIONS

The points B and C are (2, 4) and (4, 4) respectively. Which of the following lines is a part of the locus of P?

(a) $y = 4$ (b) $y = 7$
(c) $y = 5$ (d) $y = 6$

22. If PAB is a triangle in which $\angle B = 90°$ and $A(1, 1)$ and $B(0, 1)$, then the locus of P is ______.

(a) $y = 0$ (b) $xy = 0$
(c) $x = y$ (d) $x = 0$

23. The locus of P whose distance from the X-axis is thrice the distance from the line $x = 5$ is ______.

(a) $x - y - 5 = 0$
(b) $3x - y - 15 = 0$
(c) $3x + y + 15 = 0$
(d) $x + y + 5 = 0$

24. Two of the vertices of a triangle ABC are $A(1, 1)$, $B(-1, -3)$ and the area of ΔABC is 6 sq. units. If P is the centroid of the ΔABC, then find the locus of P.

(a) $2x - y + 1 = 0$ (b) $2x - y - 3 = 0$
(c) $2x + y + 3 = 0$ (d) Both (a) and (b)

25. In a circle with radius 25 cm, what is the area of the region determined by the locus of the mid-points of chords of length 48 cm?

(a) 154 cm^2 (b) 254 cm^2
(c) 72 cm^2 (d) None of these

26. The locus of the centre of a circle that touches the given circle externally is a ______.

(a) curve (b) straight line
(c) circle (d) helix.

27. The locus of a rectangle, when the rectangle is rotated about one of its sides, is a ______.

(a) plane (b) sphere
(c) cone (d) cylinder.

28. If the ortho-centre of a triangle ABC is B, then which of the following is true?

(a) $AC^2 = AB^2 + BC^2$
(b) $AC^2 > AB^2 + BC^2$
(c) $AC^2 < AB^2 + BC^2$
(d) None of these

29. In ΔABC, $\angle A = \angle B + \angle C$. The point which is equidistant from A, B and C is ______.

(a) mid-point of AB
(b) mid-point of AC
(c) mid-point of BC
(d) None of these

30. The locus of a point which is equidistant from (0, 2) and (0, 8) is ______.

(a) $y = 4$ (b) $y = 5$
(c) $x = 4$ (d) $x = 5$

Level 3

31. The area of ΔPQR is 4 sq. units Q and R are (1, 1) and (1, 0), respectively. Which of the following lines is a part of the locus of P?

(a) $x - 6 = 0$ (b) $x - 7 = 0$
(c) $x + 8 = 0$ (d) $x + 7 = 0$

32. What is the locus of the point $P(x, y)$ (where $xy > 0$), which is at a distance of 2 units from the origin?

(a) $x^2 + y^2 = 4$
(b) $x^2 + y^2 = 4$, $x > 0$, $y > 0$
(c) $x^2 + y^2 = 4$, $x < 0$, $y < 0$
(d) None of these

33. The locus of a point which is twice as far from each vertex of a triangle as it is from the mid-point of the opposite side is a/an/the ______.

(a) median (b) centroid
(c) incentre (d) angle bisector

34. The locus of a point which is collinear with the two given points is ______.

(a) a circle (b) a triangle
(c) a straight line (d) a parabola

35. The locus of a point, which is at a distance of 8 units from (0, −7), is ______.

(a) $x^2 + y^2 + 6x + 14y - 15 = 0$
(b) $x^2 + y^2 + 14y - 15 = 0$
(c) $y^2 + 14y - 8 = 0$
(d) $x^2 + y^2 + 14x + 14y - 15 = 0$

36. The area of ΔABC is 2 sq. units. If $A = (2, 4)$ and $B = (4, 4)$, then find the locus of $C(x, y)$.

(a) $y - 6 = 0$

(b) $y - 2 = 0$

(c) Both (a) and (b)

(d) None of these

37. Find the locus of a point which is a constant distance of 4 units away from the point (2, 4).

(a) $x^2 + y^2 - 4x - 8y + 4 = 0$

(b) $x^2 + 4x + 16 = 0$

(c) $y^2 - 8y + 12 = 0$

(d) None of these

38. P is the point of intersection of the diagonals of a square $READ$. P is equidistant from ______.

(a) the vertices R, E, A and D

(b) $\overline{RE}$ and $\overline{EA}$

(c) $\overline{EA}$ and $\overline{AD}$

(d) All of these

39. A coin of radius 1 cm is moving along the circumference and interior of a square of side 5 cm. Find the locus of centre of the coin.

(a) A square of side 6 cm

(b) A square of side 4 cm

(c) A square of side 3 cm

(d) A square of side 2 cm

40. ABC is a triangle in which $AB = 40$ cm, $BC = 41$ cm and $AC = 9$ cm. Then ortho-centre of ΔABC lies ______.

(a) interior of the triangle

(b) exterior of the triangle

(c) on the triangle

(d) at the mid-point of the triangle

41. O is an interior point of a rhombus, $ABCD$, and O is equidistant from BC and CD. Then O lies on ______.

(a) $\overline{AC}$

(b) $\overline{BD}$

(c) Either (a) or (b)

(d) Neither (a) nor (b)

42. In a triangle ABC, D is a point on BC, such that any point on AD is equidistant from points B and C. Which of the following is necessarily true?

(a) $AB = BC$

(b) $BC = AC$

(c) $AC = AB$

(d) $AB = BC = AC$

43. The locus of a point, equidistant from the coordinate axes is ______.

(a) $x = |y|$

(b) $y = |x|$

(c) Both (a) and (b)

(d) None of these

44. P is an interior point of an equilateral triangle ABC. If P is equidistant from AB and BC, BC and AC, then $\angle BPC =$ ______.

(a) 120° (b) 90°

(c) 60° (d) 150°

45. The locus of a point, which is equidistant from (2, 6) and (2, 8), is ______.

(a) $y = 7$ (b) $x = 7$

(c) $x = 2$ (d) $y = 2$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 2 : 1
2. equidistant
3. Another circle of radius $(p + q)$ or $(q - p)$ concentric to it.
4. circle with diameter AB excluding the points A and B.
5. circum-centre
6. angle bisectors of the two pairs of vertically opposite angles of intersecting lines.
7. altitude
8. locus
9. No
10. vertical line
11. vertex containing the right angle
12. median
13. perpendicular bisector
14. on the mid-point of the hypotenuse
15. circle

Short Answer Type Questions

16. The required locus is a line parallel to the given base.
17. The required locus is the perpendicular bisector of $\overline{AB}$, except the mid-point of $\overline{AB}$.
18. The required locus is the circle with diameter AB, excluding points A and B.
19. $5x^2 + 5y^2 - 88x + 114y + 317 = 0$ is the required locus of point P.
20. The required locus is diagonal AC.
21. According to given conditions, two equal cones of base radius BD and slant height AB or BC are formed in such a way that their bases touch one another.
23. The required locus is the angular bisector of the angle RPM.
24. The required locus is the bisector of $\angle BAC$.

Essay Type Questions

26. $y = (2, 0)$
27. $x^2 + y^2 - 10x - 14y + 49 = 0$
28. $x^2 + y^2 - 2x - y + 1 = 0$
29. $x + y - 71 = 0$; $x + y + 57 = 0$
30. $x^2 + y^2 - 2x - 6y + 8 = 0$

CONCEPT APPLICATION

Level 1

1. (b) **2.** (b) **3.** (c) **4.** (d) **5.** (a) **6.** (c) **7.** (a) **8.** (b) **9.** (d) **10.** (c)
11. (b) **12.** (d) **13.** (d) **14.** (c) **15.** (c)

Level 2

16. (d) **17.** (c) **18.** (b) **19.** (c) **20.** (c) **21.** (c) **22.** (d) **23.** (b) **24.** (d) **25.** (a)
26. (c) **27.** (d) **28.** (a) **29.** (c) **30.** (b)

Level 3

31. (d) **32.** (d) **33.** (b) **34.** (c) **35.** (b) **36.** (c) **37.** (a) **38.** (d) **39.** (c) **40.** (c)
41. (a) **42.** (c) **43.** (c) **44.** (a) **45.** (a)

CONCEPT APPLICATION

Level 1

1. Three points form a triangle.
2. Recall the definition.
4. Recall the properties of right triangle with respect to geometric centres.
6. Non-intersecting lines are parallel.
7. Equations of the lines which are k units from the Y-axis.
8. When a wheel is rolling on the straight path, the distance from centre to the path remains same.
9. Angle in semicircle is 90°.
10. $\overline{AB}$ is bisector of $\angle APB$.
12. Recall the definitions of geometric centres.
14. Required locus is a line which is in the midway of X-axis and Y-axis.
15. Points on angle bisectors.

Level 2

17. (i) When a right triangle is rotated about one of its perpendicular sides, the other perpendicular side acts as radius of the base and the hypotenuse acts as the slant height of solid.

    (ii) The top of a solid is a point (vertex).

18. $\Delta ADB \cong \Delta ADC$.

19. (i) Perpendicular distance of a point $P(x_1, y_1)$ from the line, $px + qy + r = 0$, is

$$\frac{|px_1 + qy_1 + r|}{\sqrt{p^2 + q^2}}.$$

    (ii) Perpendicular distance of the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

20. (i) Given, area of a triangle $ABC = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix} = 4.$$

    (ii) Substitute $(x_2, y_2) = (2, 5)$ and $(x_3, y_3) = (3, 4)$ and obtain the relation.

21. (i) The area of the given triangle will be half that of the given parallelogram.

    (ii) Area of ΔPBC is half of the area of parallelogram $ABCD$.

    (iii) Area of ΔPBC is given by

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$$

    and proceed same as above.

22. (i) Angle in a semicircle is 90°.

    (ii) Use, $(PA)^2 = (PB)^2 + (AB)^2$ and obtain the required locus.

23. (i) The perpendicular distance from $P(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

    (ii) The distance between P and X-axis is equal to thrice the distance between P and $x = 5$.

24. (i) As P is the centroid of ΔABC, area of ΔPAB is $\frac{1}{3}$(area of ΔABC).

    (ii) Area of ΔPAB is given by

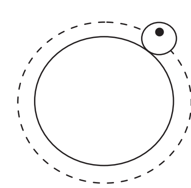
$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}.$$

25. (i) Locus of the mid-points of equal chords in a circle forms a concentric circle with the given circle.

    (ii) The required locus is a circle of radius 7 cm.

    (iii) Now, find its area.

26. The required locus is a circle.



27. The required locus is a cylinder.
28. In a right triangle, the orthocentre lies at the vertex which contains right angle.

    ∴ The given triangle is a right triangle, where $\angle B = 90°$.

    ∴ $AC^2 = AB^2 + BC^2$.

29. Given, in a ΔABC, $\angle A = \angle B + \angle C$

$\Rightarrow \angle A = 90°$, and BC is the hypotenuse.

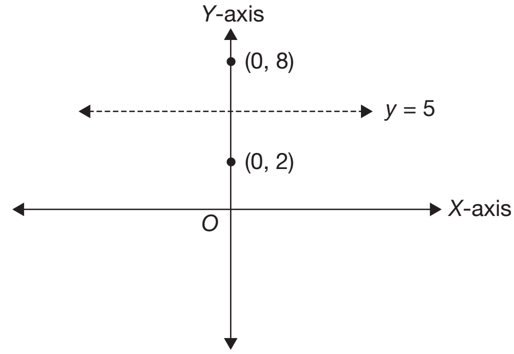
The point equidistant from the vertices of a triangle is circum-centre of the triangle.

For a right triangle, circum-centre lies at the mid-point of the hypotenuse.

$\therefore$ The required point is the mid-point of BC.

30. The locus of a point which is equidistant from two points is the perpendicular bisector of the line segment joining the points.

$\therefore$ The required locus is $y = 5$.



Level 3

31. (i) Area of triangle is given by

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}.$$

(ii) Substitute $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (1, 0)$ and obtain the relation in terms of x_1 and y_1.

32. (i) If $xy > 0$, then $x > 0$, $y > 0$ (or) $x < 0$, $y < 0$.

(ii) Given, $OP = 2$, where $O = (0, 0)$ and $P = (x, y)$.

(iii) Substitute the values and obtain the locus.

33. (i) Recall the concept of medians in a triangle.

(ii) The point of concurrence of medians is the centroid.

(iii) Centroid divides each median in the ratio 2 : 1.

36. Area of $A(2, 4)$, $B(4, 4)$ and $C(x, y)$ is 2 sq. units.

$$\frac{1}{2}\begin{vmatrix} 2-4 & 4-x \\ 4-4 & 4-y \end{vmatrix} = 2$$

$$\begin{vmatrix} -2 & 4-x \\ 0 & 4-y \end{vmatrix} = 4 \Rightarrow |2y - 8| = 4$$

$$\Rightarrow |y - 4| = 2 \Rightarrow y - 4 = \pm 2$$

$$\Rightarrow y - 6 = 0 \quad \text{or} \quad y - 2 = 0.$$

37. The required locus is a circle of radius 4 units and passing through point (2, 4).

Let $P(x, y)$ be any point on the locus.

$$\therefore \sqrt{(x-2)^2 (y-4)^2} = 4$$

$$x^2 + y^2 - 4x - 8y + 4 = 0.$$

38. Given, P is the point of intersection of the diagonals of a square $READ$.

In a square, diagonals are equal and bisect to each other.

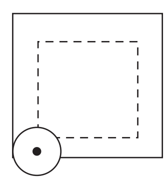
$\therefore$ P is equidistant from the vertices.

As P lies on the diagonals, P is equidistant from any two adjacent sides.

$\therefore$ Hence, the correct answer is option (d).

39. When a coin is moving on the circumference of a square, the path of the center of the coin is a square.

$\therefore$ The required locus is a square of side $(5 - 2)$ cm, i.e., 3 cm.



40. Given, $AB = 40$ cm, $BC = 41$ cm and $AC = 9$ cm.

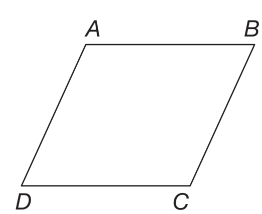
$\therefore$ ABC is right triangle, right angled at A.

$\therefore$ Ortho-centre lies at A.

$\therefore$ Hence, the correct answer is option (c).

41. Given, O is equidistant from BC and CD.

$\therefore$ O lies on the bisector of $\angle BCD$, i.e., $\overline{AC}$.



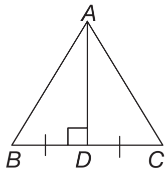
42. Given, any point on AD is equidistant from B and C.

$\therefore$ $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$.

By SAS congruence property,

$\Delta ADB \cong \Delta ADC$.

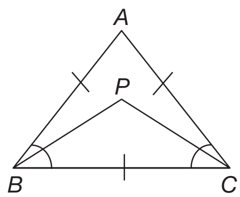
By CPCT, $AB = AC$.



43. The required locus is the union of the locus passing through the origin and making angles of 45° and 135° with the X-axis in positive direction.

That is, $x = |y|$ or $y = |x|$.

44. Here, P is the point of intersection of bisectors of $\angle B$ and $\angle C$.



$\therefore \angle PBC = \angle PCB = 30°$

$\Rightarrow \angle BPC = 180° - (30° + 30°) = 120°$.

45. Let $A = (2, 6)$, $B = (2, 8)$.

The required locus is the perpendicular bisector of $\overline{AB}$.

Let K be the point of intersection of $\overline{AB}$ and its perpendicular bisector.

$$K = \left(\frac{2+2}{2}, \frac{8+6}{2}\right) = (2, 7). \quad (1)$$

Slope of $\overline{AB} = \dfrac{8-6}{2-2}$ is not defined.

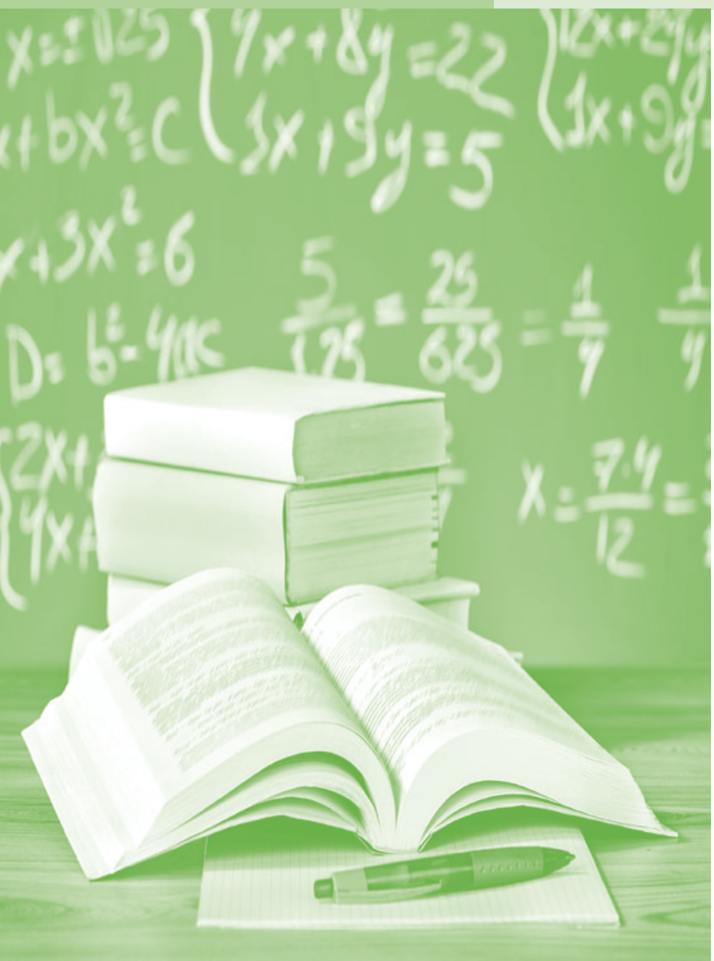
$\therefore$ $\overline{AB}$ is parallel to Y-axis. The required line is parallel to X-axis (2)

$\therefore$ The required line is $y = 7$ (From Eqs. (1) and (2)).

Chapter

16

Trigonometry



REMEMBER

Before beginning this chapter, you should be able to:

- Know the different types of angles and their values
- Understand the definitions of triangles and their types

KEY IDEAS

After completing this chapter, you should be able to:

- Know the systems of measurement of angle and their inter-relations
- Understand trigonometric ratios and identities
- Learn the trigonometric ratios of compound angles

INTRODUCTION

The word trigonometry is originated from the Greek word 'tri' means three, 'gonia' means angle and 'metron' means measure. Hence, the word trigonometry means three angle measure, i.e., it is the study of geometrical figures which have three angles, i.e., triangles.

The great Greek mathematician Hipapachus of 140 BCE gave relation between the angles and sides of a triangle. Further trigonometry is developed by Indian (Hindu) mathematicians. This was migrated to Europe via Arabs.

Trigonometry plays an important role in the study of Astronomy, Surveying, Navigation and Engineering. Now a days it is used to predict stock market trends.

ANGLE

A measure formed between two rays having a common initial point is called an angle. The two rays are called the arms or sides of the angle and the common initial point is called the vertex of the angle.

In the Fig. 16.1, OA is said to be the initial side and the other ray OB is said to be the terminal side of the angle.

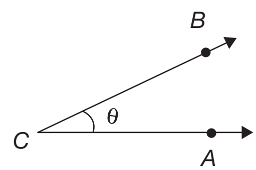


Figure 16.1

The angle is taken positive when measured in anti-clockwise direction and is taken negative when measured in clockwise direction (see Fig. 16.2).

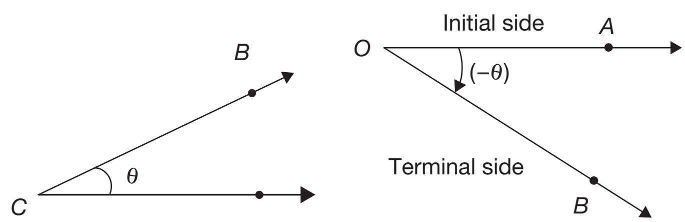


Figure 16.2

Systems of Measurement of Angle

We have the following systems of the measurement of angle.

Sexagesimal System

In this system, the angle is measured in degrees (°).

Degree When the initial ray is rotated through $\left(\frac{1}{360}\right)$ of one revolution, we say that an angle of one degree (1°) is formed at the initial point. A degree is divided into 60 equal parts and each part is called one minute (1 m).

Further, a minute is divided into 60 equal parts called seconds (″).
So, 1 right angle $= 90°$
$1° = 60'$ (minutes) and
$1' = 60''$ (seconds)

Note This system is also called the British system.

Centesimal system

In this system, the angle is measured in grades.

Grade When the initial ray is rotated through $\left(\frac{1}{400}\right)$ of one revolution, an angle of one grade is said to be formed at the initial point. It is written as 1^g.

Further one grade is divided into 100 equal parts called minutes and one minute is further divided into 100 equal parts called seconds.

So, 1 right angle = 100^g
$1^g = 100'$ (minutes) and
$1' = 100''$ (seconds)

Note This system is also called the French system.

Circular System

In this system, the angle is measured in radians.

Radian The angle subtended by an arc of length equal to the radius of a circle at its centre is said to have a measure of one radian (see Fig. 16.3). It is written as 1^c.

Note This measure is also known as radian measure.

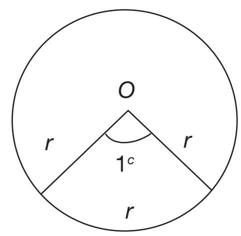


Figure 16.3

Relation Between the Units of the Three Systems

When a rotating ray completes one revolution, the measure of angle formed about the vertex is $360°$ or 400^g or $2\pi^c$.

So,

$$360° = 400^g = 2\pi^c$$

(or)

$$90° = 100^g = \frac{\pi^c}{2}.$$

For convenience, the above relation can be written as,

$$\frac{D}{90} = \frac{G}{100} = \frac{R}{\frac{\pi}{2}}.$$

Where, D denotes degrees, G grades and R radians.

Remember

1. $1° = \frac{\pi}{180}$ radians = 0.0175 radians (approximately).
2. $1^c = \frac{180}{\pi}$ degrees = $57°17'44''$ (approximately).

Notes

1. The measure of an angle is a real number.
2. If no unit of measurement is indicated for any angle, it is considered as radian measure.

EXAMPLE 16.1

Convert 60° into circular measure.

SOLUTION

Given, $D = 60°$

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$

So, $\dfrac{60}{90} = \dfrac{R}{\frac{\pi}{2}}$

$$\frac{2}{3} \times \frac{\pi}{2} = R$$

$$R = \frac{\pi}{3}.$$

Hence, 60° in circular measure is $\dfrac{\pi}{3}$.

EXAMPLE 16.2

Convert 180^g into sexagesimal measure.

SOLUTION

Given, $G = 180^g$

We have, $\dfrac{D}{90} = \dfrac{G}{100}$

So, $\dfrac{D}{90} = \dfrac{180}{100}$

$$D = \frac{9}{5} \times 90 = 162.$$

Hence, sexagesimal measure of 180^g is 162°.

EXAMPLE 16.3

The angle measuring $\dfrac{\pi}{4}$ when expressed in sexagesimal measure is _________.

SOLUTION

Given, $R = \dfrac{\pi}{4}$.

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$.

So, $\dfrac{D}{90} = \dfrac{\frac{\pi}{4}}{\frac{\pi}{2}}$

$$D = \frac{2}{4} \times 90 = 45°$$

Hence, the sexagesimal measure of $\dfrac{\pi^c}{4}$ is 45°.

TRIGONOMETRIC RATIOS

Let AOB be a right triangle with $\angle AOB$ as 90°. Let $\angle OAB$ be θ. Notice that $0° < \theta < 90°$. That is, θ is an acute angle (see Fig. 16.4).

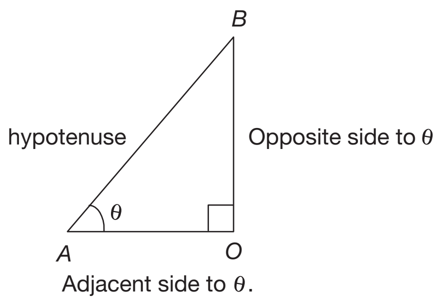


Figure 16.4

We can define six possible ratios among the three sides of the triangle AOB, known as trigonometric ratios. They are defined as follows.

1. Sine of the angle θ or simply $\sin\theta$:

$$\sin\theta = \frac{\text{Side opposite to angle } \theta}{\text{Hypotenuse}} = \frac{OB}{AB}.$$

2. Cosine of the angle θ or simply $\cos\theta$:

$$\cos\theta = \frac{\text{Side adjacent to angle } \theta}{\text{Hypotenuse}} = \frac{OA}{AB}.$$

3. Tangent of the angle θ or simply $\tan\theta$:

$$\tan\theta = \frac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta} = \frac{OB}{OA}.$$

4. Cotangent of the angle θ or simply $\cot\theta$:

$$\cot\theta = \frac{\text{Side adjacent to } \theta}{\text{Side opposite to } \theta} = \frac{OA}{OB}.$$

5. Cosecant of the angle θ or simply $\operatorname{cosec}\theta$:

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \theta} = \frac{AB}{OB}.$$

6. Secant of the angle θ or simply $\sec\theta$:

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta} = \frac{AB}{OA}.$$

Observe that,

1. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$.

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$.

EXAMPLE 16.4

If $\cos\theta = \frac{3}{5}$, then find the values of $\tan\theta$, $\operatorname{cosec}\theta$.

SOLUTION

Given, $\cos\theta = \frac{3}{5}$

Let PQR be the right triangle such that $\angle QPR = \theta$ (see Fig. 16.5)

Assume that $PQ = 3$ and $PR = 5$.

Then, $QR = \sqrt{PR^2 - PQ^2} = \sqrt{25-9} = 4$.

So, $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{QR}{PQ} = \frac{4}{3}$ and $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{PR}{QR} = \frac{5}{4}$.

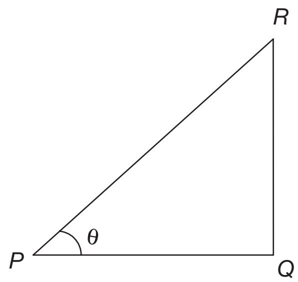


Figure 16.5

Pythagorean Triplets

1. 3, 4, 5
2. 5, 12, 13
3. 8, 15, 17
4. 7, 24, 25
5. 9, 40, 41

Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Values of Trigonometric Ratios for Specific Angles

Trigonometric Ratios	Angle				
	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

From the above table, we observe that

1. $\sin\theta = \cos\theta$, $\tan\theta = \cot\theta$ and $\sec\theta = \operatorname{cosec}\theta$, if $\theta = 45°$.
2. $\sin\theta$ and $\tan\theta$ are increasing functions in $0° \leq \theta \leq 90°$.
3. $\cos\theta$ is a decreasing function in $0° \leq \theta \leq 90°$.

EXAMPLE 16.5

Find the value of $\sin 60° + 2\tan 45° - \cos 30°$.

SOLUTION

$$\sin 60° + 2\tan 45° - \cos 30°$$

$$= \frac{\sqrt{3}}{2} + 2(1) - \frac{\sqrt{3}}{2} = 2$$

$$\therefore\ \sin 60° + 2\tan 45° - \cos 30° = 2.$$

EXAMPLE 16.6

Using the trigonometric table and evaluate

(a) $\sin^2 45° + \cos^2 45°$

(b) $\sec^2 30° - \tan^2 30°$

SOLUTION

(a) $\sin^2 45° + \cos^2 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} = 1.$$

Hence, $\sin^2 45° + \cos^2 45° = 1$.

(b) $\sec^2 30° - \tan^2 30°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1.$$

Hence, $\sec^2 30° - \tan^2 30° = 1$.

EXAMPLE 16.7

Find the values of $\dfrac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°}$ and $\tan 90°$; what do you observe?

SOLUTION

$$\frac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°}$$

$$= \frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{\frac{1+3}{\sqrt{3}}}{1-1} = \frac{4}{0}$$

= not defined or ∞.

$= \tan 90°$

Hence, $\dfrac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°} = \tan 90°$.

Trigonometric Ratios of Compound Angles

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and
   $\sin(A-B) = \sin A \cos B - \cos A \sin B$.
2. $\cos(A+B) = \cos A \cos B - \sin A \sin B$ and
   $\cos(A-B) = \cos A \cos B + \sin A \sin B$.
3. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ and

   $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

Also, by taking $A = B$ in the above relations, we get,

1. $\sin 2A = 2\sin A \cos A$.
2. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.
3. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$.

EXAMPLE 16.8

Find the value of $\sin 15°$.

SOLUTION

We have, $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\therefore \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

EXAMPLE 16.9

Find the value of $\tan 75°$.

SOLUTION

We have, $\tan 75° = \tan(45° + 30°)$

$$\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}.$$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{3+1+2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$$\therefore\ \tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } 2 + \sqrt{3}.$$

EXAMPLE 16.10

Eliminate θ from the equations $a = x \sec\theta$ and $b = y \tan\theta$.

SOLUTION

We know that trigonometric ratios are meaningful when they are associated with some θ, i.e., we cannot imagine any trigonometric ratio without θ. Eliminate θ means, eliminating the trigonometric ratios by using suitable identity.

Given, $a = x \sec\theta$ and $b = y \tan\theta$

$$\frac{a}{x} = \sec\theta \text{ and } \frac{b}{y} = \tan\theta.$$

We know that, $\sec^2\theta - \tan^2\theta = 1$.

So,
$$\left(\frac{a}{x}\right)^2 - \left(\frac{b}{y}\right)^2 = 1$$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1.$$

Hence, the required equation is $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

EXAMPLE 16.11

Find the relation obtained by eliminating θ from the equations $x = a \cos\theta + b \sin\theta$ and $y = a \sin\theta - b \cos\theta$.

SOLUTION

Given, $x = a \cos\theta + b \sin\theta$

$$x^2 = (a \cos\theta + b \sin\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab \cos\theta \sin\theta.$$

Also $y = a \sin\theta - b \cos\theta$

$$y^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab \sin\theta \cos\theta$$

$$x^2 + y^2 = a^2 (\sin^2\theta + \cos^2\theta) + b^2 (\cos^2\theta + \sin^2\theta)$$

$$= a^2 + b^2.$$

Hence, the required relation is $x^2 + y^2 = a^2 + b^2$.

EXAMPLE 16.12

Eliminate θ from the equations $P = a \operatorname{cosec}\theta$ and $Q = a \cot\theta$.

SOLUTION

Given, $P = a \operatorname{cosec}\theta$ and $Q = a \cot\theta$

$$\frac{P}{a} = \operatorname{cosec}\theta \text{ and } \frac{Q}{a} = \cot\theta.$$

We know that, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\left(\frac{P}{a}\right)^2 - \left(\frac{Q}{a}\right)^2 = 1$$

$$P^2 - Q^2 = a^2.$$

Hence, the required relation is $P^2 - Q^2 = a^2$.

EXAMPLE 16.13

Eliminate θ from the equations $s = \sin\theta + \operatorname{cosec}\theta$ and $r = \sin\theta - \operatorname{cosec}\theta$.

SOLUTION

Given, $s = \sin\theta + \operatorname{cosec}\theta$ (1)

$r = \sin\theta - \operatorname{cosec}\theta$ (2)

Adding Eqs. (1) and (2), we get

$$s + r = 2 \sin\theta$$

$$\sin\theta = \frac{s + r}{2}. \quad (3)$$

Subtracting Eq. (2) from Eq. (1), we get

$$s - r = 2 \operatorname{cosec}\theta$$

$$\operatorname{cosec}\theta = \frac{s - r}{2}. \quad (4)$$

Multiplying Eqs. (3) and (4), we get

$$\sin\theta \cdot \operatorname{cosec}\theta = \left(\frac{s+r}{2}\right)\left(\frac{s-r}{2}\right)$$

$$\sin\theta \cdot \frac{1}{\sin\theta} = \frac{s^2 - r^2}{4}$$

$$1 = \frac{s^2 - r^2}{4} \quad \text{(or)} \quad s^2 - r^2 = 4.$$

Hence, by eliminating θ, we obtain the relation $s^2 - r^2 = 4$.

EXAMPLE 16.14

If $\sin(A+B) = \dfrac{\sqrt{3}}{2}$ and cosec $A = 2$, then find A and B.

SOLUTION

Given, $\sin(A+B) = \dfrac{\sqrt{3}}{2}$

$$\sin(A+B) = \sin 60^\circ$$
$$A + B = 60^\circ \quad (1)$$
$$\text{cosec } A = 2 = \text{cosec } 30^\circ$$
$$A = 30^\circ \quad (2)$$

From Eqs. (1) and (2), we have

$A = 30^\circ$ and $B = 30^\circ$.

EXAMPLE 16.15

Find the length of the chord which subtends an angle of 90° at the centre 'O' and which is at a distance of 6 cm from the centre.

SOLUTION

Let the chord be PQ and OR be the distance of chord from the centre of circle (see Fig. 16.6).

Given $\angle POQ = 90^\circ$ and $OR = 6$ cm.

Clearly, $\Delta POR \cong \Delta QOR$, by SSS axiom of congruency.

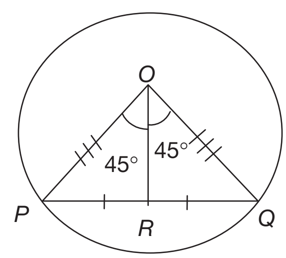


Figure 16.6

$$\angle POR = \angle QOR = \frac{1}{2}\angle POQ = 45^\circ.$$

$$\text{In } \Delta POR, \tan 45^\circ = \frac{PR}{OR}.$$

$$1 = \frac{PR}{6}$$

$$PR = 6 \text{ cm.}$$

$\therefore$ The length of the chord $PQ = 2\ PR = 12$ cm.

EXAMPLE 16.16

Evaluate $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$.

SOLUTION

Given $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

Rationalize the denominator, i.e.,

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1+\cos^2\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \quad (\because \sin^2\theta+\cos^2\theta=1)$$

$$=\frac{1-\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$

$$=\operatorname{cosec}\theta-\cot\theta\,.$$

$$\therefore\ \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta.$$

EXAMPLE 16.17

If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2-bx-1=0$, then find the relation between a and b.

SOLUTION

The given equation is $ax^2-bx-1=0$.

Here, $a=a$, $b=-b$ and $c=-1$.

$$\sin\alpha+\cos\alpha=\frac{-b}{a}=\frac{-(-b)}{a}=\frac{b}{a}$$

$$\sin\alpha\cdot\cos\alpha=\frac{c}{a}=\frac{-1}{a}.$$

Consider,

$$\sin\alpha+\cos\alpha=\frac{-b}{a}$$

Squaring on both sides,

$$(\sin\alpha+\cos\alpha)^2=\left(\frac{-b}{a}\right)^2$$

$$=1+2\left(\frac{-1}{a}\right)=\frac{b^2}{a^2}$$

$$=1-\frac{b^2}{a^2}=\frac{2}{a}$$

$$\therefore\ a^2-b^2=2a.$$

EXAMPLE 16.18

If $\cos\alpha = \frac{2}{3}$ and $\sin\beta = \frac{1}{4}$, then find $\cos(\alpha - \beta)$.

SOLUTION

Given,

$$\cos\alpha = \frac{2}{3} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{3}$$

$$\sin\beta = \frac{1}{4} \Rightarrow \cos\beta = \frac{\sqrt{15}}{4}$$

$$\cos(\alpha - \beta) - \cos\,\alpha \cdot \cos\,\beta + \sin\,\alpha \cdot \sin\,\beta = \frac{2}{3} \times \frac{\sqrt{15}}{4} + \frac{\sqrt{5}}{3} \times \frac{1}{4}$$

$$\cos(\alpha - \beta) = \frac{2\sqrt{15} + \sqrt{5}}{12}.$$

EXAMPLE 16.19

Express the following as a single trigonometric ratio:

(a) $\sqrt{3}\cos\theta - \sin\theta$

(b) $\sin\theta - \cos\theta$

SOLUTION

(a) Given,

$$\sqrt{3}\cos\theta - \sin\theta$$

$$= 2\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$$

$$= 2(\cos\theta \cdot \cos 30^\circ - \sin\theta \cdot \sin 30^\circ)$$

$$= 2(\cos(\theta + 30^\circ))$$

$$\Rightarrow \sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + 30^\circ).$$

(b) Here,

$$\sin\theta - \cos\theta$$

$$= \sqrt{2}\left(\frac{\sin\theta - \cos\theta}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right)$$

$$= \sqrt{2}\left[\sin\theta\cos\left(\frac{\pi}{4}\right) - \cos\theta\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right). \quad (\because \sin(A - B) = \sin A\cos B - \cos A\sin B)$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. The value of 144° in circular measure = _______.

2. The value of sin 30° · sin 45° · cosec 45° · cos 30° = _______.

3. If $\dfrac{1-\tan^2 60°}{1+\tan^2 60} = \cos X$, then the value of X is _______.

4. The value of log [(sec θ + tan θ) (sec θ – tan θ)] is _______.

5. If $\dfrac{\cos 13° + \sin 13°}{\cos 13° - \sin 13°} = \tan A$, then $A =$ _______.

6. If $\sin\theta = \dfrac{1}{2}$ and 0° < θ < 90°, then cos 2θ = _______.

7. If cosec θ – cot θ = x, then cosec θ + cot θ = _______.

8. The value of $\dfrac{\sin 20° \cos 70° + \cos 20° \sin 70°}{\sin 23° \operatorname{coses} 23° + \cos 23° \sec 23°}$ is _______.

9. If $A + B = 45°$, then (1 + tan A)(1 + tan B) = _______.

10. If $\tan\theta = \dfrac{5}{6}$ and $\tan\phi = \dfrac{1}{11}$, then $\theta + \phi =$ _______.

11. If $A - B = 45°$ and $\tan A - \tan B = \sqrt{3}$, then $\tan A \cdot \tan B =$ _______.

12. If $\cos A + \sin A = \dfrac{1}{2(\cos A - \sin A)}$; (0° < A < 90°), then $\sin^2 A =$ _______.

13. The value of $(\sin A - \cos A)^2 + (\sin A + \cos A)^2$ is _______.

14. If $A + B = 60°$, then the value of $\sin A \cos B + \cos A \sin B =$ _______.

15. In ΔABC, the value of cos (A – B) – cos (C) is _______.

16. ABC is right triangle, right angled at A, then $\tan B \cdot \tan C =$ _______.

17. $\tan(A+B) = \sqrt{3}$ and $\sin A = \dfrac{1}{\sqrt{2}}$, then the value of B in radians is _______.

18. In ΔABC, the lengths of the three sides AB, BC and CA are 28 cm, 96 cm and 100 cm respectively. Find the value of cos C.

19. If sin A = cos B, where A and B are acute angles, then $A + B =$ _______.

20. ABC is a right isosceles triangle, right angled at B. Then $\sin^2 A + \cos^2 C =$ _______.

21. Evaluate: $\sin^2 60° \cos^2 45° \cos^2 60° \operatorname{cosec}^2 90°$.

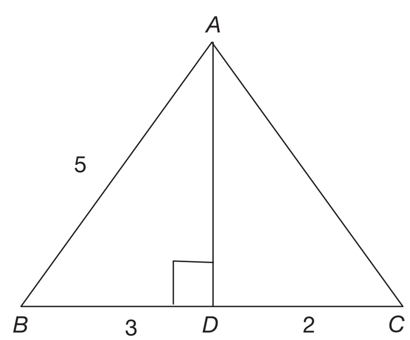
22. If $\sin\theta = \dfrac{3}{5}$ and θ is acute, then find the value of $\dfrac{\tan\theta - 2\cos\theta}{3\sin\theta + \sec\theta}$.

23. Find the values of the cos 15°.

24. Evaluate: $\operatorname{cosec}^2 30° + \sec^2 60° + \tan^2 30°$.

25. Convert $\dfrac{\pi^c}{15}$ into the other two systems.

26. In the adjoining figure, find the values of tan B.



27. Simplify and express $\sec^4\alpha - \tan^4\alpha$ in their least exponents.

28. Convert $\left(\dfrac{200}{3}\right)^g$ into other two systems.

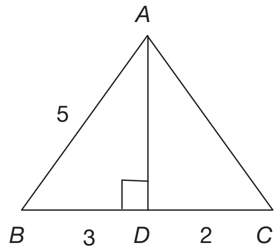
29. Convert 270° into other two systems.

30. Evaluate: $\cos 0° + \sqrt{2} \sec 45° - \sqrt{3} \tan 30°$.

PRACTICE QUESTIONS

Short Answer Type Questions

31. In the adjoining figure, find the value of sin C.



32. A wheel makes 200 revolutions in 2 minutes. Find the measure of the angle it describes at the centre in 24 seconds.

33. Find the length of the chord subtending an angle of 120° at the centre of the circle whose radius is 4 cm.

34. Find the value of tan 75°.

35. If $\sec^2\alpha + \cos^2\alpha = 2$, then find the value of $\sec\alpha + \cos\alpha$.

36. Eliminate θ from the equations, $a = x\sin\theta - y\cos\theta$ and $b = x\cos\theta + y\sin\theta$.

37. If $\cos\alpha = \dfrac{12}{13}$ and $\sin\beta = \dfrac{4}{5}$, then find $\sin(\alpha + \beta)$.

38. Express $\sin\theta$ in terms of $\cot\theta$.

39. If $\sin\alpha = \dfrac{4}{5}$, then find the value of $\sin 2\alpha$.

40. Find the value of $\sin 2\alpha$, if $\sin\alpha + \cos\alpha = \dfrac{1}{3}$.

41. The length of the minutes hand of a wall clock is 36 cm. Find the distance covered by its tip in 35 minutes.

42. If $\tan 2\alpha = \dfrac{3}{4}$, then find $\tan\alpha$.

43. If $A + B = 45°$, then find the value of $\tan A + \tan B + \tan A \tan B$.

44. If $\sin(A + B) = \dfrac{\sqrt{3}}{2}$ and $\cot(A - B) = 1$, then find A and B.

45. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $lx^2 - mx - n = 0$, then find the relation between l, m and n.

Essay Type Questions

46. Show that $3(\sin x + \cos x)^4 - 6(\sin x + \cos x)^2 + 4(\sin 6x + \cos 6x) = 1$.

47. If $\cos^2\alpha + \cos\alpha = 1$, then find the value of $4\sin^2\alpha + 4\sin^4\alpha + 2$.

48. Obtain the relation by eliminating θ from the equations, $x = a + r\cos\theta$ and $y = b + r\sin\theta$.

49. One of the angles of a rhombus is 60° and the length of the diagonal opposite to it is 6 cm. Find the area of the rhombus (in sq.cm).

50. If $\alpha + \beta + \gamma = 90°$, then $\tan\alpha + \tan\beta + \tan\alpha\tan\beta\cot\gamma$ is _______.

CONCEPT APPLICATION

Level 1

1. The length of the minutes hand of a wall clock is 6 cm. Find the distance covered by the tip of the minutes hand in 25 minutes.

   (a) $\dfrac{270}{7}$ cm (b) 110 cm
   (c) $\dfrac{88}{7}$ cm (d) $\dfrac{110}{7}$ cm

2. The value of $\tan 10° \cdot \tan 20° \cdot \tan 45° \cdot \tan 70° \cdot \tan 80° =$

   (a) 1 (b) $\dfrac{1}{\sqrt{3}}$
   (c) 0 (d) 8

3. The value of $\sin\theta$ in terms of $\tan\theta$ is _______.

   (a) $\dfrac{\tan\theta}{\sqrt{1-\tan^2\theta}}$ (b) $\dfrac{\tan^2\theta}{\sqrt{1+\tan^2\theta}}$
   (c) $\dfrac{\tan^2\theta}{\sqrt{1-\tan^2\theta}}$ (d) $\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

4. The value of $\sin^2 60° + \cos^2 30° - \sin^2 45°$ is ______.

(a) 1 (b) sin 90°

(c) $\frac{1}{2}$ (d) Both (a) and (b)

5. The simplified value of $\operatorname{cosec}^2\alpha\left(1+\frac{1}{\sec\alpha}\right)\left(1-\frac{1}{\sec\alpha}\right)$ is ________.

(a) 1 (b) 0

(c) 2 (d) −1

6. A wheel makes 240 revolutions in one minute. Find the measure of the angle it describes at the centre in 15 seconds.

(a) $60\,\pi^c$ (b) $120\,\pi^c$

(c) $8\,\pi^c$ (d) None of these

7. If $3\tan A = 4$, then find the value of $\frac{2\sin A - 7\cos A}{3\cos A + 4}$.

(a) $\frac{-13}{29}$ (b) $\frac{13}{11}$

(c) ∞ (d) $\frac{29}{13}$

8. If $\tan\alpha = 3$ and $\tan\beta = \frac{1}{2}$, then which of the following is true?

(a) $\alpha+\beta = \frac{\pi^c}{4}$ (b) $\alpha\beta = \frac{\pi^c}{4}$

(c) $\alpha-\beta = \frac{\pi^c}{4}$ (d) None of these

9. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then which of the following is true?

(a) $A+B = \frac{\pi^c}{4}$ (b) $A-B = \frac{\pi^c}{4}$

(c) $2A+B = \frac{\pi^c}{4}$ (d) $A+2 = \frac{\pi^c}{4}$

10. If $\sin A = \frac{1}{2}$ and $90° < A < 180°$, then the value of A in circular measure is ______.

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{7\pi}{6}$

11. The value of $\frac{3\pi^c}{5}$ in sexagesimal measure is ______.

(a) 216° (b) 144°

(c) 128° (d) 108°

12. The value of $\log_{\sec\theta}(1-\sin^2\theta)$ is ______.

(a) 2 (b) −2 (c) 0 (d) 1

13. If $\sin A = \frac{\sqrt{3}}{2}$ and A is an acute angle, then find the value of $\frac{\tan A - \cot A}{\sqrt{3} + \operatorname{cosec} A}$.

(a) $\frac{-2}{5}$ (b) $\frac{2}{5}$

(c) $\frac{2}{3+2\sqrt{3}}$ (d) −2

14. The length of an arc, which subtends an angle of 30° at the centre of the circle of radius 42 cm is ______.

(a) 22 cm (b) 44 cm

(c) 11 cm (d) $\frac{22}{7}$ cm

15. If $\sin^4\theta + \cos^4\theta = \frac{1}{2}$, then find $\sin\theta\cos\theta$.

(a) $\pm\frac{1}{8}$ (b) $\pm\frac{1}{4}$

(c) ±1 (d) $\pm\frac{1}{2}$

16. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ and $\tan\theta$ in terms of a and b are

(a) $\frac{\sqrt{b^2-a^2}}{b}$ and $\frac{b}{\sqrt{b^2-a^2}}$

(b) $\frac{b}{\sqrt{b^2-a^2}}$ and $\frac{a}{\sqrt{b^2-a^2}}$

(c) $\frac{\sqrt{a^2-b^2}}{a}$ and $\frac{b}{\sqrt{a^2-b^2}}$

(d) $\frac{\sqrt{b^2-a^2}}{b}$ and $\frac{a}{\sqrt{b^2-a^2}}$

PRACTICE QUESTIONS

17. The distance covered by the tip of a minute hand in 35 minutes is 33 cm. What is the length of the minute hand?

(a) 6 cm (b) 9 cm
(c) 10 cm (d) 12 cm

18. If $\sin\alpha = \frac{4}{5}$, where $(0° \leq \alpha \leq 90°)$, then find $\sin 2\alpha$.

(a) $\frac{12}{25}$ (b) $-\frac{24}{25}$
(c) $\frac{25}{24}$ (d) $\frac{24}{25}$

19. In a ΔABC, $\cos\left(\frac{A+B}{2}\right) =$ _____.

(a) $\cos\frac{C}{2}$ (b) $-\sin\frac{C}{2}$
(c) $\cos\left(\frac{A-B}{2}\right)$ (d) $\sin\frac{C}{2}$

20. $\sin^4\theta - \cos^4\theta =$

(a) -1 (b) $\cos 2\theta$
(c) $2\sin^2\theta - 1$ (d) $\sin 2\theta$

21. If $P : Q = \tan 2A : \cos A$ and $Q : R = \cos 2A : \sin 2A$, then $P : R$ is _________.

(a) $\tan 2A$ (b) $2\sin A$
(c) 1 (d) $\sec A$

22. If $A = \sin\theta + \cos\theta$ and $B = \sin\theta - \cos\theta$, then which of the following is true?

(a) $A^2 + B^2 = 1$
(b) $A^2 - B^2 = 2$
(c) $A^2 + B^2 = 2$
(d) $2A^2 + B^2 = 4$

23. If $\cot(A - B) = 1$ and $\cos(A + B) = \frac{1}{2}$, then find B.

(a) $42\frac{1°}{2}$
(b) $7\frac{1°}{2}$
(c) $15\frac{1°}{2}$
(d) 60°

24. Find the measure of angle A, if $(2\sin A + 1)(2\sin A - 1) = 0$.

(a) 75° (b) 60°
(c) 45° (d) 30°

25. The value of $\cos 2\theta$ in terms of $\cot\theta$ is _______.

(a) $\frac{\cot^2\theta + 1}{\cot^2\theta - 1}$
(b) $-\frac{1+\cot^2\theta}{\cot^2\theta - 1}$
(c) $\frac{\cot^2\theta - 1}{\cot^2\theta + 1}$
(d) $\frac{1-\cot^2\theta}{1+\cot^2\theta}$

26. The simplified value of $\sin^4\alpha + \cos^4\alpha + \frac{1}{2}\sin^2 2\alpha$ is

(a) -1 (b) $\sin\alpha + \cos\alpha$
(c) 0 (d) 1

27. $\sqrt{\frac{(1+\sin 2\theta)}{1-\cos^2\theta}}\left[\text{where } \theta \in \left[0, \frac{\pi}{4}\right]\right] =$

(a) $\operatorname{cosec}^2\theta$ (b) 1
(c) $1 + \cot\theta$ (d) $1 + \tan\theta$

28. If A and B are complementary angles, then the value of $\frac{\sin^2 A + \sin^2 B}{\operatorname{cosec}^2 A - \tan^2 B}$ is _______.

(a) 0 (b) 1
(c) -1 (d) 2

29. Find the value of $4(\sin^4 30° + \cos^4 30°) - 3(\cos^2 45° + \sin^2 90°)$.

(a) $-\frac{1}{2}$ (b) -2
(c) 2 (d) $\frac{1}{2}$

30. If $\sec\theta + \tan\theta = \frac{4}{3}$, then $\sec\theta\tan\theta =$

(a) $\frac{175}{24}$ (b) $\frac{25}{576}$
(c) $\frac{27}{576}$ (d) $\frac{175}{576}$

Level 2

31. If $\sin\alpha + \cos\alpha = n$, then $\sin^6\alpha + \cos^6\alpha$ in terms of n is _______.
    (a) $4 + 3(n-1)^2$ (b) $\frac{4+3(n^2-1)}{4}$
    (c) $\frac{4-3(n^2-1)^2}{4}$ (d) None of these

32. Find the value of $\frac{\sin 25°}{\cos 35°} - \frac{\cos 25°}{\sin 35°}$.
    (a) cosec 70° (b) sin 70°
    (c) –sin 70° (d) –cosec 70°

33. The value of $\frac{3\tan 30° - \tan^3 30°}{1 - 3\tan^2 30°}$ is _______.
    (a) tan 90° (b) tan 60°
    (c) tan 45° (d) tan 30°

34. If $\cot^4 x - \cot^2 x = 1$, then the value of $\cos^4 x + \cos^2 x$ is _______.
    (a) –1 (b) 0
    (c) 2 (d) 1

35. If $\frac{\cos(A-B)}{\cos(A+B)} = \frac{8}{3}$, then $\tan A \cdot \tan B$ is _______.
    (a) $\frac{5}{11}$ (b) $\frac{7}{13}$
    (c) $\frac{8}{5}$ (d) $\frac{11}{5}$

36. If $A + B + C = 45°$, then the value of $\sum(\tan A + \tan A \tan B)$ is _______.
    (a) $1 - \pi\tan A$ (b) 1
    (c) $1 + \pi\tan A$ (d) $1 + \Sigma\tan A$

37. If $\cot\theta + \tan\theta = 2$, then the value of $\tan^2\theta - \cot^2\theta$ is _______.
    (a) 1 (b) 0
    (c) –1 (d) 2

38. The value of $\cot 5° \cdot \cot 15° \cdot \cot 25° \cdot \cot 35° \cdot \cot 45° \cdot \cot 55° \cdot \cot 65° \cdot \cot 75° \cdot \cot 85°$ is _______.
    (a) 0 (b) –1
    (c) –2 (d) 1

39. The simplified form of $\sqrt{1 + \sin\left(\frac{x}{8}\right)}$ is _______.
    (a) $\sin\left(\frac{x}{8}\right) + \cos\left(\frac{x}{8}\right)$
    (b) $\sin\left(\frac{x}{16}\right) + \cos\left(\frac{x}{16}\right)$
    (c) $\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)$
    (d) None of these

40. $\sin^4\theta + \cos^4\theta$ in terms of $\sin\theta$ is _______.
    (a) $2\sin^4\theta - 2\sin^2\theta - 1$
    (b) $2\sin^4\theta - 2\sin^2\theta + 1$
    (c) $2\sin^4\theta + 2\sin^2\theta - 1$
    (d) $2\sin^4\theta - 2\sin^2\theta$

41. If $\tan(A - B) = 1$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, then find B.
    (a) $42\frac{1°}{2}$ (b) $7\frac{1°}{2}$
    (c) $15\frac{1°}{2}$ (d) 60°

42. If $\sin\beta + \cos\beta = \frac{5}{4}$, then find the value of $\sin\beta \cdot \cos\beta$.
    (a) $\frac{1}{4}$ (b) $\frac{9}{32}$
    (c) $\frac{5}{16}$ (d) $\frac{11}{32}$

43. If $\sin^2\alpha + \sin\alpha = 1$, then the value of $\cos^4\alpha + \cos^2\alpha$ is _______.
    (a) 0 (b) –1
    (c) 1 (d) 2

44. If $\tan P + \cot P = 2$, then the value of $\tan^n P + \cot^n P$ is _______.
    (a) 2 (b) 2^n
    (c) 2^{n-1} (d) 2^{n+1}

45. The value of $\sqrt{3}\tan 10° + \sqrt{3}\tan 20° + \tan 10° \cdot \tan 20°$ is _______.
    (a) –1 (b) 0
    (c) 1 (d) 2

PRACTICE QUESTIONS

46. If tan $(A+B)=1$ and $(A-B)=\dfrac{1}{\sqrt{2}}$, then find A and B.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\tan(A+B)=1 \Rightarrow \tan(A+B)=\tan 45°$ and

$\sin(A-B)=\dfrac{1}{\sqrt{2}} \Rightarrow \sin(A-B)=\sin 45°$.

(B) $2A=90° \Rightarrow A=45°$.

(C) $A+B=45°$ and $A-B=45°$.

(D) $\therefore A=45°$ and $B=0°$.

(a) DBCA (b) CABD

(c) ACDB (d) ACBD

47. If $\cos(A-B)=\dfrac{1}{2}$ and $\sin(A+B)=\dfrac{\sqrt{3}}{2}$, then find A and B.

The following are the steps involved in solving the following problem. Arrange them in sequential order.

(A) $2A=120° \Rightarrow A=60°$.

(B) $\therefore A=60°, B=0°$.

(C) $\cos(A-B)=\dfrac{1}{2} \Rightarrow \cos(A-B)=\cos 60°$ and

$\sin(A+B)=\dfrac{\sqrt{3}}{2} \Rightarrow \sin(A+B)=\sin 60°$.

(D) $A+B=60°$ and $A-B=60°$.

(a) DCAB (b) CADB

(c) DCBA (d) CDAB

48. If $\sin\alpha+\sin\beta+\sin\gamma=3$, then $\sin^3\alpha+\sin^3\beta+\sin^3\gamma=$ ________.

(a) 0 (b) 2

(c) 3 (d) 1

49. $\sec^4\theta-\sec^2\theta=$ ________.

(a) $\tan^2\theta\sec^2\theta$ (b) $\dfrac{\tan^2\theta}{\sec^2\theta}$

(c) $\operatorname{cosec}^2\theta\cot^2\theta$ (d) $\dfrac{\cot^2\theta}{\operatorname{cosec}^2\theta}$

50. $\sin\theta+\cos\theta=\sqrt{2}$, then $\sin^{16}\theta=$ ________.

(a) $\dfrac{\cos^{16}\theta}{2^{16}}$ (b) $\dfrac{\sec^{16}\theta}{2^8}$

(c) $\dfrac{1}{2\sec^{16}\theta}$ (d) $\dfrac{1}{2^{16}\cos^{16}\theta}$

Level 3

51. If $\dfrac{\sin(x-y)}{\sin(x+y)}=\dfrac{3}{5}$, then $\tan x\cdot\cot y$ is ________.

(a) 1 (b) 2

(c) 3 (d) 4

52. $\sqrt{\sqrt{16\sin^4\theta+\operatorname{cosec}^4\theta+8}-4}=$

(a) $2\sin\theta-\operatorname{cosec}\theta$

(b) $2\sin\theta+\operatorname{cosec}\theta$

(c) $2\operatorname{cosec}\theta+\cos\theta$

(d) $2\operatorname{cosec}\theta+\sin\theta$

53. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $px^2+qx+r=0$ $(p\neq 0)$, then which of the following relation holds good?

(a) $q^2-p^2=2pr$

(b) $p^2-q^2=2pr$

(c) $p^2+q^2+2pr=0$

(d) $(p-q)^2=2pr$

54. If $\sin\alpha-\cos\alpha=m$, then the value of $\sin^6\alpha+\cos^6\alpha$ in terms of m is ________.

(a) $1+\dfrac{3}{4}(1+m^2)^2$ (b) $1-\dfrac{4}{3}(m^2-1)^2$

(c) $1-\dfrac{3}{4}(1-m^2)^2$ (d) $1-\dfrac{3}{4}(1+m^2)^2$

55. The value of $\dfrac{8\sec^4\theta-8\tan^4\theta}{4+8\tan^2\theta}-\dfrac{2\cos^6\theta+2\sin^6\theta}{1-3\sin^2\theta\cos^2\theta}$ is ________.

(a) 0 (b) 1

(c) −1 (d) 3

56. If $\dfrac{1+\tan\theta}{1-\tan\theta}=\sqrt{3}$, then find the value of θ.

(a) 30° (b) 25°

(c) 15° (d) 45°

PRACTICE QUESTIONS

57. If $A \times B = 1$, $A + B = \text{cosec}\,\theta \cdot \sec\theta$ then $\dfrac{A}{B}$ can be ______.

(a) $\tan^2\theta$ (b) $\sec^2\theta$

(c) $\sin^2\theta\cos^2\theta$ (d) $\text{cosec}^2\theta\sec^2\theta$

58. If $7\sin^2\theta + 3\cos^2\theta = 4$, then find $\tan\theta$.

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{2}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) 1

59. If $\sin(A+B) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ and $\sec A = 2$, then the value of B in circular measure is ______.

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{3\pi}{5}$

(c) $\dfrac{7\pi}{5}$ (d) $\dfrac{5\pi}{12}$

60. If $\tan\theta - \cot\theta = 7$, then the value of $\tan^3\theta - \cot^3\theta$ is

(a) 250 (b) 354

(c) 343 (d) 364

61. If $x^n = a^m\cos^4\theta$ and $y^n = b^m\sin^4\theta$, then

(a) $\dfrac{x^{n/2}}{a^{m/2}} + \dfrac{y^{n/2}}{b^{m/2}} = 1$ (b) $\dfrac{x^n}{a^m} + \dfrac{y^n}{b^m} = 1$

(c) $\dfrac{x^{n/2}}{y^{n/2}} + \dfrac{a^{m/2}}{b^{m/2}} = 1$ (d) None of these

62. If $\sin^2\theta + 2\cos^2\theta + 3\sin^2\theta + 4\cos^2\theta + \cdots +$ 40 terms $= 405$ where θ is acute, then find the value of $\tan\theta$.

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) ∞

63. If $\sin\alpha + \sin\beta = 2$, then find the value of $\cos^2\alpha + \cos^2\beta$.

(a) 0 (b) 1

(c) 2 (d) 3

64. If $x = a^2\cos 3\theta$ and $y = b^2\sin 3\theta$, then

(a) $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$

(b) $\left(\dfrac{x}{a^2}\right)^{1/3} + \left(\dfrac{y}{b^2}\right)^{1/3} = 1$

(c) $\left(\dfrac{x^2}{a^2}\right)^{2/3} + \left(\dfrac{y^2}{b^2}\right)^{2/3} = 1$

(d) $\left(\dfrac{x}{a^2}\right)^{2/3} + \left(\dfrac{y}{b^2}\right)^{2/3} = 1$

65. If $\cos^2\theta + 2\sin^2\theta + 3\cos^2\theta + 4\sin^2\theta + \cdots$ (200 terms) $= 10025$, where θ is acute, then the value of $\sin\theta - \cos\theta$ is

(a) $\dfrac{1-\sqrt{3}}{2}$

(b) $\dfrac{1+\sqrt{3}}{2}$

(c) $\dfrac{\sqrt{3}-1}{2}$

(d) 0

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\frac{4\pi}{5}$
2. $\frac{\sqrt{3}}{4}$
3. $120°$
4. 0
5. $58°$
6. $\frac{1}{2}$
7. $\frac{1}{x}$
8. $\frac{1}{2}$
9. 2
10. $45°$
11. $\sqrt{3}-1$
12. $\frac{1}{4}$
13. 2
14. $\frac{\sqrt{3}}{2}$
15. $2\cos A \cos B$
16. 1(B and C are complementary)
17. $\frac{\pi}{12}$
18. $\cos C = \frac{24}{25}$
19. $A + B = 90°$
20. $1(A = 45°,\ C = 45°)$

Shot Answer Type Questions

21. $\frac{3}{32}$
22. $\frac{-17}{61}$
23. $\frac{\sqrt{3}+1}{2\sqrt{2}}$
24. $8\frac{1}{3}$
25. $\frac{40^g}{3}$
26. $\frac{4}{3}$
27. $\sec^2\alpha + \tan^2\alpha$
28. $60°, \frac{\pi}{3}$
29. $300^g, \frac{3\pi^c}{2}$
30. 2
31. $\frac{2\sqrt{5}}{5}$
32. $80\pi^c$
33. $4\sqrt{3}$ cm
34. $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
35. ± 2
36. $x^2 + y^2 = a^2 + b^2$
37. $\frac{63}{65}$
38. $\sqrt{\frac{1}{1+\cot^2\theta}}$
39. $\frac{24}{25}$
40. $-\frac{1}{X}$
41. 132 cm
42. $\frac{1}{3}$ or -3
43. 1
44. $A = 52\frac{1}{2}°$ and $B = 7\frac{1}{2}°$
45. $l^2 - m^2 = 2nl$

ANSWER KEYS

Essay Type Questions

47. 6

48. $(x - a)^2 + (y - b)^2 = r^2$

49. $18\sqrt{3}$

50. cot r

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (c)
21. (d)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (d)

Level 2

31. (c)	**32.** (d)	**33** (a)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (b)
41. (b)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (c)	**46.** (d)	**47.** (d)	**48.** (c)	**49.** (a)	**50.** (d)

Level 3

51. (d)	**52.** (c)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (a)	**60.** (d)
61. (a)	**62.** (b)	**63.** (a)	**64.** (d)	**65.** (a)					

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (i) The minutes hand covers an angle of 6° per minute.
   (ii) Use $l = r \times \theta$.
2. Use $\tan\theta \cdot \tan(90 - \theta) = 1$.
3. Use $\sec^2\theta - \tan^2\theta = 1$.
4. Take the values of trigonometric ratios from the table.
5. Use $(a + b)(a - b) = a^2 - b^2$ and $\cos^2\theta + \sin^2\theta = 1$.
6. Use, 1 revolution = 2π.
7. Find $\sin A$, $\cos A$ using triangle ABC.
8. Use the identity $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.
9. Apply $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$.
10. Find the value of A and then multiply it by $\dfrac{\pi}{180}$.
11. Replace π^c by 180°.
12. Use $\cos^2\theta + \sin^2\theta = 1$.
13. Observe the values of trigonometric ratios from table.
14. Use $l = r \times \theta$, where θ is in radians.
15. Use the identity $a^2 + b^2 = (a + b)^2 - 2ab$.
16. Apply $\cos\theta = \dfrac{\text{Side adjacent to } \theta}{\text{Hypotenuse}}$,
    $\tan\theta = \dfrac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta}$.
17. (i) Minutes hand moves 6° in one minute.
    (ii) Use $l = r \times \theta$.
18. Use $\sin 2\alpha = 2\sin\alpha\cos\alpha$.
19. In triangle ABC, $\dfrac{A + B}{2} = \dfrac{180 - C}{2}$.
20. Apply $a^2 - b^2 = (a + b)(a - b)$.
22. Apply $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ and $\sin^2\theta + \cos^2\theta = 1$.
23. Use $\tan 45° = 1$ and $\sin 60° = \dfrac{\sqrt{3}}{2}$.
24. Use $(a + b)(a - b) = a^2 - b^2$ and solve.
25. Use $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$.
26. Use $a^2 + b^2 = (a + b)^2 - 2ab$.
27. $1 + \sin 2\theta = (\sin\theta + \cos\theta)2$, when $\theta \in \left[0, \dfrac{\pi}{4}\right]$.
28. (i) $\sin^2 A = \sin^2(90 - B)$. If A and B complementary angles.
    (ii) If $A + B = 90°$, then $\sin A = \cos B$ and $\tan B = \cot A$.
29. Use trigonometric ratios values from table.
30. Squaring both sides of given equation.

Level 2

31. (i) $\sin^6\alpha + \cos^6\alpha = (\sin^2\alpha)^3 + (\cos^2\alpha)^3$. Use the formula $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.
    (ii) Find $\sin\alpha \cdot \cos\alpha$ by substituting $\sin\alpha + \cos\alpha = n$ in the above equation.
32. (i) Use $\cos A\cos B - \sin A\sin B = \cos(A + B)$ and $\sin 2A = 2\sin A\cos A$.
    (ii) Find the LCM of denominators and simplify.
    (iii) Use the formula $\cos(A + B) = \cos A\cos B - \sin A\sin B$.
33. (i) Use $\tan 30° = \dfrac{1}{\sqrt{3}}$.
    (ii) Substitute the value of $\tan 30°$ and simplify.
    (iii) Then check from the options.
34. Use $\operatorname{cosec}^2 x = 1 + \cot^2 x$ and $\sin^2 x + \cos^2 x = 1$.
35. (i) Use componendo and dividendo theorem, i.e., $\dfrac{a}{b} = \dfrac{a + b}{a - b}$.
    (ii) Use the formula $\cos(A - B)$ and $\cos(A + B)$.
    (iii) Take cross multiplication and find $\tan A + \tan B$.

36. Use $\tan(A + B) = \tan(45° - C)$ and proceed.

37. (i) Put $\theta = 45°$.

(ii) If $\tan\theta + \cot\theta = 2$, then $\theta = 45°$.

38. Use $\cot A \cdot \cot(90 - A) = 1$.

39. (i) Apply $\sqrt{1+\sin 2A} = \sin A + \cos A$.

(ii) $\sin\frac{x}{8} = \sin 2\left(\frac{x}{16}\right)$ and $\sin 2\theta = 2\sin\theta\cos\theta$.

(iii) Use the identity $1 = \sin^2\theta + \cos^2\theta$.

40. (i) Use $(a + b)^2 = a^2 + b^2 + 2ab$ and $\cos^2\theta = 1 - \sin^2\theta$.

(ii) Take $\cos 4\theta$ as $(1 - \sin^2\theta)^2$ and simplify.

41. (i) Use $\tan 45° = 1$ and $\sin 60° = \frac{\sqrt{3}}{2}$.

(ii) If $\tan(A - B) = 1$, then $A - B = 45°$.

(iii) If $\sin(A+B) = \frac{\sqrt{3}}{2}$, then $A + B = 60°$.

(iv) Subtract the above two equations and find B.

42. (i) By squaring on both the sides of the given equation we can obtain.

(ii) Square on both sides of $\sin\beta + \cos\beta = \frac{5}{4}$.

43. (i) Use $\sin^2 x + \cot^2 x = 1$.

(ii) $\cos^4\alpha = (1 - \sin^2\alpha)^2$.

(iii) $\cos^2\alpha = (1 - \sin^2\alpha)$.

(iv) Substitute the above values in the given expression.

44. If $x + \frac{1}{x} = 2$, then $x^n + \frac{1}{x^n} = 2$.

45. (i) Use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

(ii) Take $\tan(10° + 20°) = \tan 30°$, i.e.,

$\frac{\tan 10° + \tan 20°}{1 - \tan 10° \cdot \tan 20°} = \frac{1}{\sqrt{3}}$ and simplify.

46. ACBD is the required sequential order.

47. CDAB is the required sequential order.

48. Given, $\sin\alpha + \sin\beta + \sin\gamma = 3$.

This is possible only if $\sin\alpha = \sin\beta = \sin\gamma = 1$, i.e., if $\alpha = \beta = \gamma = 90°$.

$\therefore \sin^3\alpha + \sin^3\beta + \sin^3\gamma = 1^3 + 1^3 + 1^3 = 3$.

49. $\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1) = \sec^2\theta\tan^2\theta$.

50. $\sin\theta + \cos\theta = \sqrt{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow 2\sin\theta\cos\theta = 1$

$\sin\theta\cos\theta = \frac{1}{2}$

$\sin^{16}\theta = \frac{1}{2^{16}\cos^{16}\theta}$.

Level 3

51. (i) Apply componendo and dividendo rule, i.e., $\frac{a}{b} = \frac{a+b}{a-b}$.

(ii) Use the formula $\sin(x - y)$ and $\sin(x + y)$.

(iii) And then apply cross multiplication.

52. Use $a^2 + b^2 + 2ab = (a + b)^2$ and $\sin\theta \cdot \operatorname{cosec}\theta = 1$.

53. (i) For the equation $ax^2 + bx + c = 0$, the sum of roots is $\frac{-b}{a}$, and product of the roots is $\frac{c}{a}$.

(ii) Sum of the roots, $\sin\theta + \cos\theta = \frac{-q}{p}$.

(iii) Product of the roots $\sin\theta \cdot \cos\theta = \frac{r}{p}$.

(iv) Eliminate θ, by using the formula $(a + b)^2 = a^2 + b^2 + 2ab$ from the above equations.

54. (i) Use the identity $a^3 + b^3 = (a + b)^3 = 3ab(a + b)$.

(ii) Find the value of $\sin\alpha\cos\alpha$ by squaring $\sin\alpha - \cos\alpha = m$.

55. (i) Put $\theta = 0$ and simplify.

(ii) $\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$.

(iii) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.

56. $\frac{1+\tan\theta}{1-\tan\theta} = \sqrt{3}$

$\frac{\tan 45 + \tan\theta}{1 - \tan 45 \cdot \tan\theta} = \sqrt{3}$

$\tan(45° + \theta) = \tan 60°$

$\Rightarrow 45° + \theta = 60°$

$\theta = 60° - 45° = 15°$.

HINTS AND EXPLANATION

57. $AB = 1 \Rightarrow A = \dfrac{1}{B}$

$$A + B = \frac{1}{\sin\theta}\,\frac{1}{\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$A + B = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$A + B = \tan\theta + \cot\theta.$

Since, $\tan\theta \times \cot\theta = 1 = AB.$

$\therefore\ A = \tan\theta,\ B = \cot\theta.$

$$\frac{A}{B} = \frac{\tan\theta}{\cot\theta} = \tan\theta \cdot \tan\theta = \tan^2\theta.$$

$$A = \cot\theta,\ B = \tan\theta,\ \frac{A}{B} = \cot^2\theta.$$

58. $3\sin^2\theta + 4\sin^2\theta + 3\cos^2\theta = 4$

$3 + 4\sin^2\theta = 4$

$4\sin^2\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$\theta = 30°.$

$$\therefore \tan\theta = \frac{1}{\sqrt{3}}.$$

59. $\sin(A + B) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin(A + B) = \sin 75°$

$A + B = 75°.$ (1)

$\sec A = 2$

$\sec A = \sec 60°$

$A = 60°.$

Substitute the value of A in Eq. (1),

$60° + B = 75°$

$B = 15°.$

In circular measure,

$$B = 150° \times \frac{\pi}{180°} = \frac{\pi^c}{12}.$$

60. Given, $\tan\theta - \cot\theta = 7.$

We know that,

$a^3 - b^3 = (a - b)^3 + 3ab\,(a - b)$

$\tan^3\theta - \cot^3\theta = (\tan\theta - \cot\theta)^3 + 3\tan\theta\cot\theta(\tan\theta - \cot\theta)$

$= 7^3 + 3(7) = 343 + 21 = 364.$

61. $x^n = a^m\cos^4\theta$, and $y^n = b^m\sin^4\theta$

$$\Rightarrow \cos^4\theta = \frac{x^n}{a^m} \text{ and } \sin^4\theta = \frac{y^n}{b^m}$$

$$\Rightarrow \cos^2\theta = \frac{x^{n/2}}{a^{m/2}},\ \sin^2\theta = \frac{y^{n/2}}{b^{m/2}}.$$

But, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore\ \frac{x^{n/2}}{a^{m/2}} + \frac{y^{n/2}}{b^{m/2}} = 1.$$

62. Given,

$\sin^2\theta + 2\cos^2\theta + 3\sin^2\theta + \ldots$ 40 terms $= 405$

$\Rightarrow (\sin^2\theta + 3\sin^2\theta + \ldots$ 20 terms$) + (2\cos^2\theta + 4\cos^2\theta + \ldots$ 20 terms$) = 405$

$\Rightarrow 20^2\sin^2\theta + (20^2 + 20)\cos^2\theta = 405$

$\Rightarrow 400(\sin^2\theta + \cos^2\theta) + 20\cos^2\theta = 405$

$\Rightarrow 400 + 20\cos^2\theta = 405 = 20\cos^2\theta = 5$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2}\ (\because \theta \text{ is acute})$$

$\Rightarrow \theta = 60°.$

$\therefore\ \tan 60° = \sqrt{3}.$

63. Given, $\sin\alpha + \sin\beta = 2$

$\alpha = \beta = 90°.$

$\therefore\ \cos^2\alpha + \cos^2\beta = \cos^2 90° + \cos^2 90° = 0.$

64. Given, $x = a^2\cos^3\theta$ and $y = b^2\sin^3\theta.$

$$\Rightarrow \frac{x}{a^2} = \cos^3\theta \text{ and } \frac{y}{b^2} = \sin^3\theta$$

$$\Rightarrow \left(\frac{x}{a^2}\right)^{2/3} = \cos^2\theta;\ \left(\frac{y}{b^2}\right)^{2/3} = \sin^2\theta.$$

$$\sin^2\theta + \cos^2\theta = \left(\frac{x}{a^2}\right)^{2/3} + \left(\frac{y}{b^2}\right)^{2/3}$$

$$\therefore\ \left(\frac{x}{a^2}\right)^{2/3} + \left(\frac{y}{b^2}\right)^{2/3} = 1.$$

65. Given,

$\cos^2\theta + 2\sin^2\theta + 3\cos^2\theta + 4\sin^2\theta + \cdots + 200$ terms $= 10025$

$\Rightarrow (\cos^2\theta + 3\cos^2\theta + 5\cos^2\theta + \cdots$ 100 terms$) + (\sin^2\theta + 2\sin^2\theta + \cdots + 100$ terms$) = 10025$

$\Rightarrow 100^2\cos^2\theta + (100^2 + 100)\sin^2\theta = 10025$

$\Rightarrow 10000(\cos^2\theta + \sin^2\theta) + 100\sin^2\theta = 10025$

$\Rightarrow 100\sin^2\theta = 25$

$\Rightarrow \sin^2\theta = \dfrac{1}{4}$

$\Rightarrow \sin\theta = \dfrac{1}{2}$ ($\because \theta$ is acute)

$\Rightarrow \theta = 30°$.

$\therefore \sin\theta - \cos\theta = \sin 30° - \cos 30°$

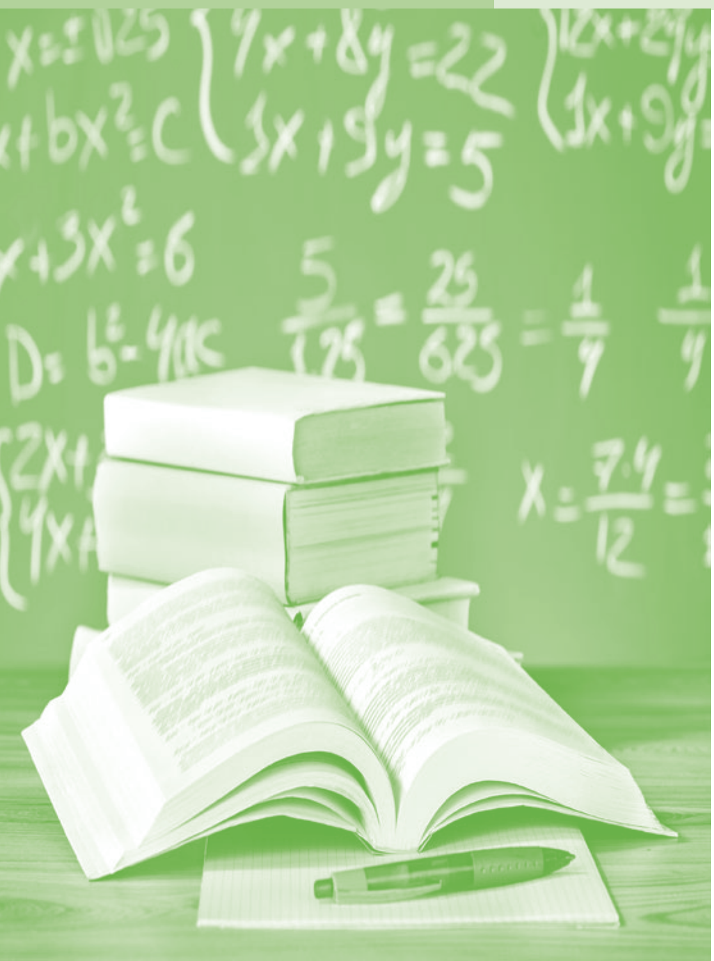
$= \dfrac{1}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{1-\sqrt{3}}{2}$.

HINTS AND EXPLANATION

Chapter

17

Percentages, Profit and Loss, Discount and Partnership



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the concepts of percentage
- Know the terms *profit* and *loss* in mathematics
- Understand the concepts of discount and partnership

KEY IDEAS

After completing this chapter, you should be able to:

- Calculate percentage of absolute values given
- Express percentage as fraction and fraction as percentage and decimal
- Solve numerical problems on percentage
- Make comparison of different percentages in a given problem
- Understand terms, like selling price, cost price, profit and loss
- Know about partnership and its types

INTRODUCTION

In this chapter, we shall learn the concepts of percentage and their wider applications in day-to-day life situations. In order to solve problems in chapters, like profit and loss, simple interest, compound interest, it is essential to have a thorough understanding of this topic.

PERCENTAGE

In mathematics, 'per cent' means 'for every hundred'.

The result of any division in which the divisor is 100, is a percentage. The divisor, that is, 100 is denoted by a special symbol '%', read as per cent.

For example, $\frac{10}{100} = 10\%$

$$\frac{25}{100} = 25\%$$

$$\frac{x}{100} = x\%$$

Since any ratio is also a division, each ratio can also be expressed as a percentage.

For example, the ratio $\frac{1}{2}$ can be converted into a percentage value:

$$\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100} = 50 \text{ per cent} = 50\%.$$

Expressing *x*% as a Fraction

Any percentage can be expressed as a decimal fraction by dividing the percentage figure by 100.

For example $$37\% = \frac{37}{100} = 0.37.$$

$$75\% = 75 \text{ out of } 100 = \frac{75}{100} = \frac{3}{4} \text{ or } 0.75.$$

Expressing a Fraction *a*/*b* as a Decimal and as a Percentage

Any fraction can be expressed as a decimal (i.e., terminating or non-terminating, but recurring), and any decimal fraction can be converted to percentage by multiplying it with 100.

$$\frac{1}{2} = 0.5 = 50\%$$

$$\frac{1}{4} = 0.25 = 25\%$$

$$\frac{1}{5} = 0.2 = 20\%$$

$$\frac{1}{3} = 0.33\ldots = 33.33\ldots\%.$$

Problems Based on Basic Concepts

EXAMPLE 17.1

Express 36% as a fraction.

SOLUTION

$$36\% = \frac{36}{100} = \frac{9}{25}$$

$\therefore$ 36% as a fraction is $\frac{9}{25}$.

EXAMPLE 17.2

Express 64% as a decimal.

SOLUTION

$$64\% = \frac{64}{100} = 0.64$$

$\therefore$ 64% as a decimal is 0.64.

EXAMPLE 17.3

Express $\frac{3}{20}$ as per cent.

SOLUTION

$$\frac{3}{20} = \left(\frac{3}{20} \times 100\right)\% = 15\%$$

$\therefore$ $\frac{3}{20}$ as per cent is 15%.

EXAMPLE 17.4

Find 36% of 30.

SOLUTION

$$36\% \text{ of } 30 = \frac{36}{100} \times 30 = 10.8$$

$\therefore$ 36% of 30 is 10.8.

Percentage: A Relative Value

When you score 18 marks out of 20 marks in your maths unit test, then it is an absolute value. Let us say that you have scored 90% in the maths unit test; it is understood that you obtained 90 marks out of 100 marks. However, if the maximum marks for the unit test is 50, then the marks you obtained are 90% of 50, or $\frac{90}{100} \times 50 = 45$.

Hence, the actual score depends upon the maximum marks of the unit test. It varies with the maximum marks. For example, if the maximum marks are 60, then 90% of 60 = 54. If the maximum marks are 70, then 90% of 70 = 63. As the maximum marks vary, your marks also vary. Hence, percentage is a relative or comparative value. That means, in relation to the total marks, or when compared with the total marks, you score 90% marks.

Comparison of Percentages

Let us say that in your class 30% of the students are girls, and in Class IX 40% of the students are girls. Can you say that the number of girls in your class is less than the number of girls in Class IX? The answer depends on the total number of students in each class. If there are 50 students in your class, the number of girls in your class $= \frac{30}{100} \times 50 = 15$.

If there are 30 students in Class IX, then the number of girls in Class IX $= \frac{40}{100} \times 30 = 12$.

Although the percentage of girls in your class is less than the percentage of girls in Class IX, the number of girls in your class may be more than that of Class IX.

However, you can say that the percentage of girls in your class is more than the percentage of girls in Class IX. But the number of girls in the two classes cannot be compared, if the total number of students in each class is not known.

However, when you say that the percentage of girls in your class is less than the percentage of girls in Class IX, you can specify that it is less by 10 percentage points (i.e., 40% − 30% = 10% points).

Percentage points is the difference between two percentage values. It is not equal to either percentage increase or percentage decrease.

When two absolute values are given, different percentage values can be calculated involving the two values. Let one value be greater than the other. The percentage values involved are:

1. One value as a percentage of the other.

EXAMPLE 17.5

x is what per cent of y?

SOLUTION

Let $x = k\%$ of y

$$x = \frac{k}{100} \text{ of } y$$

$$\Rightarrow k = \frac{x}{y} \times 100.$$

EXAMPLE 17.6

y is what percentage of x?

SOLUTION

Let $y = p\%$ of x

$$\Rightarrow p = \frac{y}{x} \times 100.$$

EXAMPLE 17.7

What percent of 3.6 km is 360 metres?

SOLUTION

We know that 1 km = 1000 metres.

$\Rightarrow$ 3.6 km = 3.6 × 1000 = 3600 metres

$\therefore$ The required percentage $=\left(\frac{360}{3600}\times 100\right)\% = 10\%$.

EXAMPLE 17.8

Find the number whose 30% is 36.

SOLUTION

Let the number be x.

Given that 30% of the number is 36.

$\Rightarrow$ 30% of $x = 36$

$\Rightarrow \frac{30}{100}\times x = 36$

$\Rightarrow x = \frac{36\times 100}{30} \Rightarrow x = 120$

$\therefore$ The required number is 120.

2. By what per cent is the greater quantity more than the smaller?

$$\text{Percentage more} = \frac{\text{Greater} - \text{Smaller}}{\text{Smaller}}\times 100\%$$

EXAMPLE 17.9

By what per cent is the sum of ₹100 more than the sum of ₹90?

SOLUTION

$$\frac{(100-90)}{90}(100)\% = 11\frac{1}{9}\%$$

That is, the sum of ₹100 is more than ₹90 by $11\frac{1}{9}\%$.

EXAMPLE 17.10

If Anil's salary is 20% less than Raju's salary, then by what per cent is Raju's salary more than that of Anil?

SOLUTION

Let Raju's salary be ₹100.

Anil's salary is 20% less than Raju's salary.

$\Rightarrow$ Salary of Anil = 80% of 100 = ₹80.

Raju's salary is ₹20 more than that of Anil's.

$\Rightarrow$ Now, the required percentage $= \frac{20}{80}(100)\% = 25\%$

$\therefore$ Raju's salary is 25% more than Anil's salary.

3. By what per cent is the smaller quantity less than the greater?

$$\text{Percentage less } = \frac{\text{Greater} - \text{smaller}}{\text{greater}} \times 100\%.$$

EXAMPLE 17.11

Mohit's weight is 40 kg and Rohan's weight is 35 kg. By what per cent is Rohan's weight less than that of Mohit's?

SOLUTION

$$\left(\frac{40-35}{40}\right)(100)\% = 12.5\%$$

When a quantity changes from time to time, we find percentage change in the quantity.

For example, the price of an article is ₹20 in the year 2005. It became ₹24 in the year 2006.

The percentage in change in the article is $\frac{4}{20} \times 100 = 20\%$ increase.

Percentage change can be defined as $\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100.$

$\therefore$ Percentage change could be increase or decrease.

EXAMPLE 17.12

In an examination, Mohit secured 60% of the maximum marks which is 45 marks more than the pass marks. If the pass mark is 45%, then find the maximum marks in the examination.

SOLUTION

Marks secured by Mohit = 60%

Pass marks = 45%

Difference between the marks obtained and pass marks = (60 − 45)% = 15%

Given that Mohit obtained 45 marks more than the pass marks.

Let the maximum marks be x.

$$\Rightarrow 15\% \text{ of } x = 45$$

$$\Rightarrow x = \frac{45 \times 100}{15} = 300$$

$\therefore$ The maximum marks in the examination = 300.

EXAMPLE 17.13

The price of an article is decreased by 20%. By what percentage its consumption must be increased so that the expenditure on it increases by 10%? Choose the correct answer from the following options:

(a) 10% **(b)** 25% **(c)** $33\frac{1}{3}\%$ **(d)** 37.5%

HINTS

(i) Let the initial price and initial commodity consumption be ₹100 and 100 units.

(ii) Let the price of article be ₹100x.

(iii) New price of the article will be 80x.

(iv) From Steps (ii) and (iii), find the required percentage.

EXAMPLE 17.14

In an office, 60% of the employees are female. 30% of the female employees have children and 20% of the male employees have children. What percentage of the employees has children? Choose the correct answer from the following options:

(a) 28% **(b)** 26% **(c)** 30% **(d)** 32%

HINTS

(i) Let the total number of employees in the office be 100x.

(ii) Given, the number of female employees as 60% of 100x. Then, male employees will be 40% of 100x.

(iii) Find the number of male employees and female employees who have children.

(iv) Required percentage $= \frac{\text{Number of employees who have children}}{\text{Total number of employees}} \times 100.$

PROFIT AND LOSS

In our everyday life and in the business world, we encounter transactions involving sales and purchases. Every time such a transaction occurs, it may be observed that there is a seller and a buyer involved.

The seller sells some things/goods for a certain amount paid by the buyer. The seller eventually makes some profit or loss in the transaction. This chapter deals with various aspects relating to such transactions of sales and purchases.

Cost Price (CP)

The price at which an article is purchased is called its cost price.

Selling Price (SP)

The price at which an article is sold is called its selling price.

Profit

If the selling price of an article is greater than its cost price, we say that there is profit or gain.

Profit = selling price − cost price

Percentage of profit is always calculated on the cost price of an article.

When SP > CP

1. Profit = SP − CP
2. SP = CP + Profit
3. CP = SP − Profit
4. Profit Percentage $= \dfrac{\text{Profit}}{\text{CP}}(100)\%$
5. Profit = Profit Percentage (CP)
6. When CP and Profit Percentage are given, $\text{SP} = (\text{CP})\left(\dfrac{100 + \text{Profit Percentage}}{100}\right)$
7. When SP and Profit Percentage are given, $\text{CP} = \dfrac{100\ (\text{SP})}{100 + \text{Profit Percentage}}$

Loss

If the selling price of an article is less than its cost price, we say that there is a loss. Loss = cost price − selling price.

Percentage of loss is always calculated on the cost price of the article.

When SP < CP:

1. Loss = CP − SP
2. SP = CP − Loss
3. CP = Loss + SP
4. Loss Percentage $= \dfrac{\text{Loss}}{\text{CP}} \times 100\%$
5. Loss = Loss Percentage × CP
6. When CP and Loss Percentage are given, $\text{SP} = \text{CP}\left(\dfrac{100 - \text{Loss Percentage}}{100}\right)$
7. When SP and Loss Percentage are given, $\text{CP} = \dfrac{\text{SP}\,(100)}{(100 - \text{Loss Percentage})}$

Overheads

All the expenditure incurred on transportation, repairs, etc., (if any) are categorized as overheads. These overheads are always included in the CP of the article.

Note When there are two articles having the same cost price and, if one article is sold at a% profit and the other is sold at the same loss per cent, then effectively neither profit nor loss is made. If there are two articles having the same selling price and, one is sold at x% profit and the other is sold at x% loss, effectively, there is always a loss and the loss percentage is $\left(\dfrac{x}{10}\right)^2$ %.

EXAMPLE 17.15

A shopkeeper bought a cycle for ₹1200 and sold it for ₹1500. Find his profit or loss percentage.

SOLUTION

Cost price of the cycle = ₹1200

Selling price of the cycle = ₹1500

SP > CP ⇒ There is a gain.

⇒ Gain = SP − CP = 1500 − 1200 = ₹300

$\therefore$ Gain Percentage $= \frac{\text{Gain}}{\text{CP}}(100)\% = \frac{300}{1200}(100)\% = 25\%$

$\therefore$ The shopkeeper makes a profit of 25%.

EXAMPLE 17.16

Rakesh purchased a TV for ₹5000. He paid ₹250 for its transportation. If he sold the TV for ₹5075, then find his profit or loss percentage.

SOLUTION

Price at which the TV was bought = ₹5000

Overheads in the form of transportation = ₹250

$\therefore$ The total cost price of the TV = (5000 + 250) = ₹5250

Selling price of the TV = ₹5075

SP < CP ⇒ There is a loss.

The amount of loss = CP − SP = 5250 − 5075 = ₹175.

$\therefore$ Loss percentage $= \frac{\text{Loss}}{\text{CP}}(100)\% = \frac{175}{5250}(100)\% = \frac{10}{3}\% = 3.33\%$.

$\therefore$ Rakesh incurred a loss of 3.33%.

EXAMPLE 17.17

By selling 24 pens, Kranthi lost an amount equal to the CP of 3 pens. Find his loss percentage.

SOLUTION

Let us assume that cost price of each pen is ₹1.

⇒ CP of 24 pens = ₹24

Loss = CP of 3 pens = 3 × 1 = ₹3

$$\Rightarrow \text{Loss Percentage} = \left(\frac{\text{Loss}}{\text{CP}} \times 100\right)\%$$

$$= \frac{3}{24} \times 100\% = 12.5\%$$

$\therefore$ Kranthi's loss is 12.5%.

EXAMPLE 17.18

Naresh sold two books for ₹600 each, thereby gaining 20% on one book and losing 20% on the other book. Find his overall loss or gain percent.

SOLUTION

Selling price of the first book = ₹600; Profit = 20%.

$$\Rightarrow CP = \frac{100\text{ SP}}{(100 + \text{Profit Percentage})} = \frac{(100)(600)}{100 + 20} = ₹500$$

The selling price of the second book = ₹600; Loss = 20%

$$\Rightarrow CP = \frac{100 \times SP}{(100 - \text{Loss Percentage})} = \frac{100 \times 600}{100 - 20} = ₹750$$

So, the total cost price of the books = ₹500 + ₹750 = ₹1250.

The total selling price of the books = 2 × 600 = ₹1200.

As the total selling price of the books < the total cost price of the books, there is a loss. Loss = CP − SP = ₹1250 − ₹1200 = ₹50.

$$\Rightarrow \text{Loss Percentage} = \frac{\text{Loss}}{\text{CP}}(100)\%$$

$$= \left(\frac{50}{1250}\right)100\% = 4\%$$

∴ Naresh's loss is 4%.

EXAMPLE 17.19

By selling a ball for ₹39, a shopkeeper gains 30%. At what price should he sell it to gain 40%?

SOLUTION

The selling price of the ball = ₹39; Gain = 30%.

$$\Rightarrow CP = \left(\frac{100 \times SP}{100 + \text{Gain Percentage}}\right) = \frac{100 \times 39}{100 + 30} = ₹30$$

Now, CP = ₹30 and gain required = 40%, then

$$\Rightarrow SP = \frac{CP\,(100 + \text{Gain Percentage})}{100}$$

$$= ₹\frac{(30)(140)}{100} = ₹42.$$

∴ To gain 40%, the shopkeeper has to sell it for ₹42.

EXAMPLE 17.20

A merchant marked his product at 50% above the cost price and then allowed 50% discount before selling it. The selling price of the product was ₹225. What is the profit that he made/incurred out of this? Choose the correct answer from the following options:

(a) profit of ₹37.50 **(b)** profit of ₹75

(c) loss of ₹75 **(d)** loss of ₹37.50

SOLUTION

Let the cost price of the product be ₹100x.

$\therefore$ Marked price = ₹($100x + 50x$)

= ₹$150x$

Discount $= \frac{50}{100}$(₹$150x$) = ₹$75x$

Selling price = ₹$75x$

Given, $75x = 225$

$x = 3$

$75x < 100x$

$\therefore$ A loss was incurred.

Loss incurred = ₹$25x$ = ₹75.

EXAMPLE 17.21

The cost price and the marked price of a watch are ₹200 and ₹300. It was sold at a discount of y%. The profit percentage was $\frac{3}{2}y$%. Find the value of y from the following options:

(a) $16\frac{2}{3}$ **(b)** 25 **(c)** $33\frac{1}{3}$ **(d)** 20

SOLUTION

Selling price $=$ ₹$300\left(1-\frac{y}{100}\right) =$ ₹$(300-3y)$

Selling price = ₹200

$$\left(1+\frac{\frac{3}{2}y}{100}\right) = ₹(200+3y)$$

($\because$ '+' applies in case of a profit and '−' applies in case of a loss)

$300 - 3y = 200 + 3y$

$100 = 6y$

$\therefore\ y = 16\frac{2}{3}$.

Partnership

The total amount of money required to start a business is called its **capital**. It is not always possible for a single person to invest huge amount of money. So, two or more persons come together and start business jointly. Such a business is called **partnership**. The people who jointly runs the business are called **partners**. The money invested by the partners in the business is called **investment**.

Types of Partnership

1. In general partnership, the period of investment is the same and the partners divide profit or loss in the ratio of their investments.
2. In compound partnership, the investments and the periods of investment differ. Then their investments reduce to investments per month or year and the profit or loss is divided in the ratio of these converted investments.

EXAMPLE 17.22

Satish and Kranthi started a business with capitals of ₹12,000 and ₹18,000, respectively. The business made a profit of ₹3500. Find the share of Kranthi and Satish in the profit at the end of the year.

SOLUTION

Investment made by Satish = ₹12,000

Investment made by Kranthi = ₹18,000

Ratio of the investments of Satish and Kranthi = 12,000 : 18,000 = 2 : 3.

As the period of investment is the same, profit is to be divided in the ratio of their investments.

⇒ Ratio in which the profit is divided = 2 : 3

Profit = ₹3500

∴ Satish's share in the profit $= \dfrac{3500 \times 2}{5} =$ ₹1400.

Kranthi's share in the profit $= \dfrac{3500 \times 3}{5} =$ ₹2100.

EXAMPLE 17.23

Rakesh set up a factory with a capital of ₹90,000. Ramesh joined him later with an investment of ₹50,000. The total profit earned at the end of the year was ₹68,000. Find when Ramesh joined Rakesh as the partner, if Rakesh's share in the profit is ₹48,000.

SOLUTION

Investment of Rakesh = ₹90,000.

Investment period of Rakesh = 12 months.

Investment of Ramesh = ₹50,000.

Let investment period of Ramesh be 'x' months.

⇒ Ratio of their investments = $90,000 \times 12 : 50,000 \times x = 108 : 5x$.

Total profit at the end of the year = ₹68,000.

Share of Rakesh in the profit = ₹48,000.

⇒ Share of Ramesh in the profit = ₹(68,000 − 48,000) = ₹20,000.

⇒ Ratio of their profits = 48000 : 20000 = 12 : 5

Ratio of investments = Ratio of profits

$\Rightarrow 108 : 5x = 12 : 5$

$\Rightarrow \dfrac{108}{5x} = \dfrac{12}{5}$

$\Rightarrow x = 9$

$\Rightarrow$ Investment period of Ramesh = 9 months.

$\therefore$ Ramesh joined Rakesh as a partner after (12 − 9) months from the commencement of the business, i.e., he joined after 3 months.

EXAMPLE 17.24

Naresh, Gopi and Sarath started a business with initial investments of ₹10,000, ₹20,000, and ₹20,000, respectively. After 6 months, Gopi withdrew an amount of ₹5000 from his investment. After 3 more months, Sarath brought in ₹10,000. If at the end of the year, the total amount of profit earned is ₹36,000, then find the share of each partner.

SOLUTION

(a) Investment of Naresh = ₹10,000

Period of investment = 12 months

$\Rightarrow$ Total investment made by Naresh = ₹(12 × 10,000) (1)

(b) Investment of Gopi = ₹20,000

Period of investment = 6 months

Amount withdrawn = ₹5000

$\Rightarrow$ Investment for the remaining 6 months = ₹(20,000 − 5000) = ₹15,000

$\Rightarrow$ Total investment made by Gopi = ₹(6 × 20,000 + 6 × 15,000) (2)

3. Investment of Sarath = ₹20,000

Period of investment = (6 + 3) = 9 months

Additional investment = ₹10,000

$\Rightarrow$ Investment for the remaining 3 months = ₹(20,000 + 10,000) = ₹30,000

$\Rightarrow$ Total investment made by Sarath = ₹(9 × 20,000 + 3 × 30,000)

From Eqs. (1), (2) and (3), we find the ratio of investments of Naresh, Gopi and Sarath as (12 × 10,000) : (6 × 20,000 + 6 × 15,000) : (9 × 20,000 + 3 × 30,000) = 12 : 21 : 27 = 4 : 7 : 9.

Total profit at the end of the year = ₹36,000.

Total profit is divided in the ratio of their investments.

$\therefore$ Naresh's share in the profit $= \dfrac{4}{20} \times 36{,}000 =$ ₹7200

Gopi's share in the profit $= \dfrac{7}{20} \times 36{,}000 =$ ₹12,600

Sarath's share in the profit $= \dfrac{9}{20} \times 36{,}000 =$ ₹16,200

EXAMPLE 17.25

In a business, P, Q, R and S are four partners. P's investment is twice R's investment, Q's investment is $\frac{1}{4}$ of S's investment and P and Q invested equally. If the total profit at the end of the year is ₹13,000, then find the sum of the shares of P and R in the profit.

(a) ₹10,000 **(b)** ₹3000 **(c)** ₹4000 **(d)** ₹5000

HINTS

(i) Using the given data, find the ratio of the investments of P, Q, R and S.

(ii) Let the investment of A and B be ₹x each.

(iii) Now, find the investments of C and D in terms of x.

(iv) Ratio of shares of profits is equal to the ratio of their investments.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 26% of 640 is ________.
2. If X is 20% more than Y, then what per cent of Y is X?
3. Find the value of 99% of 1500 + 45% of 360 + 55.99% of 3600.
4. If $\frac{1}{4}$ is 25%, then $\frac{9}{16}$ is _______.
5. If 13% of a number is 28, then 26.52% of the same number is _______.
6. The ratio of incomes of Akshay and Aravind is $4 : 3\frac{2}{3}$. Income of Aravind is less than that of Akshay by ________%.
7. Koutons, an apparel showroom, offers two successive discounts of 50% and 20%. What is the single equivalent discount percentage?
8. If the cost price of an article is ₹64 and its selling price is ₹80, then profit percentage is ________.
9. If cost price is ₹100, marked price is 200% above the cost price, and discount offered is 50%, then profit per cent is ______.
10. If Nityananda scored 25% more than Advaita, then Advaita scored ______ % less than Nityananda.
11. If $2a = 3b$, $b = \frac{3}{4}c$, and $c = 0.8d$, then find by what per cent is a less/more than d.
12. If the cost price of five apples is equal to the selling price of 3 apples, then find the profit percentage.
13. If the cost price is obtained by multiplying the selling price by $\frac{7}{8}$, then find the profit percentage.
14. This year, the number of children in a colony is three times that of the previous year. What is the percentage increase in the number of children in the colony?
15. A number becomes 63 after it is reduced by 12.5%. The original number is _______.
16. 50% of 50% of 50% of 10% of 560 is ________.
17. When calculated on selling price, profit is found to be 25%, find the actual profit percent.
18. If Krishna and Balaram share profits in the ratio 3 : 5 at the end of the year, then the share of Krishna in the total investment of ₹16,000 crores is ₹ ________.
19. If Nanda sold an article at ₹40 and earned 100% profit, then what should be the selling price to earn a profit of 300%?
20. An article is marked up by 100%. If a discount of ₹12 on it is equal to 10%, then the cost price of the article is ______.
21. An increase in the side of a square by 20% results in ______ % increase in its area.
22. When the selling price is ₹80, there is a loss of 20%. Find the selling price to earn 40% profit.
23. In a class, 40% of students are girls. Of the total number of boys, 50% failed in an exam. What per cent of the boys passed in the exam?
24. Ram and Lakshman started a business by investing ₹30 lakhs and ₹48 lakhs, respectively. The ratio in which they share the profits is __________.
25. Santan sells a diamond at a loss of 10%. If he had sold it at a profit of 5% he would have earned ₹37.5 lakhs more. What is the cost price of the diamond?
26. The cost price of a machine is $66\frac{2}{3}\%$ of its selling price, then profit per cent is ______.
27. A person sells $33\frac{1}{3}\%$ of his property at 20% loss. At what profit per cent should he sell the remaining property to earn 20% profit on the whole?
28. If 30% of 40% of x = 60% of 12.5% of 96, then x is _________.
29. If two bikes are sold at the same price so that there is a profit of 45% on one bike and a loss of 45% on the other. The net result of the transaction is _______ (loss/profit) and it is ______%.
30. Every year there is a 30% hike in Rama's salary. If his present salary is ₹2000, then his salary after two years will be ₹ _______.

Short Answer Type Questions

31. The selling price and the marked price of an article are ₹132x and ₹165x, respectively. If the article is marked 65% above the cost price and a discount of ₹198 is given, then find the profit on the article.

32. The cost of a piece of land increases by 50% every year from 2005 onwards. If the difference in the cost price of the land between the third and the second year is ₹67,500, then find the cost of the land in 2005.

33. In an election, there were only two contestants. The democrats secured 35% valid votes more than the republicans. If 6% of votes were invalid, then what percentage of the total votes polled was in favour of the democrats?

34. By selling an article at $\frac{1}{4}$ of its actual selling price, a trader incurs a loss of 50%. What will be the profit per cent if the trader sells the article at its actual selling price?

35. An electric room heater is sold at \$880, after a reduction of 12% in its price. What was the original price, in dollars, before the reduction?

36. A dishonest trader declares to sell his goods at cost price, but gives only 960 gms for every 1 kg. Find the error percentage.

37. A mock IIT-JEE exam constitutes of 150 questions. Each correct answer fetches 4 marks. There is a penalty of 2 marks for every incorrect answer and a penalty of 1 mark for every unattempted question. Find the percentage of marks scored by the students by attempting 120 questions of which $33\frac{1}{3}\%$ are incorrect.

38. Noel and Mathew save 20% and 30% of their income. If their expenditures are equal and Mathew's income is \$5600, then find Noel's savings (in dollars).

39. In an election, there were only two candidates, A and B. A secured 60% valid votes more than B, and 9% of the total votes were invalid. What percentage of the total votes was in favour of A?

40. A trader suffered a loss of 15% by selling an article. Had he sold it for ₹100 more, he would have made a profit of 5%. Find the article's cost price.

41. The price of a commodity is decreased by 25%. By what percentage must its consumption increase so that the expenditure on it remains unchanged?

42. Find the single discount equivalent to successive discounts of 25% and 20%.

43. Ajay marked an article at 60% above its cost price. Find the maximum percentage of discount he can offer and still avoid incurring a loss.

44. Find the single discount equivalent to successive discounts of 20% and 10%.

45. Profit percentage is numerically equal to the cost price of an article on selling it for ₹31.25. Find the cost price.

Essay Type Questions

46. Anthony spends x% of his salary towards expenditure on food, $2x$% of the remaining on transport, $3x$% of the remaining towards various bills, $4x$% of the remaining towards loan repayments. If the expenditures are denoted as F, T, B and L, respectively, if x is 10, then what is the ascending order of these amounts?

47. Three merchants A, B and C marked three identical articles each at ₹2000. A sold his article after two successive discounts of 30% each. B sold his article after successive discounts of 40% and 20%. C sold his article after successive discounts of 50% and 10%. Find the highest selling price.

48. In a business, A, B, C and D are the four partners. A's investment is thrice C's investment, B's investment is $\frac{1}{3}$ of D's investment. A and B invested equally. The total profit at the end of the year is ₹16,000. Find the sum of the shares of A and C in the profit.

49. Amar, Bhuvan and Chetan have some marbles with them. Bhuvan has 20% more marbles than Amar, and Chetan has 10% less marbles than Bhuvan. If the total number of marbles with them is 328, then find the number of marbles with Bhuvan.

50. In a business, Gita and Sita invested in the ratio of 4 : 5. At the end of the year, they earned a profit of ₹72,000. If Sita's salary is included, the ratio of their earning is 1 : 2. Find the salary of Sita.

PRACTICE QUESTIONS

CONCEPT APPLICATION

Level 1

1. The population of a city increases by 20% at the end of every year. During which of the following years, the population would be doubled?

(a) second (b) third
(c) fourth (d) fifth

2. If 20% of a and 40% of b is 230, and 40% of a and 20% of b is 190, then b is what percentage more or less than a?

(a) 80% (b) 60%
(c) 50% (d) 40%

3. In all the three sections of Woodland High School, there are 50, 70 and 80 students. Those who secured F grade are 10%, 20%, and 30%, respectively. What percentage of the total students in the school secured F grade?

(a) 12.5 (b) 15.6
(c) 21.5 (d) 23.4

4. In a test, Arun scored 20% and failed by 10 marks. Bala scored 40% in the same test and obtained 10 marks more than the pass mark. Find the maximum marks.

(a) 100 (b) 300
(c) 400 (d) 200

5. On an occasion, 40% of the young men wear grey hats, 60% of the remaining young men wear black hats. What percentage of the young men wears neither grey hats nor black hats? (No young man wears both grey hat and black hat)

(a) 12% (b) 18%
(c) 24% (d) 25%

6. If a number x is increased by 20% and then reduced by 20, it results in 160. Instead, if the number x is reduced by 20% and increased by 20, then what will be the result?

(a) 140 (b) 144
(c) 148 (d) 152

7. The price of an article is increased by 25%. By what percentage must its consumption decrease so that the expenditure on it remains unchanged?

(a) 25% (b) 20%
(c) $33\frac{1}{3}\%$ (d) 10%

8. If the cost price of 15 articles is equal to the selling price of 20 articles, find the profit or loss percentage.

(a) 25% profit (b) 20% profit
(c) 33.33% loss (d) 25% loss

9. In a test, by scoring $41\frac{2}{3}\%$ of the maximum marks, a student obtains 10 marks more than the pass marks. By scoring 30% of the maximum marks, another student scores 4 marks less than the pass mark. Find the actual pass percentage.

(a) $11\frac{2}{3}\%$ (b) $22\frac{1}{6}\%$
(c) $33\frac{1}{3}\%$ (d) $66\frac{2}{3}\%$

10. The profit made in selling 25 m of a cloth equals the selling price of 5 m of that cloth. Find the profit percentage.

(a) 25% (b) 20%
(c) $33\frac{1}{3}\%$ (d) 15%

11. Ajay sold two motorbikes for ₹40,000 each. He sold one at 20% profit and the other at 20% loss. Find the profit or loss percentage in the whole transaction.

(a) 2% profit (b) 3% loss
(c) 4% loss (d) No profit, no loss

12. A sold an article to B at 30% loss, B sold it to C at 20% profit and C sold it to D at 10% profit. If D bought it for ₹924, then the cost price for A. (in ₹)

(a) 1200 (b) 1050
(c) 900 (d) 1000

13. Raja spent $0.5x\%$ of his monthly salary on rent. He spent $x\%$ of the remaining salary on food, $2x\%$ of the remaining salary on transport and $4x\%$ of the remaining salary on various bills. If these expenditures are denoted by R, F, T and S, respectively, and if x is 20, then find the ascending order of these expenditures.

(a) R, S, F, T (b) R, F, T, S

(c) R, T, F, S (d) R, F, S, T

14. In a certain season, the Indian cricket team had a 40% success rate in the first 80 matches it played. What is the minimum number of additional matches it must play so that it has a 60% success rate for the season?

(a) 30 (b) 40

(c) 45 (d) 35

15. If the cost price of 20 articles is equal to the selling price of 15 articles, then find the profit or loss percentage.

(a) 20% loss (b) 25% profit

(c) $33\frac{1}{3}\%$ profit (d) 25% loss

16.

Year	Percentage of First Generation Immigrants in US	Population of US (in Millions)
1990	3%	$8k$
2000	3.2%	$9k$

If x denotes the percentage increase in the number of first generation immigrants from 1990–2000, and y denotes the percentage increase in the total population of US from 1990–2000, which of the following is equal to $(x - y)$?

(a) 12.5% (b) 12.2%

(c) 9.6% (d) 7.5%

17. Trader A gives a single discount of 30% and trader B gives two successive discounts of 20% and 10% on an identical article. If the discount given by A is ₹600 more than the discount given by B, find the marked price of the article.

(a) ₹1500 (b) ₹3000

(c) ₹30,000 (d) ₹600

18. X sells an article to Y at 15% profit. Y sells it to Z at 10% profit. What is X's cost price, if Y makes a profit of ₹23?

(a) ₹230 (b) ₹200

(c) ₹150 (d) ₹180

19. When a discount of 10% is given on the marked price of an article, the gain of the trader is 20%. What will be the profit percent, if a discount of 13% is given?

(a) 12% (b) 16%

(c) 20% (d) 24%

20.

Year	Percentage of the First Generation Immigrants in UK	The Population of UK (in Millions)
1995	2%	$6x$
2005	2.4%	$8x$

If p denotes the percentage increase in the number of the first generation immigrants from 1995–2005, and q denotes the percentage increase in the total UK population from 1995–2005, find $p - q$.

(a) $13\frac{1}{3}\%$ (b) $26\frac{2}{3}\%$

(c) $33\frac{1}{3}\%$ (d) $46\frac{2}{3}\%$

21. A shopkeeper marks the price of an article 50% above the cost price and declares a discount of 20%. If profit earned is ₹30, then find the marked price.

(a) ₹150 (b) ₹180

(c) ₹225 (d) ₹250

22. In a business, P, Q and R are three partners. Thrice P's investment is equal to twice Q's investment and R's investment is equal to twice P's investment. Q's period of investment is $\frac{4}{3}$ times P's period of investment and is twice R's period of investment. If the total profit at the end of the year is ₹52,000, find the sum of the shares of P and Q in the profit. (in ₹)

(a) 32,000 (b) 36,000

(c) 40,000 (d) 28,000

23. Ratan spends 70% of his income. His income increases by 25%, and his expenditure also increased by 25%. Find the percentage increase in his savings.

(a) 25 (b) 30

(c) 10 (d) No change

24. Mahesh sold an article for ₹39 and obtained profit percentage which is numerically equal to its cost price (in rupees). Find the cost price of the article.

(a) ₹28 (b) ₹32

(c) ₹30 (d) ₹35

25. The selling price of an article after giving three successive discounts is ₹7560. If the marked price is ₹15,000, then which of the following can be the successive discounts?

(a) 8, 16, 24

(b) 5, 10, 15

(c) 10, 20, 30

(d) 15, 30, 45

26. A trader marks his product 30% above his cost price, and then offers a 30% discount. Find his cost price if he incurs a loss of ₹900. (in ₹)

(a) 12,000 (b) 10,000

(c) 9000 (d) 8000

27. The loss made in selling 20 m of a cloth equals the cost price of 4 m of that cloth. Find the loss percentage.

(a) 20% (b) 25%

(c) $33\frac{1}{3}\%$ (d) 40%

Level 2

28. Three merchants, P, Q and R marked three identical articles each at ₹4000, respectively. P sold his article after two successive discounts of 40% each. Q sold it after successive discounts of 50% and 30%. R sold it after successive discounts of 60% and 20%. Find the least selling price. (in ₹)

(a) 1320 (b) 1280

(c) 1440 (d) 1200

29. A shopkeeper marks the cost of two identical articles, one 100% above the cost price and the other 50% above the cost price. If a discount of 20% is allowed on each of them, then find the overall profit percentage.

(a) 10% (b) 20%

(c) 30% (d) 40%

30. The profit made in selling 5 m of a cloth equals the cost price of 2 m of that cloth. Find the profit percentage.

(a) 30% (b) 40%

(c) $33\frac{1}{3}\%$ (d) 45%

31. The price of an article is increased by 20%. By what percentage must its consumption be reduced so that the expenditure on it reduces by 10%?

(a) 20% (b) 30%

(c) 25% (d) 15%

32.

Name of the class	A	B
Number of girls	24	36
Number of girls as a percentage of total number of students	60	80

As per the information given in the above table, approximately by what percentage is the total number of students in class A is less than those in class B?

(a) 9% (b) 10%

(c) 11% (d) 13%

33. A person bought two articles for ₹1800. He sold the first article at a profit of 25% and the second at a loss of 20%. On the whole there is neither loss nor gain. Find the cost price of the second article.

(a) ₹800 (b) ₹1000

(c) ₹900 (d) ₹1200

34. The ratio of the marked price and the cost price of an article is 5 : 3. If a loss of 2*a*% is obtained after giving a discount of 4*a*%, then find *a*.

(a) $14\frac{6}{7}$ (b) $12\frac{3}{7}$

(c) $13\frac{2}{7}$ (d) $14\frac{2}{7}$

35. Three persons X, Y and Z started a business with investments in the ratio of 3 : 2 : 4. The ratio of their periods of investments is 5 : 6 : 7. The difference in the shares of profits of X and Z is what

PRACTICE QUESTIONS

percentage of the share of profit of Y at the end of the year? (Approximately)

(a) 120% (b) 108.33%

(c) 115.25% (d) 125.66%

36. Ajay invested ₹m for 7 months and ₹n for the remaining period. Sohail invested ₹n for the first 9 months and ₹m for the remaining period. If at the end of the year, they share profits equally, then what is the relation between m and n?

(a) $m = n + 1$ (b) $m - n = 2$

(c) $m = n$ (d) $m + n = 2$

37. Ramesh spends 60% of his income. His income increases by 40%, and also his expenditure by 40%. The percentage of increase in his savings is _______.

(a) 20% (b) 30%

(c) 40% (d) 50%

38. A dishonest shopkeeper sells his items at cost price. But, for every kg he gives 200 gm less. His profit percentage is _______.

(a) 20% (b) 25%

(c) 30% (d) 15%

39. Preetam bought two mobile phones for ₹4800. He sold the first mobile phone at a loss of 15%, and the second at a profit of 25%. On the whole, there was neither loss nor gain. The cost price of the second mobile phone is _______.

(a) ₹1800 (b) ₹3000

(c) ₹1200 (d) ₹2400

40. The selling price of a TV, after giving two successive discounts, is ₹15,840. If the marked price is ₹20,000, then which of the following could be the successive discounts given on the marked price of the TV?

(a) 10, 15 (b) 12, 15

(c) 10, 12 (d) 15, 18

41. In a business, A, B and C are three partners. Twice A's investment is equal to C's investment. Also, B's investment is equal to $\frac{1}{2}$ of A's investment. A's period of investment is equal to 4 times C's period of investment, and A and B invested for the same period. If the total profit at the end of the year is ₹44,000, then find the sum of the shares of A and B in the profit.

(a) ₹22,000 (b) ₹11,000

(c) ₹16,500 (d) ₹33,000

42. Ramapada has just enough money to purchase either 30 pens or 50 pencils. He decides to spend only 80% of his money and buys 10 pens. At the maximum, how many pencils can he buy with the remaining money that he has?

(a) 23 (b) 24

(c) 25 (d) 26

43. A trader claims to sell his goods at cost price. But, he gives only 900 g for every kg. Find his profit percentage.

(a) $11\frac{1}{9}\%$ (b) $9\frac{1}{11}\%$

(c) 10% (d) $12\frac{2}{3}\%$

44. Amish sold an article at two-thirds of the marked price and suffered a loss of $16\frac{2}{3}\%$. Find the percentage of profit, if he sold the article at the marked price.

(a) 20% (b) 25%

(c) $16\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$

45. Raman suffered a loss of 10% by selling an article. Had he sold it by ₹180 more, he would have made a profit of 2%. Find his cost price (in ₹).

(a) 1350 (b) 1800

(c) 1650 (d) 1500

46. Anil bought certain number of books at the rate of 12 books for ₹18 and sold them at the rate of 18 books for ₹30. Find his profit/loss percentage.

(a) $9\frac{1}{11}\%$ (b) 10%

(c) $11\frac{1}{9}\%$ (d) $12\frac{1}{2}\%$

47. In an election, there were only two contestants P and Q. 14% of the total votes polled were invalid. The number of valid votes secured by P was 15% more than that secured by Q. What percentage of the total votes was cast in favour of Q?

(a) 48% (b) 37.5%

(c) 40% (d) 31.5%

Level 3

48. Bindu goes to a shop to buy a gift costing ₹1500. On her request, the shopkeeper allows two successive discounts of $x\%$ and $y\%$. ($x < y$). Bindu buys the gift for ₹1188 including sales tax of 10%. Which of the following could be the discount rates offered by the shopkeeper?

(a) 15%, 20% (b) 20%, 25%
(c) 20%, 30% (d) 10%, 20%

49. In the year 2001, the population in a village is x. Every year, the population increases by 10%. In which year, for the first time, will the population of the village be at least 50% more than that of year 2001?

(a) 2005 (b) 2006
(c) 2004 (d) 2007

50. In a family, a person saves 30% of his income. If his salary is increased by 20% and savings are decreased by 20%, then find the percentage increase in the expenditure.

(a) $34\frac{1}{4}\%$ (b) $37\frac{1}{7}\%$
(c) $41\frac{1}{7}\%$ (d) $43\frac{1}{3}\%$

51. A sold an article to B at 10% profit. B sold it to C at 20% profit. Find the price at which A bought the article, if B's profit is ₹44. (in ₹)

(a) 150 (b) 200
(c) 250 (d) 100

52. A person spends 25% of his salary on house rent, 20% of the remaining salary on clothes, $16\frac{2}{3}\%$ of the remaining on petrol and 30% of the remaining on food. If after incurring the above expenditure his savings are ₹147, then find his salary.

(a) ₹480 (b) ₹300
(c) ₹350 (d) ₹420

53. In an office, 30% of the employees are unmarried; 20% of these employees are post-graduates. The number of married employees in the office who are post-graduates is $3\frac{1}{2}$ times that of the unmarried employees who are post-graduates. What percentage of the married employees are post-graduates?

(a) 36% (b) 32%
(c) 30% (d) 27%

54. Amar, Bhavan and Chetan divide a certain amount among themselves. The average of the amounts with them is ₹1180. Amar's share is $33\frac{1}{3}\%$ more than that of Bhavan's and $16\frac{2}{3}\%$ less than that of Chetan's. Find Amar's share. (in ₹)

(a) 1050 (b) 1200
(c) 1350 (d) 1500

55. In a certain season, the Indian cricket team had won 30% of the first 60 matches it had played. Find the minimum possible number of additional matches it should play to achieve a success rate of 44% in that season.

(a) 20 (b) 15
(c) 25 (d) 30

56. Mohan and Sohan started a business. Mohan was a sleeping partner in the business. Sohan, being the working partner, took a certain monthly salary. At the end of the first year, the total amount of profit was ₹3,00,000. The ratio of Mohan's and Sohan's shares was 9 : 11. If Sohan's annual salary was excluded, the ratio of their shares become 9 : 7. Find the monthly salary of Sohan. (in ₹)

(a) 5400 (b) 5000
(c) 7500 (d) 7800

57. X and Y are two natural numbers. The sum of thrice of X and twice of Y is 230. If $3X$ increases by 40% and $2Y$ increases by 30%, then their sum will increase by 84. If X decreases by 40% and Y decreases by 30%, then their sum will be _______.

(a) 58 (b) 66
(c) 52 (d) 46

58. Giri sold two books. The selling price of a book equals the cost price of the second book. He sold the first book at 10% profit and the second book at 10% loss. Find the overall profit/loss percentage.

PRACTICE QUESTIONS

(a) $\frac{5}{21}$% profit (b) $\frac{5}{21}$% loss

(c) $\frac{10}{21}$% profit (d) $\frac{10}{21}$% loss

59. P sold an article at 20% profit to Q. Q sold it at 10% profit to R. The profit made by Q was ₹24 less than the profit made by P. Find the cost price of the article for P. (in ₹)

(a) 250 (b) 300

(c) 360 (d) 200

60. In a class, 45% of the students drink pulpy orange and 90% of the remaining students drink Maaza. No student drinks both Maaza and pulpy orange. Find the percentage of students in the class who drink neither Maaza nor pulpy orange.

(a) 4.5%

(b) 6.5%

(c) 7.5%

(d) 5.5%

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 166.4
2. 120%
3. 3662.64
4. 56.25%
5. 57.12
6. 8.33%
7. 60%
8. 25%
9. 50%
10. 20%
11. 10% less
12. $66\frac{2}{3}\%$
13. $14\frac{2}{7}\%$
14. 200%
15. 72
16. 7
17. $33\frac{1}{3}\%$
18. ₹6000 crores
19. ₹80
20. ₹60
21. 44%
22. ₹140
23. 30%
24. 5 : 8
25. 250 lakhs
26. 50%
27. $x = 40\%$
28. 60
29. loss, 20.25%
30. ₹3380

Short Answer Type Questions

31. 192
32. ₹60,000
33. 54
34. 100%
35. 1000
36. 4%
37. 35%
38. $980
39. 56
40. 500
41. $33\frac{1}{3}\%$
42. 40%
43. 37.5%
44. 28%
45. 25%

Essay Type Questions

46. *F*, *T*, *L*, *B*
47. 980
48. 4000
49. 120
50. ₹18,000

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (a)
11. (c)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (a)			

Level 2

28. (b)	**29.** (d)	**30.** (b)	**31.** (c)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (c)
38. (b)	**39.** (a)	**40.** (c)	**41.** (b)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (c)

Level 3

48. (d)	**49.** (b)	**50.** (b)	**51.** (b)	**52.** (d)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (b)	**57.** (a)
58. (d)	**59.** (b)	**60.** (d)							

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. Use the formula, $A = P\left(1+\frac{r}{100}\right)^n$.

2. 20% a + 40% b = 230.

   40% a + 20% b = 190. Find a and b.

3. $\frac{\text{Number of students who obtained grade } F}{\text{The total number of students}} \times 100.$

4. Difference in the percentages = Difference in the marks.

5. (i) Let the total number of young men be 100.

   (ii) Let the number of young men who attend the party be $100x$.

   (iii) Find number of young men who wear grey hats, and also find that of who wear black hats.

6. Find x from the first statement.

7. Let the initial price be ₹100 and initial consumption be 100 units.

8. (i) Find SP and CP of 20 articles by taking CP of 1 article = ₹x.

   (ii) As the CP of 15 articles is equal to the SP of 20 articles, trader suffers loss.

   (iii) Let $C = 15$ and $S = 20$. Loss % $= \left(\frac{S-C}{S}\right)\times 100.$

9. Let m be the maximum marks and p be the pass marks.

   (ii) Given, $\frac{41\frac{2}{3}\times m}{100} = p + 10$ and $\frac{30\times m}{100} = p - 4.$

   From the above equations, we obtain the values of m and p.

10. (i) Let SP of 1 m of cloth be ₹x.

    (ii) Let SP of 25 m of a cloth be ₹$25x$.

    ∴ Profit = ₹$5x$.

    (iii) Now find the CP, and then the profit %.

11. (i) Whenever SP is same and % of loss = % of profit = x%, there is always loss, and loss% $= \frac{x^2}{100}$.

    (ii) In the given conditions, trader suffer loss.

    (iii) Loss percentage $= \frac{(\text{Profit \% or loss \%})^2}{100}$

12. (i) Let CP of A be ₹100.

    (ii) Let CP for A be ₹x.

    (iii) $x\left(\frac{70}{100}\right)\left(\frac{120}{100}\right)\left(\frac{110}{100}\right) = 924.$

13. (i) Let Raja's monthly salary be ₹100.

    (ii) Let the salary of Raja be ₹$100x$.

    (iii) R = 10% of ₹$100x$ = ₹$10x$.

    (iv) Similarly, find F, T and S, and then compare.

14. (i) Find the number of matches won out of 80 matches. Minimum number of additional matches is equal to the number of matches to be won.

    (ii) First, find the number of matches won out of 80 matches played.

    That is, $\frac{40}{100}\times 80 = 32.$

    (iii) To get minimum number of matches to be played, it has to win all the additional matches.

    (iv) Let the required number of matches = x.

    $\Rightarrow \frac{(32+x)}{(8+x)}\times 100 = 60.$

15. (i) Find SP and CP of 15 articles by taking CP of one article as ₹x.

    (ii) As the CP of 20 articles is equal to SP of 15 articles, trader earns profit.

    (iii) Let C = 20, and S = 15.

    Profit% $= \left(\frac{S-C}{S}\right)\times 100.$

16. (i) Find x and y using the given information.

    (ii) First, find population of the first generation immigrants in 1990 and 2000, i.e., 3% of $8k$ and 3.2% of $9k$.

    (iii) Then find percentage increase (x).

    (iv) $y = \frac{1k}{8k}(100)$, and then find $(x - y)$.

17. (i) Let MP of A be ₹100, then find the difference in the discount given by A and B.

(ii) Let the MP be ₹x.

(iii) Successive discounts 20% and 10% is equivalent to a single discount of 28%.

(iv) Now, 30% of x − 28% of x = ₹600.

18. (i) Let CP of x be ₹100, then find SP of x.

(ii) Let CP for X be ₹100x.

(iii) Find CP and SP for Y. Then find profit earned by Y (in x) and compare it with ₹23.

(iv) CP of Y = SP of X.

19. Assume the cost price be ₹100.

$$m\left(\frac{90}{100}\right) = 120.$$

20. (i) Find p and q using the given information.

(ii) First, find the population of the first generation immigrants in 1995 and 2005, i.e., 2% of 6x and 2.4% of 8x.

(iii) Then find percentage increase (q).

(iv) $P = \frac{2x}{6x}(100)$, and then find $(p - q)$.

21. Assume the cost price be ₹100, then marked price is ₹150.

22. (i) Find the ratio of investments and the ratio of time periods.

(ii) Let P's investment be ₹x.

(iii) Now, find investments of Q and R in terms of x with the given data.

(iv) Let the time period of investment of R be y months.

(v) Now, find time periods of investment of P and Q.

(vi) Ratio of shares of profits is equal to the ratio of product of investments and investment periods.

26. (i) Let CP of the product be ₹100, then find MP and SP.

(ii) Let the CP be ₹100x.

(iii) MP = 130x and SP = 70% of 130x.

(iv) CP − SP = 900, find x.

27. (i) Let CP of 1 m cloth be ₹x, then the amount of loss and SP.

(ii) Let the CP of 20 m of a cloth be ₹20x.

(iii) So, loss = ₹4x.

(iv) Now, find loss percentage.

Level 2

28. (i) Find the discounts given by P, Q and R.

(ii) Find the SP for P, Q and R by using the following formulae.

$$\text{SP} = \frac{(100 - d)}{100}\text{MP (OR)}$$

$$\text{SP} = \frac{(100 - d_1)}{100}\frac{(100 - d_2)}{100}\text{MP}$$

(iii) Compare the SP's.

29. Marked price of the first article = ₹200.

Marked price of the second article = ₹150.

30. (i) Let CP of 1 m cloth be ₹x.

(ii) Let the CP of 5 m of a cloth be ₹5x.

(iii) So, profit = ₹2x.

(iv) Now, find the profit percentage.

31. (i) Let the initial price and initial consumption be ₹100 and 100 units, respectively.

(ii) (Consumption) (Price of an article) = Total expenditure.

(iii) Let consumption be 100 articles and rate be ₹100.

(iv) Given, rate = ₹80 and total expenditure = ₹9000. Find the consumption by using the formula which is mentioned in (i).

(v) Find the percentage decrease in the consumption.

32. (i) Find the total number of students in class A and class B.

(ii) Total number of students in class

$$\text{A} = \left(\frac{24}{60}\right) \times 100.$$

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(iii) Similarly, find the total number of students in class B.

(iv) Find the difference between the total number of students in both the sections, and then find percentage difference with respect to the total number of students in class B.

41. Ratio of profits = (Ratio of investments) × (Ratio of time periods)

42. Assume that Ramapada have ₹150, then find the CP of pen and pencil.

43. Let the cost price be ₹$1000x$/kg.

The selling price of 900 g = The cost price of 1000 g.

$\therefore$ The selling price of 900 g = ₹$1000x$.

The cost price of 900 g $= \frac{900}{1000}$ (₹$1000x$) = ₹$900x$

Profit in selling 900 g = ₹$100x$.

Profit % $= \frac{100x}{900x}(100\%) = 11\frac{1}{9}\%$.

44. Let the marked price of the article be ₹$100x$.

$\therefore$ Selling price $= \frac{2}{3} \times (₹100x) = ₹\frac{200}{3}x$ (1)

Let the cost price be ₹$100y$.

$$\text{Loss} = \frac{16\frac{2}{3}}{100} \times (₹100y) = ₹\frac{100y}{6}$$

$$\Rightarrow \text{Selling price} = ₹100y - ₹\frac{100y}{6}$$

$$= \frac{₹500y}{6} \quad (2)$$

$$\frac{500y}{6} = \frac{200x}{3} \text{(from Eqs. (1) and (2))} y = \frac{4}{5}x$$

$$\therefore \text{Cost price} = 100\left(\frac{4x}{5}\right) = ₹80x$$

If he sold the article at marked price.

$$\therefore \text{Profit percentage} = \frac{100x - 80x}{80x} \times (100\%) = 25\%.$$

45. Let his cost price be ₹$100x$.

Actual SP = 90% of $(100x)$

Considered SP = 102% of $(100x)$

102% of $(100x)$ = 90% of $(100x)$ + 180

12% of $(100x)$ = 180

$$\therefore 100x = \frac{180}{12}(100) = 1500$$

$\therefore$ Cost price = ₹1500.

46. While calculating the profit/loss percentage, we must consider equal number of books being bought and sold.

The cost price of each book $= ₹\frac{18}{12} = ₹\frac{3}{2}$.

The selling price of each book $= ₹\frac{30}{18} = ₹\frac{5}{3}$.

As $\frac{3}{2} < \frac{5}{3}$, a profit is made.

$$\text{Profit \%} = \frac{\frac{5}{3} - \frac{3}{2}}{\frac{3}{2}x} \times (100\%)$$

$$= \frac{\frac{1}{6}}{\frac{3}{2}} \times (100\%) = 11\frac{1}{9}\%.$$

47. Let the total number of votes casted be $100x$.

The number of invalid votes $= \frac{14}{100}(100x) = 14x$.

The number of valid votes $= 100x - 14x = 86x$.

Let the number of votes secured by Q be q.

Number of votes secured by $P = q$.

$$\left(1 + \frac{15}{100}\right) = \frac{23}{20}q \Rightarrow \frac{23}{20}q + q = 86x$$

$$\frac{43}{20}q = 86x \Rightarrow q = 40x$$

$$\therefore \text{The required percentage} = \frac{40x}{100x}(100\%) = 40\%.$$

Level 3

49. Let the population of the village in the year 2001 be 100 and proceed.

50. Assume the income of the person to be ₹100 and proceed.

51. (i) Let CP of A be ₹100.

(ii) Let CP for A be ₹$100x$.

(iii) Find CP and SP for B, and then find profit earned by B (in x) and compare it with ₹44.

(iv) CP of B = SP of A.

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52. Assume that the salary of the person to be ₹100 and follow the data given in the problem.

53. Let the total number of employees be $100x$.

The number of unmarried employees $= \frac{30}{100}(100x)$ $= 30x$.

The number of unmarried post-graduates $= \frac{20}{100}(30x) = 6x$.

The number of married post-graduates $= \left(3\frac{1}{2}\right)(6x) = 21x$.

The number of married employees = $100x - 30x$ = $70x$.

The required percentage $= \frac{21x}{70x}(100)\% = 30\%$.

54. Let the amounts with Amar, Bhavan and Chetan be ₹a, ₹b and ₹c, respectively.

$$\frac{a+b+c}{3} = 1180$$

$$a+b+c = 3540$$

$$a = b\left(1 + \frac{33\frac{1}{3}}{100}\right) = c\left(1 - \frac{16\frac{2}{3}}{100}\right)$$

$$a = \frac{4}{3}b = \frac{5}{6}c$$

$$b = \frac{3}{4}a \text{ and } c = \frac{6}{5}a$$

$$a+b+c = \frac{59}{20}a$$

$$\frac{59}{20}a = 3540 \text{ (given)}$$

$$\frac{a}{20} = 60$$

$$a = 1200.$$

55. Let the number of matches played be $60 + x$. This is minimum when x is minimum.

As the target was achieved for the minimum value of x, all the x matches must have been won.

Let the minimum $(x) = y$.

∴ The total number of matches won $= \frac{30}{100}(60) + y = 18 + y$

$$\frac{18+y}{60+y} \times (100)\% = 44\%$$

$$1800 + 100y = 2640 + 44y$$

$$56y = 840$$

$$y = 15$$

56. Let Mohan's and Sohan's shares be ₹$9x$ and ₹$11x$, respectively.

$9x + 11x = 300000$

$x = 15000$

Sohan's share excluding his annual salary = $7x$.

∴ Sohan's Annual salary = $11x - 7x = 4x$ = ₹60,000.

Sohan's monthly salary $= \frac{₹60000}{12} = ₹5000$.

57. $3X + 2Y = 230$ (1)

If $3X$ increases by 40% and $2X$ increases by 30%, total increase $= \frac{40}{100}(3X) + \frac{30}{100}(2Y)$

$$\therefore \frac{40}{100}(3X) + \frac{30}{100}(2Y) = 84$$

$$1.2X + 0.6Y = 84$$

Dividing both sides by 0.6, $2X + Y = 140$

∴ $Y = 140 - 2X$ (2)

∴ $2Y = 280 - 4X$

(1) $\Rightarrow 2Y = 230 - 3X$

∴ $280 - 4X = 230 - 3X$

$50 = X$

(2) $\Rightarrow Y = 40$

The required sum $= X\left(1 - \frac{40}{100}\right) + Y\left(1 - \frac{30}{100}\right)$ = (50)(0.6) + (40)

(0.7) = 58.

58. Let the cost price of the first book be ₹$100x$.

The profit made on it $= \frac{10}{100}(₹100x) = ₹10x$

Its selling price = ₹$110x$

The cost price of the second book = ₹$110x$

The loss suffered on it $= \frac{10}{100}(₹111x) = ₹11x$

Its selling price = ₹$99x$

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Total cost price = ₹$210x$

Total selling price = ₹$209x$

As $210x > 209x$, on overall loss is made.

Overall loss = ₹x

$\therefore$ Overall loss percentage $= \frac{x}{210x} \times (100)\% = \frac{10}{21}\%$.

59. Let the cost price for P be ₹$100x$.

P's profit $= \frac{20}{100}(₹100x) = ₹20x$

The selling price for P = ₹$120x$

The cost price for Q = The selling price for P = ₹$120x$

Q's profit $= \frac{10}{100}(₹120x) = ₹12x$

Given, $12x = 20x - 24$

$\Rightarrow 3 = x \Rightarrow 100x = 300$.

60. Let the strength of the class be $100x$.

The number of students who drink pulpy orange

$= \frac{45}{100}(100x) = 45x$.

The number of students who drink Maaza

$= \frac{90}{100}(100x - 45x)$

$= \frac{9}{10}(55x) = 49.5x$

No student drinks both Maaza and pulpy orange.

The number of students of the class who drink neither Maaza nor pulpy orange

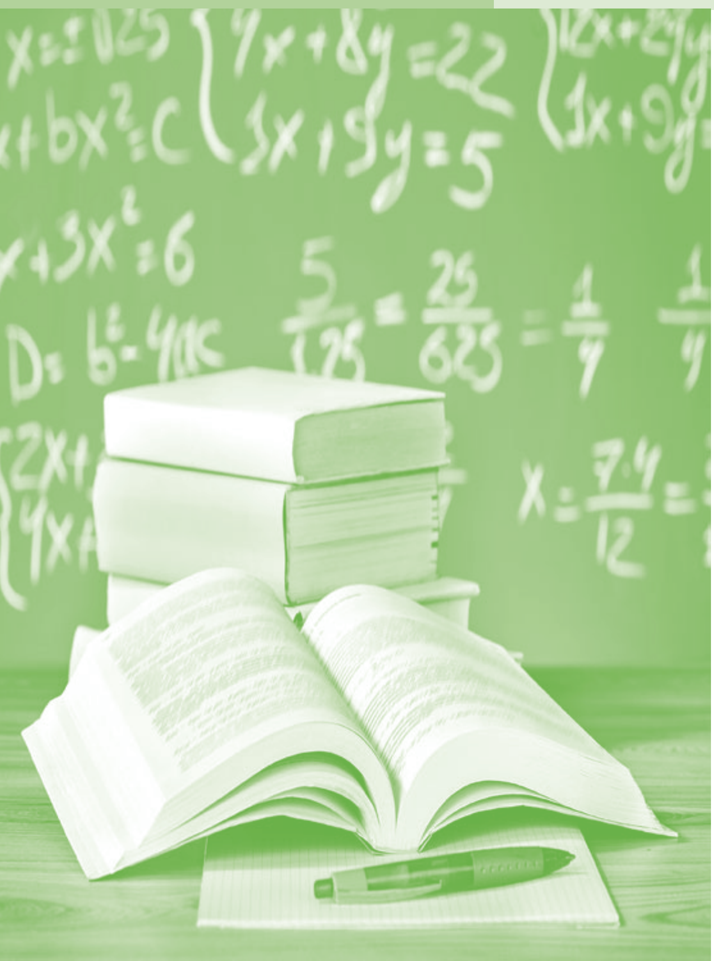
$= 100x - 94.5x = 5.5x$

The required percentage $= \frac{5.5x}{100x}(100)\% = 5.5\%$.

HINTS AND EXPLANATION

Chapter 18

Sales Tax and Cost of Living Index



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the term 'tax' and its importance
- Know development and welfare activities done by the government
- Explain the phrase 'cost of living'

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the term 'sales tax' and its functions
- Know what is value-added tax (VAT)
- Work on cost of living index
- Solve word problems based on tax

INTRODUCTION

The government provides primary health care, free education, maintenance of infrastructural facilities, like roads and bridges. For all these activities, the government needs huge funds. To meet the expenditure incurred on these developmental and welfare activities, the government levies or imposes various taxes, like central excise, sales tax, octroi and income tax.

In this chapter, we will deal with sales tax and the cost of living indexes.

Sales Tax

Sales tax is levied at a specified rate on the net selling price of a commodity. It varies from commodity to commodity, and also from state to state. The selling price is the marked price when discount offered is zero.

In case a discount is allowed on a commodity, the net selling price would be the marked price minus discount.

The traders or business organizations, which collect sales tax, submit monthly sales tax returns to the sales tax office, under whose jurisdiction their business operations are carried out. Sales tax has been replaced by value-added tax in many states.

Value-Added Tax (VAT)

VAT is a multi-point levy on the goods in a supply chain, with the facility to set-off input tax, that is, only the value addition in the hands of the entities is subject to tax. For an instance, a dealer purchases goods worth ₹75 from another dealer, and a tax of ₹3 at 4% is charged on the bill. He sells the goods for ₹100, on which he charges tax of ₹4 at 4%. Since the tax levied on ₹75, i.e., ₹3 is already paid, a tax of ₹4 – ₹3 = ₹1 has to be paid by the buyer in the second transaction.

In the second transaction, the tax is levied on the value addition, i.e., ₹100 – ₹75 = ₹25. Here, 4% of ₹25 = ₹1.

We shall learn about sales tax with the help of the following examples.

EXAMPLE 18.1

The marked price of a suitcase is ₹850. The rate of sales tax is 6%. Find the amount paid by the customer to purchase the suitcase.

SOLUTION

Marked price = ₹850

Sales tax levied $= \frac{6}{100} \times 850 =$ ₹51

Amount paid by the customer = ₹(850 + 51) = ₹901.

EXAMPLE 18.2

Sreekar purchases an article for ₹5200. If the rate of sales tax is 4%, find the marked price of the article.

SOLUTION

Let the marked price of the article be ₹100x.

Sales tax = 4% of 100x = ₹4x.

Total selling price of the article = ₹104x.

Given that $104x = 5200$

$x = 50$.

$\therefore$ Marked price = $100x$

$= 100 \times 50 =$ ₹5000.

EXAMPLE 18.3

Calculate the total sales tax levied on the following products.

(a) A cell phone worth ₹6000, sales tax at 8%.

(b) An electric lamp worth ₹300, sales tax at 6%.

(c) Readymade shirts worth ₹1500, sales tax at 8%.

SOLUTION

Sales tax on the given commodities:

Cell phone $= \dfrac{8}{100} \times 6000 =$ ₹480

Electric lamp $= \dfrac{6}{100} \times 300 =$ ₹18

Readymade shirts $= \dfrac{8}{100} \times 1500 =$ ₹120

$\therefore$ Total sales tax levied = ₹(480 + 18 + 120) = ₹618.

EXAMPLE 18.4

A customer goes to a store to buy a bag costing ₹648 at marked price. The sales tax to be levied on the bag is 8%. But the customer asks the trader to give a discount, after which she has to pay ₹648 to buy the bag. How much discount was given to the customer?

SOLUTION

Let the price of the bag after the discount be ₹100x.

Price after sales tax $= 100x + 8x$

= ₹108x

Given that $108x = 648$

$\Rightarrow x = 6$

Price after discount = ₹100x

$= 100 \times 6$

= ₹600

$\therefore$ Discount given = 648 – 600 = ₹48.

Cost of Living Index

An index is a relative number which indicates changes in prices of commodities, agricultural production, industrial production, cost of living, etc., over a certain period of time.

The changes are marked with reference to a particular year. The year is called a ‘base year’.

If the index number of agricultural production in April 2007 with reference to April 2006 (base year) is 105, it means that there is a 5% increase in the agricultural production from April 2006 to April 2007.

Some of the index numbers that are commonly used are price index number, quality index number and cost of living index number.

Though there are many index numbers that are commonly used, we focus on the cost of living index. We follow the weighted average method to find the cost of living index. In this method, the quantities of commodities consumed by a group of people are taken equal in number in both the years. These are considered as weights. The total expenditures for both the years are calculated. The year, for which the cost of living index is calculated, is referred to as the current year.

$$\text{Cost of living index} = \frac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$$

This method of finding the cost of living index can be better understood with the help of the following example.

EXAMPLE 18.5

Calculate the cost of living index for the year 2006 taking 1990 as the base year from the following information using weighted average method.

		Rate (in ₹ per kg)	
Commodity	**Quantity Consumed (kg)**	**1990**	**2006**
Wheat	15	12	18
Rice	20	13	20
Oil	5	30	50
Gram	3	30	60
Sugar	5	12	18

Total price in the year 1990, for:

Wheat = 15 × 12 = ₹180

Rice = 20 × 13 = ₹260

Oil = 5 × 30 = ₹150

Gram = 3 × 30 = ₹90

Sugar = 5 × 12 = ₹60

Total expenditure = 180 + 260 + 150 + 90 + 60 = ₹740

Similarly, total expenditure in 2006, for:

Wheat = 15 × 18 = ₹270

Rice = 20 × 20 = ₹400

Oil = 5 × 50 = ₹250

Gram = 3 × 60 = ₹180

Sugar = 5 × 18 = ₹90

Total expenditure = ₹1190

$$\therefore \text{Cost of living index in 2006} = \frac{\text{Total expenditure in the year 2006}}{\text{Total expenditure in the year 1990}} \times 100 = \frac{1190}{740} \times 100 = 160.8.$$

EXAMPLE 18.6

Item	Quantity Consumed	Cost (in ₹/kg) In 2007	Cost (in ₹/kg) In 2008
P	16	Y	50
Q	y	20	32
R	10	24	$y + 16$
S	12	45	75

The cost of the living index for 2008, taking 2007 as the base year, is 200. Find y.

(a) 10 **(b)** 12 **(c)** 14 **(d)** 8

SOLUTION

Total expenditure in 2007 (in ₹) = $(16)(y) + (y)(20) + (10)(24) + (12)(45) = 36y + 780$

Total expenditure in 2008 (in ₹) = $(16)(50) + (y)(32) + (10)(y + 16) + (12)(75) = 1860 + 42y$

$$200 = \frac{1860 + 42y}{36y + 780} \times (100)$$

$$2(36y + 780) = 1860 + 42y$$

$$30y = 300$$

$$\therefore y = 10.$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. The tax which is imposed on the sale price of an article is called _____.
2. The _____ differs from item to item and state to state. (sales tax/income tax)
3. If the list price of an article is ₹50 and the rate of sales tax is 2%, then its selling price after tax is _____.
4. Calculate the amount paid by a customer for an article, if its list price is ₹1250 and the rate of sales tax is 5%.
5. Sales tax is calculated on the list price, if no _____ is given.
6. Sales tax is calculated on the _____ price, if discount is given.
7. The sales tax given will reach the _____. (shopkeeper/government)
8. The three index numbers which are commonly used are _____, _____ and _____.
9. Akhila visits a readymade garment shop and purchases a top for ₹300, sales tax @ 3%. Calculate the total amount paid by Akhila to the shopkeeper.
10. Index number measure relative changes in industrial production, agricultural production and also in our _____ of living.
11. The year for which we calculate the cost of living index is referred as _____ year.
12. Rakesh purchased a shirt for ₹560 including sales tax. Find the rate of sales tax if its list price is ₹500.
13. Aryan purchased a Plasma TV for ₹92,000 including sales tax. Find the list price of the TV, if its sales tax is 15%.
14. Akhila visits a readymade garment shop and purchases a trouser for ₹800, sales tax @ 4%. Calculate the total amount paid by Akhila to the shopkeeper.
15. If the index number of sales prices in March 2006 compared to March 2005 is 124, there is an increase of _____ in the wholesale price from March 2005 to March 2006.

Short Answer Type Questions

16. The cost of living index in the year 2006, taking 2004 as the base year, on certain commodities is 162.75. If the total expenditure in 2004 was ₹12,500, then find the total cost for the same commodities in the year 2006.
17. Akhila visits a readymade garment shop and purchases a jean jacket for ₹600, sales tax @ 5%. Calculate the total amount paid by Akhila to the shopkeeper.
18. The reference year with respect to which we measure the changes in the current year is called _____.
19. Cost of living index = _____.
20. If the cost of living index of a certain year is 180 and the expenditure in the current year is ₹36,000, then the expenditure in the base year is _____.

Essay Type Questions

21. If the total expenditures in the base year and current year are equal, then its index number is _____.
22. Ganga visits a shopping mall to buy a necklace costing ₹42,000. The rate of sales tax is 12%. She asks the shopkeeper to allow a discount on the price such that she has to pay ₹44,240 including sales tax. Find the discount needed on the price.
23. Calculate the cost of living index for the year 2002, taking 1994 as the base year. Use weighted average method.

Commodity	Quantity Consumed (in units)	Price per unit in (₹) 1994	2002
A	18	18	24
B	22	11	16
C	13	12	20
D	10	4	10
E	6	45	80
F	5	20	45

24. Shilpa purchases a refrigerator having a list price of ₹10,500 at 12% discount. If sales tax is charged @ 5%, then find the amount Shilpa has to pay to the shopkeeper.

25. From the following data, using weighted average method, calculate the cost of living index for the year 2006, taking 2002 as the base year.

Commodity	Quantity Consumed	Price in 2002 (₹)	Price in 2006 (₹)
Rice	85 kg	10.00 per kg	16.00 per kg
Pulses	30 kg	14.00 per kg	20.00 per kg
Sugar	40 kg	12.00 per kg	15.00 per kg
Coffee	8 kg	120.00 per kg	140.00 per kg
Oil	20 litres	40.00 per litre	60.00 per litre
Cylinder	4	200.00 per filling	350.00 per filling

CONCEPT APPLICATION

Level 1

1. A shopkeeper sells an article, whose cost price is ₹400, for ₹462 including sales tax @ 5%. Find the profit percentage of the shopkeeper.

(a) 7% (b) 12%
(c) 10% (d) 8%

2. Varshit purchased a pair of shoes for ₹3360 including a VAT of 12%. Find the amount he paid towards VAT.

(a) ₹240 (b) ₹360
(c) ₹300 (d) ₹410

3. Nalini purchases a bicycle for ₹2093 inclusive of sales tax @ 15%. Find its list price.

(a) ₹1802 (b) ₹1820
(c) ₹1280 (d) ₹1208

4. On certain consumable goods, the total expenditure of a family was found to be ₹18,000 in the year 2005. If the cost of living index for the year 2006, taking 2005 as the base year, is 240, then find the expenditure of the family on the same quantity of consumable items in the year 2006.

(a) ₹43,200 (b) ₹48,000
(c) ₹24,000 (d) ₹28,000

5. If the index number of wholesale prices in May 2004 as compared to May 2003 is 91, then the expenditure in the year 2004 is less by ______.

(a) 8% (b) 7%
(c) 9% (d) 10%

6. A shopkeeper sells an article, whose cost price is ₹500, for ₹616 including a sales tax @ x%, and for a profit of ₹50. Find x.

(a) 8% (b) 10%
(c) 12% (d) 9%

7. The cost of living index in 2007, taking 2006 as the base year, is 175. The total expenditures in 2006 and 2007 are ₹$(4x + y)$ and ₹$(3x + 4y)$ respectively, where x and y are integers. Which of the following can be the total expenditure of 2007?

(a) ₹48,000 (b) ₹39,000
(c) ₹43,000 (d) ₹45,500

8. Dheeraj bought a watch whose list price is ₹6500. The shopkeeper allowed a discount of 6%, and asked to pay 10% VAT. Find the amount that Dheeraj paid for buying the watch.

(a) ₹6481 (b) ₹6625

(c) ₹6721 (d) ₹6845

9.

Item	Quantity (in kg)	Rate per kg (in ₹) In the Year 2000	In the Year 2007
A	12	X	50
B	x	16	31
C	8	20	$x + 20$
D	10	40	86

The cost of living index for the year 2007 considering the base year as 2000, is 225. Find x.

(a) 15 (b) 18

(c) 12 (d) 20

10. The total expenditure of a certain family was ₹13,090 in the year 2000. If the cost of living index in the year 2006, when compared to the cost in the year 2000, is 120, then the total expenditure in the current year is ______. (in ₹)

(a) 14,280 (b) 15,240

(c) 16,840 (d) 15,708

11. Vikram purchased a TV for ₹13,500 including sales tax. If the rate of sales tax is 8%, then the list price of the TV is ______. (in ₹)

(a) 13,100

(b) 12,800

(c) 12,500

(d) 11,950

12.

Item	Quantity Consumed (kg)	Cost per kg (in ₹) In 2001	In 2007
Rice	80	15	18
Jawar	35	20	24
Wheat	20	10	14
Vegetables	120	5	6
Coffee	10	50	80

Find the cost of living index in the year 2007, taking 2001 as the base year, based on the information given above.

(a) 127.50 (b) 128.75

(c) 132.50 (d) 135.25

13. In the year 1980, the total expenditure of a family was ₹8400. The cost of living index for the year 1980, by taking 1940 as the base year, was 240. Then the expenditure of the family in the year 1940 was ______.

(a) ₹3000 (b) ₹3500

(c) ₹3800 (d) ₹4200

14. The ratio of the total expenditure for the years 2000 and 2006 is 5 : 8. The cost of living index of 2006, taking 2000 as the base year, is ______.

(a) 125 (b) 62.5

(c) 160 (d) Cannot be determined

15. When the year 2000 is taken as the base year, the cost of living index is increased by 20% from 2006 to 2007. If the total expenditure in the base year is ₹20,000, and the cost of living index of 2007 is 210, then the total expenditures (in ₹) in the years 2006 and 2007 are______.

(a) 35,000, 42,000 (b) 30,000, 40,000

(c) 35,000, 45,000 (d) 30,000, 42,000

Level 2

16. The cost of living index of the year 2007, when compared to year 2005, is 165.7. The cost of living index for these two years is calculated considering the prices of certain essential commodities. The amount spent by Anil in year 2005 towards essential commodities is ₹35,000. What is the amount he spent in the year 2007 towards essential commodities, if the quantity of consumption in both the years remains the same?

PRACTICE QUESTIONS

(a) ₹37,762 (b) ₹48,719

(c) ₹57,995 (d) ₹46,750

17.

Item	Quantity (in kg)	Rate per kg (in ₹) In 2000	Rate per kg (in ₹) In 2007
A	105	14	20
B	15	12	X
C	27	10	20
D	36	5	10

The cost of living index for the year 2007, considering the year 2000 as the base year, is 250. Find x.

(a) 167 (b) 158

(c) 150 (d) 162

18. Sujit purchased a furniture set which is listed at ₹58,600 followed by a discount of 10%. If sales tax is charged at the rate of 5%, then the amount Sujit has to pay to buy the furniture is ____. (in ₹)

(a) 53,477 (b) 55,377

(c) 56,446 (d) 54,658

19. Ananya goes to a store to buy a mobile phone costing ₹15,675. The rate of sales tax is 10%. She asks the store keeper to allow a discount to such an extent that she has to pay ₹15,675 inclusive of sales tax. Find the amount of discount to be allowed by the shopkeeper.

(a) ₹1220 (b) ₹1567

(c) ₹1375 (d) ₹1425

20. Rishi purchased a scientific calculator for ₹736, including sales tax. If the list price of the calculator is ₹640, the rate of sales tax is ______.

(a) 12% (b) 15%

(c) 18% (d) 20%

21. Rohit went to a shop and purchased the following goods.

(i) A double cot bed listed at ₹8360, sales tax @ 10%.

(ii) A wardrobe listed at ₹8650, sales tax @ 8%.

(iii) A TV stand listed at ₹500, sales tax @ 9%.

Find the total tax paid by Rohit on the above purchases. (in ₹)

(a) 1483 (b) 1523

(c) 1653 (d) 1573

22. Alok bought a scientific calculator for ₹728, including sales tax. The list price of the calculator was ₹650. Find the rate of sales tax.

(a) 9% (b) 10%

(c) 11% (d) 12%

23. A furniture set was listed at ₹62,400. It was sold at 20% discount. The sales tax charged on it was 10%. Find the amount to be paid to purchase the furniture. (in ₹)

(a) 54,912 (b) 57,408

(c) 52,416 (d) 49,920

24. Pavan went to a mobile phone store. He wanted to buy a mobile phone listed at ₹6075. The rate of sales tax was $11\frac{1}{9}\%$. He requested the shopkeeper to give a discount such that he only needed to pay ₹6075 including sales tax. Find the discount Pavan must be given to fulfill his request. (in ₹)

(a) 607.50 (b) 596.50

(c) 580.50 (d) 616.50

25. The price of a TV, inclusive of sales tax of 12%, is ₹15,680. If the sales tax rate is increased to 14%, then what additional amount of money a customer should pay for it? (in ₹).

(a) 280 (b) 240

(c) 260 (d) 300

Level 3

26. The price of a washing machine, inclusive of sales tax at 11%, is ₹5550. If sales tax is increased to 13%, then what amount of additional money a customer should pay to buy it.

(a) ₹175 (b) ₹240

(c) ₹100 (d) ₹650

27. Three essential commodities, the quantities of their consumption and their prices in the years 2000 and 2007 are listed below. The cost of living index for the years 2000 and 2007 is calculated on the prices of these three commodities only.

Commodity	2006 Consumption (in kg)	2006 Price/kg	2007 Consumption (in kg)	2007 Price/kg
A	x	y	$2x$	y
B	y	z	$\frac{y}{2}$	$4z$
C	z	x	$2z$	x

Find the cost of living index of the year 2007, taking 2006 as the base year.

(a) 200 (b) 100
(c) 150 (d) 50

28. Ritesh went to a shopping mall to buy a trouser costing ₹3500. On his request, the shopkeeper allowed two successive discounts of x% and y% ($x > y$). If Ritesh paid ₹2618 including sales tax of 10%, then which of the following could be the discount rates given by the shopkeeper?

(a) 20%, 15% (b) 20%, 10%
(c) 20%, 25% (d) 15%, 10%

29. On certain consumable commodities, the total expenditure of a family was found to be ₹28,000 in the year 1994. If the cost of living index for the year 2000, taking 1994 as the base year, is 265, then the expenditure of the family, on the same quantities of consumable commodities, in the year 2000 is ______.

(a) ₹78,000 (b) ₹79,000
(c) ₹78,200 (d) ₹74,200

30. Anshu visits a super market and purchases the following goods:

(A) A dinner set which is listed at ₹1750, sales tax @ 5%.

(B) A computer table, for ₹2700 including sales tax @ 8%.

(C) A bag of rice which is listed at ₹4800, sales tax @ 4%.

Find the amount of tax paid by Anshu on these goods.

(a) ₹479.50 (b) ₹384.50
(c) ₹345.00 (d) ₹493.00

31. Ashok sold an article, whose cost price is ₹300, for ₹351 including sales tax @ 8%. Find his profit percentage.

(a) $13\frac{1}{3}$% (b) $8\frac{1}{3}$%
(c) $16\frac{2}{3}$% (d) $10\frac{2}{3}$%

32. Arun bought a motorbike at a discount of 10%, and paid a sales tax @ 8%. He paid it for ₹47,142. Find its list price. (in ₹)

(a) 48,500 (b) 49,500
(c) 50,500 (d) 51,500

33. A shopkeeper sold an article, costing ₹360, for ₹495 including a sales tax of y% at a profit of ₹90. Find y.

(a) 8 (b) 12.5
(c) 15 (d) 10

34. The ratio of the total expenditures of a family for the years 2002 and 2007 is 4 : 7. Find the cost of living index for the year 2007, taking 2002 as the base year.

(a) 135 (b) 155
(c) 175 (d) 195

35. The cost of living index of the year 2006, taking the year 2005 as the base year, is 225. The total expenditures in 2005 and 2006 are ₹$(2x + 7y)$ and ₹$(7x + y)$ respectively, where x and y are positive integers. Which of the following can be the total expenditure in 2005? (in ₹)

(a) 37,600 (b) 47,600
(c) 28,400 (d) 27,200

36. The total expenditure of a family 2003 was ₹15,000. Taking this year as the base year, the cost of living index in 2007 and 2008 were calculated. The cost of living index 2008 was 25% more than that in 2007. The cost of living index in 2008 was

PRACTICE QUESTIONS

160. Find the total expenditure of the family in 2007. (in ₹)

(a) 17,500 (b) 18,000

(c) 18,400 (d) 19,200

37. Ravi bought a pair of slippers for ₹944, including a VAT of 18%. Find amount of VAT he paid. (in ₹)

(a) 108 (b) 132

(c) 144 (d) 128

38. Ramesh bought a book whose list price is ₹750. The shopkeeper gave him 8% discount. If the rate of VAT is 15%, then find the amount that Ramesh paid for the book. (in ₹)

(a) 793.50 (b) 767.50

(c) 805.50 (d) 813.50

39. The list price of an AC is ₹20,000. The shopkeeper sold it by allowing a 4% discount and charged 5% sales tax. By mistake, while calculating the bill, he considered 5% discount and 4% sales tax. As a result, the customer must have paid ______.

(a) ₹200 less (b) ₹400 less

(c) ₹400 more (d) ₹200 more

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. sales tax
2. sales tax
3. ₹51
4. ₹1312.50
5. discount
6. selling
7. government
8. price index number, quality index number and cost of living index number.
9. ₹309
10. cost
11. current year
12. 12%
13. ₹80,000
14. ₹832
15. 24%

Short Answer Type Questions

16. ₹20,343.75
17. ₹630
18. Base year
19. $\dfrac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$
20. ₹20,000

Essay Type Questions

21. 100
22. ₹2500
23. 163.34
24. ₹9702
25. 145.70

CONCEPT APPLICATION

Level 1

1. (c) **2.** (b) **3.** (b) **4.** (a) **5.** (c) **6.** (c) **7.** (d) **8.** (c) **9.** (a) **10.** (d)
11. (c) **12.** (a) **13.** (b) **14.** (c) **15.** (a)

Level 2

16. (c) **17.** (c) **18.** (b) **19.** (d) **20.** (b) **21.** (d) **22.** (d) **23.** (a) **24.** (a) **25.** (a)

Level 3

26. (c) **27.** (a) **28.** (a) **29.** (d) **30.** (a) **31.** (b) **32.** (a) **33.** (d) **34.** (c) **35.** (a)
36. (d) **37.** (c) **38.** (a) **39.** (b)

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (i) Sales tax paid by the shopkeeper $= \dfrac{462 \times 100}{105}$.

(ii) Selling price = ₹(462 – sales tax).

(iii) Profit % $= \dfrac{\text{Profit}}{\text{CP}} \times 100$.

2. (i) Let the list price be ₹x.

(ii) x + 12% of x = 3360, find x, then find $(3360 - x)$.

3. List price = Selling price – Sales tax.

4. (i) Let the list price be ₹x.

(ii) x + 12% of x = 3360, find x, then find $(3360 - x)$.

5. (i) Let the list price be ₹x.

(ii) x + 12% of x = 3360, find x, then find $(3360 - x)$.

6. (i) Sales tax = [(616 – 50) – 500] = 66

(ii) Percentage of sales tax $= \dfrac{\text{Sales tax}}{\text{SP}} \times 100$.

7. (i) Apply the formula for finding the cost of living index.

(ii) $\dfrac{3x + 4y}{4x + y} \times 100 = 175$, simplify and get the relation between x and y.

(iii) Now, replace x or y in $(3x + 4y)$ and verify from the options.

8. (i) Calculate the price after 6% discount.

(ii) Find 10% VAT on the price.

(iii) Using the given information, find the total amount paid.

9. (i) Find the total expenditure in 2000 and 2007.

(ii) Use, cost of living index

$= \dfrac{\text{Expenditure in 2007}}{\text{Expenditure in 2000}} \times 100$.

10. Cost of living index

$= \dfrac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$.

11. (i) Let the list price be ₹x.

(ii) x + 8% of x = 13,500, find x.

12. Cost of Living Index

$= \dfrac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$.

13. Cost of Living Index

$= \dfrac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$.

14. (i) Let the total expenditure in 2000 and 2006 be ₹$5x$ and ₹$8x$, respectively.

(ii) Use formula to find cost of the living index.

15. (i) Cost of living index for the year 2006

$= 210 \times \dfrac{100}{120}$, where the base year's value is 100.

(ii) Use formula to find total expenditures in 2006 and 2007.

Level 2

17. Find the expenditure in the given years and cost of living index

$= \dfrac{\text{Total expenditure in the current year}}{\text{Total expenditure in the base year}} \times 100$.

18. First, deduct the discount from the list price, and then find the sales tax on the remaining amount.

19. Let the discount be ₹x, then

$(5675 - x) \times \dfrac{110}{100} = 15675$.

20. Find the sales tax in rupees.

21. Total tax (in ₹)

$= \dfrac{10}{100}(8360) + \dfrac{8}{100}(8650) + \dfrac{9}{100}(500)$

$= 836 + 692 + 45 = 1573$.

22. List price = ₹650.

Selling price including sales tax = ₹728

Let the sales tax rate be r%.

Sales tax (in ₹) = 728 – 650 = 78

The rate of sales tax $= \frac{78}{650} \times 100\% = 12\%$.

23. List price = ₹62,400

Discount (in ₹) $= \frac{20}{100}(62,400) = 12,480$

Price after discount (in ₹) = 49,920

Sales tax (in ₹) $= \frac{10}{100}(49,920) = 4992$

Amount to be paid (in ₹) = 49,920 + 4992 = 54,912.

24. Let the discount be ₹x.

Price after discount = ₹(6075 – x)

Sales tax (in ₹) $= \frac{11\frac{1}{9}}{100}(6075 - x) = \frac{1}{9}(6075 - x)$

The price of the mobile including sales tax (in ₹)

$= 6075 - x + \frac{1}{9}(6075 - x) = \frac{10}{9}(6075 - x)$

$\therefore \frac{10}{9}(6075 - x) = 6075$

$6075 = 10x$

$x = 607.50$.

25. Let the list price of the TV be ₹x.

Sales tax = ₹ $\frac{12}{100}x$

$\therefore \quad x + \frac{12}{100}x = 15,680$

$x = \frac{15,680}{1.12} = 14,000$

Additional amount to be paid (in ₹) = (14 – 12)% of x = 2% of 14,000 = 280.

Level 3

26. Find the list price and then proceed. (Refer to Q2)

27. (i) Find the total expenditures in 2006 and 2007.

(ii) Refer to the Answer Keys for Q.13.

30. Find the sales tax of each item as per their rates and proceed.

31. Let the list price be ₹x.

Sales tax (in ₹) $= \frac{8}{100}x = \frac{2}{25}x$.

Selling price including sales tax (in ₹)

$= \frac{27}{25}x$.

$\frac{27}{25}x = 351$

$\frac{x}{25} = 13$

$x = 325$

Profit percentage $= \frac{325 - 300}{300}(100)\% = 8\frac{1}{3}\%$.

32. Let its list price be ₹x.

Discount (in ₹) $= \frac{10}{100}x = 0.1x$

Price after discount = ₹$0.9x$

Sales tax (in ₹) $= \frac{8}{100}(0.9x) = 0.072x$

The price including sales tax (in ₹) $= 0.9x + 0.072x = 0.972x$

$0.972x = 47142 \Rightarrow x = 48,500$.

33. Cost price = ₹360

Profit = ₹90

The price excluding sales tax = ₹360 + 90 = ₹450

Sales tax $= \frac{y}{100}$(₹450) = ₹$4.5y$

Selling price including sales tax (in ₹) = $450 + 4.5y$

$450 + 4.5y = 495 \Rightarrow y = 10$.

34. Let the total expenditures for the year 2002 and 2007 be ₹$4x$ and ₹$7x$, respectively.

Cost of living index $= \frac{7x}{4x}(100) = 175$.

35. $225 = \frac{7x+y}{2x+7y}(100)$

$450x + 1575y = 700x + 100y$

$1475y = 250x$

$5.9y = x$

Total expenditure in 2006 (in ₹) = $2x + 7y$ = $2(5.9y) + 7y = 18.8y$

$= \frac{188y}{10} = \frac{94y}{5}$

y is a positive integer.

Only option (a) satisfies this condition.

$(\because \frac{94y}{5} = 37600 \therefore y = 2000.$ In other options, y is not a positive integer).

36. Let the cost of living index in 2007 be x.

The total expenditure of the family in 2007

$x\left(1+\frac{25}{100}\right) = 160$

$x = \frac{160}{1.25} = 128$

$\frac{y}{15,000}(100) = 128 \Rightarrow y = 19,200.$

37. Let the list price be ₹x.

VAT (in ₹) $= \frac{18}{100}x = \frac{9}{50}x$

Selling price including VAT (in ₹)

$= x + \frac{9}{50}x = \frac{59}{50}x$

$\frac{59}{50}x = 944$

$x = 800$

$\therefore$ VAT (in ₹) $= \frac{9}{50}x = 144.$

38. List price = ₹750

Discount (in ₹) $= \frac{8}{100}(750) = 60$

Price after discount = ₹690

VAT paid (in ₹) $= \frac{15}{100}(690) = 103.50$

Amount paid (in ₹) = 690 + 103.50

= 793.50.

39. List price = ₹20,000

Discount (in ₹) $= \frac{4}{100}(20,000) = 800$

Price after discount = ₹19,200

Sales tax (in ₹) $= \frac{5}{100}(19,200) = 960$

Amount to be paid (in ₹) = 19,200 + 960 = 20,160

Discount calculated (in ₹) $= \frac{5}{100}(20,000) = 1000$

Calculated price after discount = ₹19,000

Sales tax calculated (in ₹) $= \frac{4}{100}(19,000)$

= 760

Amount actually paid (in ₹) = 19,000 + 760

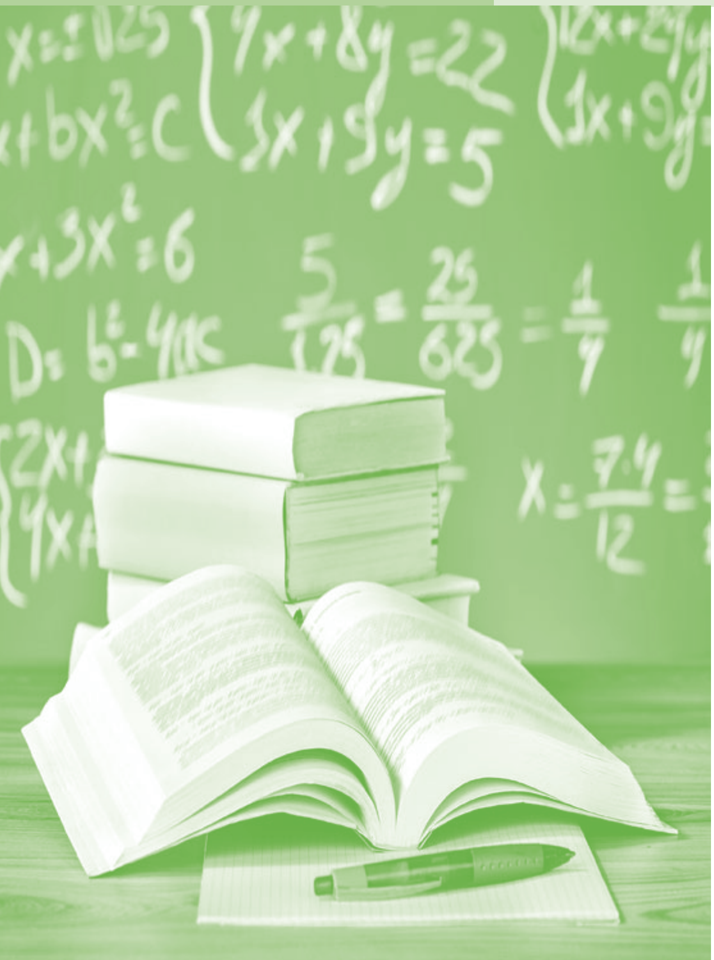
= 19,760

$\therefore$ The customer must have paid (20,160 − 19,760), i.e., ₹400 less.

Chapter

19

Simple Interest and Compound Interest



REMEMBER

Before beginning this chapter, you should be able to:

- Calculate percentage
- Understand interest rates and time period
- Compute formulae for simple interest

KEY IDEAS

After completing this chapter, you should be able to:

- Review the concept of simple interest and solve word problems on it
- Learn practical applications of simple and compound interest
- Understand functions of compound interest and calculate compound interest
- Know about compounding more than a year

INTEREST

Interest is paid to the lender by the borrower for using money for a specified period of time. Various terms and their general representations are as follows:

1. *Principal:* The original sum borrowed and denoted by P (expressed in rupees)
2. *Time:* The time for which money is borrowed. It is denoted by n. (n is expressed in number of periods, which is normally one year)
3. *Rate:* The rate at which interest is calculated on the original sum. It is denoted by r. (calculated per ₹100 per year)
4. *Amount:* The sum of principal and interest. It is denoted by A.

Simple Interest

When interest is calculated every year (or every time period) on the original principal, i.e., the sum at the beginning of the first year, such interest is called **simple interest**. Here, year after year, even though the interest gets accumulated and is due to the lender, this accumulated interest is not taken into account for the purpose of calculating interest for the later years.

$$\text{Simple interest} = \frac{PNR}{100};\ \text{where } P, N \text{ and } R \text{ are explained above.}$$

$$\text{Total amount, } A = P + \frac{PNR}{100} = P\left(1 + \frac{NR}{100}\right).$$

Compound Interest

In compound interest, the interest is added to the principal at the end of each period to arrive at the new principal for the next period.

In other words, the amount at the end of the first year (or period) will become the principal for the second year (or period); the amount at the end of the second year (or period) becomes the principal for the third year (or period), and so on.

If P denotes the principal at the beginning of period 1, then

$$P \text{ at the beginning of year } 2 = P\left(1 + \frac{r}{100}\right) = PR.$$

$$P \text{ at the beginning of year } 3 = P\left(1 + \frac{r}{100}\right)^2 = PR^2.$$

$$P \text{ at the beginning of year } (n+1) = P\left(1 + \frac{r}{100}\right)^n = PR^n;\ \text{where } R = \left\{1 + \left(\frac{r}{100}\right)\right\}.$$

Hence, the amount after n years (or periods) $= PR^n = A$.

Interest $= I = A - P = P[R^n - 1]$.

EXAMPLE 19.1

Find the simple interest on a sum of ₹1000 at an interest rate of 6% per annum, for a period of 6 years.

SOLUTION

The formula for simple interest is $I = \dfrac{PNR}{100}$; where P, the principal = ₹1000; R the rate of interest = 6%, and N is the number of years = 6.

$$\text{SI} = \frac{1000 \times 6 \times 6}{100} = ₹360.$$

EXAMPLE 19.2

If ₹5000 becomes ₹5700 in a year's time at simple interest, what will ₹7000 become at the end of 5 years at the same rate of interest?

SOLUTION

Amount = Principal + Interest

Principal = ₹5000

Therefore, 5700 = 5000 + Interest

Interest = ₹700

$$I = \frac{PNR}{100} \Rightarrow 700 = \frac{5000 \times R \times 1}{100} \Rightarrow R = 14\%.$$

Therefore, the rate of interest = 14% per annum

∴ The simple interest on ₹7000 at the rate of 14% for a period of 5 years is (7000 × 14 × 5)/100 = ₹4900. Therefore, ₹7000 at the end of 5 years, amounts to ₹7000 + 4900 = ₹11,900.

The following table gives an example of how simple interest and compound interest operate, i.e., how the principal is for various years under simple interest and compound interest. A principal at the beginning of 1st year, of ₹100 and a rate of 10% per annum are considered. The details are worked out for three years and shown below:
(All figures pertaining to principal, interest and amount are in rupees)

	Under Simple Interest				Under Compound Interest			
Year	Principal at the Begining of the Year (₹)	Interest for the Year (₹)	Interest till the End of the Year (₹)	Amount at the End of the Year (₹)	Principal at the Begining of the Year (₹)	Interest for the Year (₹)	Interest till the End of the Year (₹)	Amount at the End of the Year (₹)
1	100	10	10	110	100	10	10	110
2	100	10	20	120	110	11	21	121
3	100	10	30	130	121	12.1	33.1	133.1

As we observe in the table:

In case of simple interest,

- The principal remains same every year.
- The interest for any year is same as that for any other year.

In case of compound interest,

- The amount at the end of a year is the principal for the next year.
- The interest for different years is not the same.

The compound interest for the first year (where compounding is done every year) is the same as simple interest for one year.

EXAMPLE 19.3

The cost of an electronic device reduces at the rate of 5% per annum. If its present cost is ₹32,490, what was its cost before two years? Choose the correct answer from the following options:

(a) ₹37,000 **(b)** ₹35,400 **(c)** ₹36,000 **(d)** ₹37,200

SOLUTION

$$32{,}490 = P\left[1 - \frac{R}{100}\right]^2$$

$$32{,}490 = P\left[1 - \frac{5}{100}\right]^2$$

$$\Rightarrow 32{,}490 = P\left[\frac{20-1}{20}\right]^2$$

$$\Rightarrow 32{,}490 = P \times \frac{19}{20} \times \frac{19}{20}$$

$\therefore P =$ ₹36,000.

EXAMPLE 19.4

A certain sum lent for a period of $2\frac{1}{2}$ years under simple interest at 9% per annum earned an interest of ₹234. From the following options, find the sum that was lent.

(a) ₹960 **(b)** ₹1040 **(c)** ₹1246 **(d)** ₹1146

SOLUTION

$$\frac{PTR}{100} = 1$$

$$\frac{P \times \frac{5}{2} \times 9}{100} = 234$$

$$\Rightarrow \frac{P \times 45}{200} = 234$$

$$\Rightarrow P = \frac{234 \times 200}{45}$$

$\therefore P =$ ₹1040.

Compounding More Than Once a Year

We just looked at calculating amount and interest when compounding is done once in a year. But, compounding can also be done for multiple times in a year. For example, interest can be added to the principal in every six months or every four months, and so on.

If interest is added to the principal in every six months, we say that compounding is done twice a year. If interest is added to the principal in every four months, we say that compounding is done thrice a year. If interest is added to the principal in every three months, we say that compounding is done four times a year.

The formula that we have discussed above for calculating the amount will essentially be the same,

That is, Amount $= P\left(1 + \frac{r}{100}\right)^n$, but the rate ($r$) will <u>not</u> be for one year but for the time period over which compounding is done and the power to which the term inside the bracket is

raised (n in the above case) will <u>not</u> be the number of years, but the number of years multiplied by the number of times compounding is done per year (this product is referred to as the total number of time periods).

For example, if a sum of ₹10,000 is lent at the rate of 10% per annum and compounding is done in every four months (thrice a year), then the amount for two years will be equal to:

$$10{,}000\left(1+\frac{10}{3}\times\frac{1}{100}\right)^{2\times 3}$$

Here, the dividing factor of 3 in the rate and the multiplying factor of 3 in the exponent (multiplying the number of years)—both shown by arrow marks—are nothing, but the number of times compounding is done in a year.

If compounding is done k times a year (i.e., once in every $\frac{12}{k}$ months), at the rate of r% per annum, then in n years the principal (P) will amount to $= P\left(1+\frac{r}{k\times 100}\right)^{kn}$.

When compounding is done more than once a year, the rate of interest given in the problem is called nominal rate of interest.

We can also calculate the rate of interest which will yield simple interest in one year equal to the interest obtained under the compound interest at the given nominal rate of interest. The rate of interest so calculated is called effective rate of interest.

The following points helpful in solving problems should also be noted.

The difference between compound interest and simple interest on a certain sum for two years is equal to the interest calculated for one year on one year's simple interest.

In mathematical terms, the difference between compound interest and simple interest for two years will be equal to $P(r/100)^2$, which can be written as $P(r/100)\,(r/100)$. In this, $Pr/100$ is the simple interest for one year. When this is again multiplied by $r/100$, it gives interest for one year on $Pr/100$, i.e., interest for one year on one year's simple interest.

The difference between the compound interest for k years and the compound interest for $(k + 1)$ years is the interest for one year on the amount at the end of kth year.

This can also be expressed as follows:

The difference between the amount for k years and the amount for $(k + 1)$ years under compound interest is the interest for one year on the amount at the end of the kth year.

The difference between the compound interest for the kth year and the compound interest for the $(k + 1)$th year is equal to the interest for one year on the compound interest for the kth year.

EXAMPLE 19.5

At what rate of interest per annum, under compound interest, will ₹5120 amount to ₹7290 in 3 years?

SOLUTION

P = ₹5120

A = ₹7290

N = 3 years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$7290 = 5120\left(1+\frac{r}{100}\right)^3 \Rightarrow \frac{7290}{5120} = \left(1+\frac{r}{100}\right)^3$$

$$\left(\frac{9}{8}\right)^3 = \left(1+\frac{r}{100}\right)^3 \Rightarrow 1+\frac{r}{100} = \frac{9}{8}$$

$$\frac{r}{100} = \frac{1}{8} \Rightarrow r = \frac{25}{2} = 12\frac{1}{2}\%.$$

EXAMPLE 19.6

A sum of money triples itself in 3 years at compound interest. In how many years will it become 9 times itself?

SOLUTION

Let the sum be ₹p, rate be r% per annum.

$$3p = p\left(1+\frac{r}{100}\right)^3 \Rightarrow \left(1+\frac{r}{100}\right) = 3^{1/3}.$$

$$\text{Now, } 9p = p\left(1+\frac{r}{100}\right)^n \Rightarrow \left(1+\frac{r}{100}\right)^n = 9$$

$$3^{\frac{n}{3}} = 3^2 \Rightarrow \frac{n}{3} = 2 \Rightarrow n = 6.$$

∴ In 6 years, the sum becomes 9 times itself.

EXAMPLE 19.7

The difference between the compound interest and the simple interest for 2 years at 8% per annum on a certain sum of money is ₹120. Find the sum.

SOLUTION

Let sum be ₹P.

$$\text{CI} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right] = P\left[\left(1+\frac{8}{100}\right)^2 - 1\right]$$

$$\text{CI} = \frac{104P}{625}$$

$$\text{SI} = \frac{P \times 8 \times 2}{100} = \frac{4P}{25}.$$

Given, CI – SI = ₹120

$$= \frac{104P}{625} - \frac{4P}{25} = 120 \Rightarrow \frac{4P}{625} = 120$$

$$P = ₹18,750$$

$$\text{Sum} = ₹18,750.$$

EXAMPLE 19.8

The population of certain type of bacteria grows at 4%, 5% and 8% during first, second and third years, respectively. Find the population of the bacteria after 3 years, if the present population is 100,000.

SOLUTION

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$$

$$A = 100,000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{8}{100}\right)$$

$$A = 100,000\left[\frac{26}{25}\right]\left(\frac{21}{20}\right)\left(\frac{27}{25}\right) \Rightarrow A = 117,936.$$

$\therefore$ The population of the bacteria after 3 years is 117,936.

EXAMPLE 19.9

A man borrowed ₹10,000 at 12% per annum, interest compounded quarterly. Find the amount that he has to pay after 9 months.

SOLUTION

P = ₹10,000

R = 12% per annum

$R = \frac{12}{4}\%$ per quarter

Time period = 9 months

$\therefore\ n = \frac{9}{3} = 3$

$$A = 10,000\left(1+\frac{3}{100}\right)^3$$

$$= 10,000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} = ₹10,927.27.$$

EXAMPLE 19.10

Find the compound interest on ₹50,000 for 3 years, compounded annually, and the rate of interest being 10%, 12% and 15% for the three successive years, respectively.

SOLUTION

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$$

$$A = 50,000\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)\left(1+\frac{15}{100}\right)$$

$$A = 50,000\left(\frac{11}{10}\right)\left(\frac{112}{100}\right)\left(\frac{115}{100}\right)$$

$$A = ₹70,840.$$

$$\therefore \text{CI} = A - P = ₹20,840.$$

EXAMPLE 19.11

Two persons P and Q lent certain amounts at the same rate of interest for 2 years and 3 years, respectively, under compound interest. If their final amounts are in the ratio of 3 : 5, Q's amount at the end of the first year being ₹8500 and P earned an interest of ₹510 for the first year, then find the ratio of their principals.

(a) 33 : 50 **(b)** 27 : 47 **(c)** 35 : 61 **(d)** 22 : 51

SOLUTION

P_1, P_2 are the principals of P and Q.

$$\frac{P_1\left(1+\frac{R}{100}\right)^2}{P_2\left(1+\frac{R}{100}\right)^3} = \frac{3}{5}$$

$$\Rightarrow \frac{P_1}{P_2\left(1+\frac{R}{100}\right)} = \frac{3}{5} \Rightarrow \frac{P_1}{8500} = \frac{3}{5}$$

$$\Rightarrow P_1 = 5100.$$

$$\frac{P_1 \times 1 \times R}{100} = 510$$

$$\frac{5100 \times R}{100} = 510$$

$$R = 10\%$$

$$P_2\left(1+\frac{R}{100}\right) = 8500.$$

$$\Rightarrow P_2\left(1+\frac{10}{100}\right) = 8500 \Rightarrow P_2 \times \frac{11}{10} = 8500$$

$$\Rightarrow P_2 = 8500 \times \frac{10}{11}.$$

$$\therefore P_1 : P_2 = 5100 : 8500 \times \frac{10}{11}$$

$$= 3 : \frac{50}{11} = 33 : 50.$$

EXAMPLE 19.12

The difference between simple interest and compound interest on a sum of ₹20,000 for two years is ₹112.50. What is the annual rate of interest? Choose the correct answer from the following options:

(a) 6% **(b)** 7.5% **(c)** 8.5% **(d)** 8%

SOLUTION

The difference between simple interest and compound interest on a sum $(x) = x\left(\frac{r}{100}\right)^2$.

Therefore,
$$20{,}000\left(\frac{r}{100}\right)^2 = 112.5$$
$$\left(\frac{r}{100}\right)^2 = \frac{112.5}{20{,}000}$$
$$\left(\frac{r}{100}\right)^2 = 0.005625$$
$$\left(\frac{r}{100}\right) = 0.075$$
$$\therefore r = 7.5\%$$

EXAMPLE 19.13

Ravi borrowed ₹15,000 at the rate of 15% per annum for 2 years under simple interest. As he could not repay the loan after two years, the moneylender lender increased the rate of interest to 20% per annum for the further period. If Ravi wants to repay the entire amount at the end of a total period of 3 years and 4 months, then how much he has to pay. (in ₹)

(a) 24,500 **(b)** 24,000 **(c)** 23,500 **(d)** 23,000

SOLUTION

Case 1: For the first two years

$P = 15{,}000$, $R = 15\%$, $T = 2$

$$I = \frac{15{,}000 \times 15 \times 2}{100} = ₹4500.$$

Case 2: For the remaining time period

$P = 15{,}000$, $R = 20\%$, $T = 16$ months

$$I = \frac{15{,}000 \times 20 \times \frac{16}{12}}{100}$$
$$\Rightarrow I = \frac{15{,}000 \times 20 \times 16}{1200} = ₹4000.$$

$\therefore$ Total amount that Ravi has to pay =15,000 + 4500 + 4000 = ₹23,500.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Bank P provides loan at 10% per annum simple interest, and bank Q provides loan at 20% per annum simple interest. Vinay borrowed ₹1000 for 2 years from bank P, Kumar borrowed ₹1000 for 2 years from bank Q. How much extra interest Kumar have to pay?
2. The difference between the compound interest for the $(p + 1)$th year and compound interest for $(p + 2)$th year is equal to the interest for one year on the compound interest for the $(p + 1)$th year. (True/False)
3. Compound interest for one year on ₹500 calculated half yearly at 20% per annum is ₹100. (True/False)
4. Find the compound interest on ₹5000 at 10% per annum for 1 year, interest compounded annually?
5. A certain sum doubles in 3 years under simple interest. In how many years will the sum become 5 times itself?
6. Simple interest on a sum of ₹10,000 for 3 years at the rate of 20% per annum is ______.
7. In what time will a sum become three times itself at 20% per annum, at simple interest?
8. A certain sum becomes 16 times in 4 years at compound interest, compounded annually. What is the rate of interest?
9. If compounding is done p times a year, at the rate of 5% per annum for n years, the principal x will amount to ______.
10. A certain sum amounts to ₹73,255 in 3 years, and ₹84,525 in 5 years at simple interest. Find the sum.
11. If a certain sum becomes 3 times itself in 4 years at compound interest, in how many years will the sum become 81 times itself at the same rate of interest?
12. Raju borrowed ₹15,000 from Mahesh at the rate of 15% per annum under simple interest for 3 years. Raju lent some part of money at 20% per annum at simple interest for 3 years and the remaining part at 12% per annum at simple interest for 3 years. If the interest received by Raju on the money lent is equal to the interest payable, then find the sum he lent at 20% simple interest?
13. Farheen borrowed a sum of ₹18,000 at the rate of 20% per annum, interest compounded semi-annually. Find the amount and compound interest for a period of $1\frac{1}{2}$ years.
14. Find the rate of simple interest per annum, if the sum borrowed becomes 3 times itself in 12 years.
15. Find the simple interest on ₹5600 at 20% per annum from May 22 to November 5 of the same year.
16. If ₹6000 becomes ₹6720 in 2 years at simple interest, how much does a sum of ₹10,000 become in 5 years at the same rate of simple interest?
17. The compound interest for 2 years and simple interest for 1 year on a certain sum at certain rate of interest are ₹3780 and ₹1800, respectively. Find the principal amount and interest rate.
18. The compound interest on a certain sum at a certain rate of interest for the 2nd year is ₹720, and for the 3rd year is ₹864. Find the principal and rate of interest.
19. Farhan borrowed ₹25,000 at 10% per annum under compound interest. He repaid a certain amount at the end of the first year and paid ₹24,750 at the end of 2nd year to completely disburse the loan. What amount did Farhan repay at the end of the first year?
20. The compound interest on ₹1000 is ₹331 for 3 years at certain rate of interest. What is the rate of interest?
21. If ₹6000 is lent at 10% per annum, interest being compounded annually, then what is the interest for the 3rd year?
22. What sum would amount to ₹17,280 in 3 years, at an interest of 20% per annum rate, interest compounded annually?
23. If the rate of interest is 20% per annum compounded in every 6 months, then what is the effective rate of interest per annum?
24. The difference between the compound interest and the simple interest on a certain sum for 2 years is equal to the interest calculated for 1 year on one year's simple interest. (True/False)
25. Shyam borrowed ₹18,000 at 15% per annum at compound interest compounded annually. He repaid ₹10,700 at the end of the 1st year. What is the amount he should pay at the end of 2nd year to completely disburse the loan?

26. Rajesh borrows ₹50,000 at simple interest, but the rate of interest is not constant for the entire period. For the first three years it is 10% per annum, for the next two years it is 5% per annum and for next three years it is 8% per annum He repaid the entire amount after 8 years. How much he have to repay to clear the debt?

27. Janardhan deposited a certain sum of money in fixed deposit account at k% per annum interest being compounded annually. If the amount of interest accrued for the 3rd and the 4th years is ₹5000 and ₹6250 respectively, what is the total interest accrued for the first two years?

28. The difference between compound interest at 10% per annum and simple interest at 8% per annum on a certain sum for 3 years is ₹910. Find the sum.

29. Find the simple interest on ₹3750 at $5\frac{1}{2}\%$ per annum for the period beginning 3rd February 2007 to 29th June 2007.

30. A sum at $12\frac{1}{2}\%$ per annum amounts to ₹8723 in 5 years at simple interest. Find the sum.

Short Answer Type Questions

31. The population of a town increases at the rate of 5% every year. Find the population of the town in the year 2008, if it's population in 2005 was 200,000.

32. Suman borrowed ₹8000 from Mahesh at 20% per annum at simple interest. After 3 years when Suman wanted to clear the debt, Mahesh insisted to pay the amount at compound interest. How much more Suman have to pay?

33. Jahangir borrowed ₹80,000 at the rate of 7% per annum at compound interest, interest being compounded annually. How much should he repay at the end of the first year, so that by repaying ₹48,792 at the end of second year he can clear the loan?

34. Sunil borrowed a certain sum from a moneylender under compound interest, interest being annually. If the interest for the 2nd year is 2 times the interest for the first year, then what is the rate of interest?

35. The difference between the simple interest received from two different persons on ₹1800 for 4 years is ₹36. The difference between their rates of interest is ______.

36. The compound interest is earned on a sum of money at a rate of 8% per annum for the first year and 10% per annum for the second year. Find the single equivalent rate of interest on the sum for the two years.

37. Akhil invested ₹8000 in a bank, which pays compound interest, compounded semi-annually. He receives ₹9261 after 18 months from the bank. Find the rate of interest per annum.

38. A moneylender found that a fall in the annual rate of simple interest from 7% to 6% resulted in his amount of income being reduced by ₹212.50. Find his capital. (in ₹)

39. A sum doubles itself in 4 years at simple interest. How many times will it amount in 8 years at the same rate of simple interest?

40. Mahesh deposited ₹5000 in Syndicate Bank for 6 months. If the bank pays compound interest at 12% per annum, reckoned quarterly, find the interest received by him.

41. The difference between SI and CI (compounded annually) on a sum of ₹64,000 for 2 years is ₹1000. What is the rate of interest per annum?

42. A sum of ₹150 is borrowed at simple interest at 4% per annum for the first month, 8% per annum for the second month, 12% per annum for the third month, and so on. What is the total amount of interest to be paid at the end of 6 months?

43. The difference between the CI and SI on a sum of ₹7200 for two years is ₹72. Find the rate of interest per annum.

44. Roja invested ₹6000 in a bank, which paid compound interest, interest being compounded semi-annually. She received ₹10,368 after 18 months from the bank. Find the rate of interest. (per annum)

45. A sum of ₹40 is borrowed at simple interest. It is borrowed at 2% per annum for the first month, 4% per annum for the second month, 6% per annum for the third month, and so on. What is the total interest to be paid at the end of 6 months? (in ₹)

Essay Type Questions

46. A certain sum of money at simple interest increases by 50% in 5 years. What will be the compound interest, compounded annually on ₹13,000 for 3 years at the same rate?

47. A sum of ₹100,000 amounts to ₹171,600 in 3 years under compound interest, interest being compounded annually. It is lent at 10% per annum for the first year, 20% per annum for the second year and x% per annum for the third year. Find x.

48. Two persons each lent ₹2000 at simple interest for 2 years. The difference between the simple interests received by them is ₹20. Find the difference between the rates of interest.

49. P and Q borrowed ₹600 each for a period of 3 years. P paid simple interest at 25% per annum while Q paid compound interest at 20% per annum, interest being compounded annually. Who paid more interest and by how much?

50. What is the ratio of compound interest accrued on a certain sum in two years at 20% per annum to the compound interest accrued on the same amount in three years at 10% per annum?

CONCEPT APPLICATION

Level 1

1. A sum of money at simple interest amounts to ₹800 in 2 years and to ₹1200 in 6 years. The sum is ______.

(a) ₹600 (b) ₹1000
(c) ₹400 (d) ₹500

2. A certain sum of money becomes ₹2100 in 4 years and ₹2550 in 7 years. Find the rate of simple interest per annum.

(a) 10% (b) 8%
(c) 12% (d) 15%

3. There are three amounts a, b and c, such that b is the simple interest on c and c is the simple interest on a. Which of the following must always be true?

(a) $a^2 = bc$ (b) $b^2 = ac$
(c) $c^2 = ab$ (d) $a^2 + b^2 = c^2$

4. If the compound interest on a certain sum of money for 2 years is ₹3280. What would the corresponding simple interest be, given the rate of interest is 5% per annum?

(a) ₹3150 (b) ₹3200
(c) ₹3100 (d) ₹3050

5. If the compound interest on a certain sum at 8% per annum, interest compounded annually for 2 years is ₹2496, then find the simple interest on the same amount at the same rate and for the same period.

(a) ₹2300 (b) ₹2450
(c) ₹2400 (d) ₹2375

6. A sum of ₹22000 is divided into three parts such that the corresponding interests earned after 1 year at 2%, per annum, 4 years at 2% per annum and 16 years at 1% per annum simple interest are equal. Find the least of the sums which was lent. (in ₹)

(a) 1000 (b) 2000
(c) 5000 (d) 4000

7. The compound interest on a sum of money for two years is ₹459 and the corresponding simple interest is ₹450. What is the amount under simple interest on the same amount at the same rate of interest at the end of two years?

(a) ₹6325 (b) ₹6084
(c) ₹6075 (d) ₹5524

8. A sum of ₹1750 is lent out at simple interest into two parts, the smaller part being lent at 7% per annum and the larger part at 5% per annum. If the total amount of interest in one year is ₹98, then find the part which was lent at 5% per annum.

(a) ₹525 (b) ₹975
(c) ₹1225 (d) ₹1350

9. A sum of money invested at compound interest doubles itself in six years. In how many years

will it become 64 times itself at the same rate of compound interest?

(a) 30 (b) 36
(c) 42 (d) 48

10. Two equal sums are lent at simple interest. The first sum is recovered in 3 years and the second sum in 6 years. The rate of interest per annum on the first sum is 2% more than that of the second sum. Find the total sum lent if the amount in each case is ₹560.

(a) ₹530 (b) ₹500
(c) ₹1480 (d) ₹1000

11. A sum was borrowed at simple interest at R% per annum for 2 years. If it had been borrowed at $(R + 5)$% per annum it would have become ₹200 more. Find the sum (in ₹).

(a) 2500 (b) 2000
(c) 3000 (d) 1500

12. The ratio of the interest accrued on a sum, when invested at simple interest for 2 years and the interest accrued on it, if it is invested at compound interest, interest being compounded annually for 3 years at the same rate of interest is 50 : 91. Find the rate of interest (per annum).

(a) 10% (b) 15%
(c) 25% (d) 20%

13. Ravi took a certain amount from Raju at the rate of 8% per annum at simple interest and lent half of the amount to Ramu at 8% per annum at simple interest and the remaining amount to Raghu at 10% per annum at simple interest. If at the end of 10 years, Ravi made a profit of ₹1250 in the deal, then find the amount that Ravi had taken from Raju.

(a) ₹12,500 (b) ₹13,125
(c) ₹2245.50 (d) Data insufficient

14. Two equal sums were lent at the same time at simple interest rates of 6% and 4% per annum. The first sum was recovered 2 years earlier than the second sum, and the amount in each case was ₹930. What was the sum lent?

(a) ₹820 (b) ₹780
(c) ₹690 (d) ₹750

15. A person invested three different amounts at 3%, 5% and 6% per annum at simple interest. At the end of the year, he received the same interest in each case. If the person's net investment is ₹4200, then the money invested at 5% is ______.

(a) ₹2000 (b) ₹1000
(c) ₹1500 (d) ₹1200

16. A sum of ₹3000 is lent out in two parts. The smaller part is lent at 10% per annum and the larger part is lent at 20% per annum. If the total interest in a year is ₹500, then find the sum lent at 10% per annum (in ₹).

(a) 800 (b) 1000
(c) 1200 (d) 900

17. The integral number of years in which a sum of money at 25% per annum under compound interest will become more than twice itself is at least.

(a) 3 (b) 2
(c) 4 (d) 1

18. Find the least integral number of years in which a sum at 20% per annum compound interest will be more than double.

(a) 4 (b) 5
(c) 3 (d) 6

19. A sum is split into two equal parts. One of the parts is lent at simple interest at 20% per annum for 6 years. The other part is lent at 40% per annum simple interest for 2 years. The difference in the interests is ₹72. Find the total sum (in ₹).

(a) 180 (b) 360
(c) 240 (d) 270

20. A sum is split into five equal parts. Each part is lent at annual rates of simple interests of 8%, 7%, 5%, 3% and 2%. They are lent for 1 year, 2 years, 3 years, 4 years and 5 years. On how many parts is the simple interest at least 25% of the total interest?

(a) 1 (b) 2
(c) 3 (d) 4

21. A sum of ₹2500 is split into two parts, one part being lent at simple interest and the other part being lent at compound interest, interest being compounded annually. At the end of two years, the total amount of interest earned on the sum is ₹201. Find the sum lent at simple interest, if both the parts are lent at an interest rate of 4% per annum.

(a) ₹1250 (b) ₹1625
(c) ₹1875 (d) ₹1500

PRACTICE QUESTIONS

22. A sum is split into five equal parts. They are lent at annual rates of simple interests of 1%, 2%, 3%, 10% and 5%. They are lent for 6 years, 4 years, 3 years, 1 year and 1 year, respectively. The simple interest on how many parts is at least 20% of the total interest?

(a) 1 (b) 2
(c) 3 (d) 4

23. A sum is split into two equal parts. One of the parts is lent at simple interest at 4% per annum for one year. The other part is further split into three equal parts. These are lent at compound interests, interests being compounded annually. These are lent at 10% per annum for 4 years, 20% per annum for 2 years and 40% per annum for 1 year. The difference between the total compound interest and simple interest is ₹1184.10. Find the sum. (in ₹)

(a) 3000 (b) 4000
(c) 5000 (d) 6000

24. Sreedhar borrowed ₹3500 at 6% per annum for 3 years under simple interest. But, after one year he was asked to pay compound interest for the remaining two years on the sum borrowed initially. How much additional interest Sreedhar has to pay?

(a) ₹10 (b) ₹14.50
(c) ₹12.60 (d) ₹12

25. Ajay borrowed ₹4000 at 10% per annum for 4 years under simple interest. But, after 2 years he was asked to pay compound interest for the remaining 2 years on the initially borrowed amount. How much Ajay have to pay additionally if interest was compounded annually? (in₹)

(a) 40 (b) 20
(c) 80 (d) 160

26. A sum of ₹10,500 is divided into two parts *X* and *Y*, such that the interest calculated on *X* for 4 years is equal to the interest on *Y* for 6 years. If the rates of interest on *X* and *Y* are 8% and 4%, respectively,then find the larger part between *X* and *Y*.

(a) ₹5000 (b) ₹6000
(c) ₹6500 (d) ₹7000

27. A borrowed a certain sum of money from B at the rate of 10% per annum under simple interest and lent one-fourth of the amount to C at 8% per annum under simple interest and the remaining amount to D at 15% per annum under simple interest. If at the end of 15 years, A made profit of ₹5850 in the deal, then find the sum that A had lent to D.

(a) ₹24,500 (b) ₹12,000
(c) ₹9000 (d) ₹18,600

28. A sum of ₹62,000 is divided into three parts such that the corresponding interests earned for 3 years, 5 years and 6 years are equal. If the rates of simple interest are 5% per annum, 4% per annum and 3% per annum, then what is the greatest of the sums that were lent?

(a) ₹18,000 (b) ₹22,000
(c) ₹24,000 (d) ₹26,000

29. The difference between the compound interest on a sum of ₹4000, interest being compounded annually and the simple interest on it for two years is ₹250. Find the rate of interest. (per annum)

(a) 15% (b) 10%
(c) 25% (d) 20%

30. A sum of ₹4000 is split into two parts. One part is lent at simple interest and the other at compound interest, interest being compounded annually. At the end of two years, the total amount of interest earned is ₹1720. Find the sum lent at simple interest, if each part is lent at 20% per annum (in ₹).

(a) 1200 (b) 1000
(c) 800 (d) 1500

Level 2

31. A sum of ₹1500 amounts to ₹1680 in 3 years at simple interest. If the interest rate is increased by 2%, it would amount to ______.

(a) ₹1770 (b) ₹1815
(c) ₹1590 (d) ₹1850

32. A sum of money doubles itself in 3 years. At same rate of simple interest, for a period of 9 years, how many times will the sum become?

(a) 4 (b) 6
(c) 8 (d) 9

PRACTICE QUESTIONS

33. A and B borrowed ₹600 and ₹500 respectively for a period of 3 years. A paid simple interest at the rate of 10% per annum, while B paid compound interest at the rate of 10% per annum compounded annually. Who paid more interest and by how much?

(a) A paid more interest by ₹14.50.
(b) B paid more interest by ₹14.50.
(c) A and B both paid same amount of interest.
(d) None of the above.

34. Due to a fall in the annual rate of interest from 6% to 5%, a person's yearly income reduces by ₹245.25. His capital is ______.

(a) ₹24,525 (b) ₹24,600
(c) ₹23,675 (d) ₹24,000

35. The cost of a television is ₹15625. Its value depreciates at the rate of 8% per annum. Calculate the total depreciation in its value at the end of 3 years.

(a) ₹3458 (b) ₹3748
(c) ₹3548 (d) ₹3845

36. The value of an ornament decreases every year at the rate of 5% over that of the previous year. If its value at the end of 2 years is ₹9025, then what was its original value at the beginning of these two years?

(a) ₹12,000 (b) ₹11,000
(c) ₹10,000 (d) ₹13,000

37. A certain sum of money triples itself in 6 years at compound interest. In how many years will it become 27 times at the same rate of compound interest?

(a) 27 (b) 30
(c) 24 (d) 18

38. A sum of ₹24,000 is divided into two parts P_1 and P_2, such that the simple interest calculated on P_1 for 3 years and on P_2 for 4 years are equal. If the rates of interest on P_1 and P_2 are 4% and 5% respectively, then find the smaller of the parts P_1 and P_2.

(a) ₹10,000 (b) ₹12,000
(c) ₹8000 (d) ₹9000

39. A person borrowed two equal sums for two years at the rate of 10% per annum, from two persons. He borrowed the first sum at simple interest and the second sum at compound interest, compounded annually. The difference between the amounts paid by him is ₹15. Find each equal sum.

(a) ₹1800 (b) ₹2000
(c) ₹1500 (d) ₹2500

40. A certain sum of money amounts to ₹4200 in 3 years and to ₹6000 in 6 years at simple interest. Find the rate of interest.

(a) $12\frac{1}{2}\%$ (b) 20%
(c) 25% (d) 30%

41. A sum of money becomes four times itself in 5 years at a certain rate of interest, compounded annually. In how many years will it become 16 times itself at the same rate of interest?

(a) 20 (b) 16
(c) 12 (d) 10

42. The difference between simple interest and compound interest on a sum of ₹40,000 for two years is ₹900. What is the annual rate of interest?

(a) 20% (b) 10%
(c) 12% (d) 15%

43. The difference between the compound interest (compounded annually) and simple interest, for two years on the same sum at the amount rate of interest is ₹370. Find the rate of interest if the simple interest on the amount at the same rate of interest for 1 year is ₹3700.

(a) 10% (b) 12%
(c) 16% (d) 15%

44. A certain sum amounts to ₹13,310 after 3 years and to ₹16,105.10 after 5 years under compound interest. Find the sum borrowed, if the interest is compounded annually. (in ₹)

(a) 12,000 (b) 8,000
(c) 10,000 (d) 16,000

45. A certain sum amounts to ₹77,000 in 5 years and to ₹68,200 in 3 years, under simple interest. If the rate of interest is increased by 2%, then in how many years will it double itself?

(a) 8 (b) 9
(c) 10 (d) 12

46. Find the compound interest on ₹40,000 at 12% per annum for a period of 2 years. (in ₹)

(a) 10,176 (b) 8000
(c) 9176 (d) 10,000

47. A person borrowed a certain sum at 25% per annum compound interest (compounded annually) and paid ₹10,000 at the end of 4 years. Find the sum borrowed.

(a) ₹4096 (b) ₹5000
(c) ₹5016 (d) ₹4960

48. Find the amount on the sum of ₹15,625 for 18 months under compound interest, compounded half yearly at the rate of 16% per annum.

(a) ₹19,683 (b) ₹19,625
(c) ₹20,504 (d) ₹19,625

Level 3

49. Rakesh borrowed ₹42,000 from Rajnikant at 6% per annum simple interest. He lent the same sum to Kishore at 10% per annum compound interest, compounded annually, for 2 years. Find the amount earned by Rakesh in the transaction.

(a) ₹3614 (b) ₹3550
(c) ₹3610 (d) ₹3780

50. A certain sum amounts to ₹7935 in 2 years and ₹9125.25 in 3 years, under compound interest. Find the sum borrowed if interest is compounded annually. (in₹)

(a) 6000 (b) 7500
(c) 8000 (d) 10,000

51. Ravi lent Ramu a certain sum of money at the rate of $2\frac{1}{2}\%$ per annum, interest compounded annually. After two years, Ramu paid a sum of ₹2560 to Ravi. What amount of money did Ramu borrow from Ravi?

(a) ₹2036.64 (b) ₹2236.64
(c) ₹2436.64 (d) ₹2636.64

52. In how many years will a sum of ₹25,600, interest compounded quarterly, at the rate of 25% per annum, amount to ₹28,900?

(a) 1 year (b) $\frac{1}{2}$ year
(c) 2 years (d) 4 years

53. The value of an old bike decreases every year at the rate of 4% over that of the previous year. If its value at the end of three years is ₹13824, then find its present value.

(a) ₹15,625 (b) ₹14,525
(c) ₹16,625 (d) ₹15,425

54. At compound interest of 8%, 10% and 12% for three consecutive years, the interest earned in the 3rd year is ₹891. Find the principal amount. (in ₹)

(a) 6250 (b) 6050
(c) 6200 (d) 6225

55. A certain amount of money doubles in 4 years under compound interest. In how many additional years will it become 4 times of the principal amount under the same conditions.

(a) 8 (b) 4
(c) 12 (d) 16

56. A sum of money was lent in two parts which were in the ratio of 2 : 3 for 2 years and 3 years respectively, both at the rate of 10% per annum at simple interest. If the difference between the interest earned is ₹6000, then find the total sum that was lent.

(a) ₹24,000 (b) ₹36,000
(c) ₹60,000 (d) ₹84,000

57. The total simple interest at R_s% per annum and the total compound interest at R_c% per annum for 2 years on ₹10,000 are equal. If R_s, R_c are integers, then find the minimum difference between R_s and R_c.

(a) 2 (b) 4
(c) 8 (d) 10

58. Find the amount on ₹9900 at 20% per annum for 2 years at compound interest (compounded annually).

(a) ₹12,946 (b) ₹13,548
(c) ₹14,256 (d) ₹15,678

59. Find the amount when ₹9999 is lent at simple interest for 3 years and 4 months at 10% per annum. (in ₹)

(a) 13,332 (b) 12,332
(c) 12,232 (d) 13,333

60. The compound interest on a certain sum for 2 years is ₹882, whereas the simple interest on it is ₹840. Find the rate of interest.

(a) 10% (b) 12%
(c) 8% (d) 15%

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. ₹200
2. True
3. False
4. ₹500
5. 12
6. ₹6000
7. 10 years
8. 100% per annum
9. $x\left(1+\frac{5}{100p}\right)^{pn}$
10. ₹56,350
11. 16 years
12. ₹5625
13. ₹23,958, ₹5958
14. $16\frac{2}{3}$% per annum
15. ₹512.40
16. ₹13,000
17. r = 10% per annum
    Principal = ₹18,000
18. r = 20%
    Principal = ₹3000
19. ₹5000
20. 10% per annum
21. ₹726
22. ₹10,000
23. 21% per annum
24. True
25. ₹11,500
26. ₹82,000
27. ₹7200
28. ₹10,000
29. ₹82.50
30. ₹5,368

Shot Answer Type Questions

31. 231,525
32. ₹1024
33. ₹40,000
34. 100%
35. 0.5%
36. 18.8%
37. 10%
38. 21250
39. 3
40. ₹304.50
41. $12\frac{1}{2}$%
42. ₹10.50
43. 10%
44. 40%
45. 1.40

Essay Type Questions

46. ₹4303
47. ₹30
48. 0.5%
49. P paid an additional amount of interest of ₹13.20 than Q
50. $\frac{440}{331}$

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (d)
11. (b)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)
21. (c)	**22.** (c)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (c)	**28.** (c)	**29.** (c)	**30.** (b)

Level 2

31. (a)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (d)	**39.** (c)	**40.** (c)
41. (d)	**42.** (d)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (a)		

Level 3

49. (d)	**50.** (a)	**51.** (c)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (c)
59. (a)	**60.** (a)								

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (i) Interest for 4 years
= ₹(1200 − 800) = ₹400.
(ii) Interest for 2 years = ₹200.
(iii) Sum = 800 − Interest for 2 years.

2. (i) Interest for 3 years = ₹2550 – ₹2100 = 450.
(ii) Interest for 4 years = ₹600.
(iii) Sum = 2100 − Interest for 4 years.

3. (i) Let rate of interest be r% and time period be t years.
(ii) $b = \frac{c \times r \times t}{100}$ and $c = \frac{a \times r \times t}{100}$.
(iii) Using the above, find the relation among a, b and c.

4. (i) Find the principal from the given data.
(ii) Use $\text{CI} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$,
i.e., $3280 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$.
(iii) Find P from the above information.
(iv) Now, use $\text{SI} = \frac{P \times R \times T}{100}$.

5. (i) Find the principal from the given data and proceed.
(ii) Let the sum be ₹P.
(iii) $\text{CP} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$, find P by substituting R, n and CI values.
(iv) Now, find SI on P at same rate for same period.

6. (i) Find the interests in each case and equate them.
(ii) Let the sums lent be ₹x, ₹y and ₹22,000 − $(x + y)$.
(iii) $\frac{x \times 2}{100} = \frac{y \times 2 \times 4}{100}$
$= \frac{[22{,}000 - (x+y)] \times 1 \times 6}{100}$.
(iv) Solve the above equation and find the least of the sums.

7. (i) Find the difference between the two interests and proceed.
(ii) Use $\text{CI} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$ and $\text{SI} = \frac{PRT}{100}$.
(iii) $459 = P\left[\left(1+\frac{R}{100}\right)^2 - 1\right]$ and $450 = \frac{P \times R \times 2}{100}$, solve these two equations and find R, then find P.

8. (i) Let, ₹P be lent at 7% and ₹(1750 − P) be lent at 5%.
(ii) Take $\frac{P \times 7 \times 1}{100} + \frac{(1750 - P) \times 5 \times 1}{100} = 98$.
(iii) Use the above information to find P and (1750 − P).

9. (i) ₹P becomes ₹2P in 6 years.
(ii) $2P = P\left(1+\frac{R}{100}\right)^6$, find $\left(1+\frac{R}{100}\right)$ value.
(iii) Now, $64P = P\left(1+\frac{R}{100}\right)^n$ and substitute $\left(1+\frac{R}{100}\right)$ value to find n.

10. (i) Let the rate of interest on the second sum be R%.
(ii) Let the sum be ₹P in each case and rate of interest on second sum be r% per annum.
(iii) Take $P\left[1+\frac{(r+2)3}{100}\right] = P\left[1+\frac{r \times 6}{100}\right] = 560$.
Find P and then 2P.

11. (i) Find the difference between the simple interests and proceed.
(ii) Let the sum be ₹P.
(iii) Take $\frac{P \times (R+5) \times 2}{100} - \frac{P \times R \times 2}{100} = 200$, solve for P.

12. (i) Find the ratio of SI for 2 years and CI for 3 years and equate it to the given ratio.
(ii) $\left(\frac{P \times r \times 2}{100}\right) \div P\left[\left(1+\frac{r}{100}\right)^3 - 1\right] = 50:91$.
(iii) Solve the above and find r.

HINTS AND EXPLANATION

13. (i) Let the money lent to Ramu and Raghu be ₹$\frac{P}{2}$ each.

(ii) $\frac{P\times 10\times 10}{2\times 100}+\frac{P\times 8\times 10}{2\times 100}-\left(\frac{P\times 8\times 10}{100}\right)=1250.$

14. (i) Let the time period for the sum lent at 6% be T years.

(ii) Let the sum be ₹P in each case and second sum recovered in x years.

(iii) $P\left[1+\frac{6(x-2)}{100}\right]=P\left[1+\frac{4(x)}{100}\right]=930$. Find x.

(iv) Using the above information, find P.

15. (i) Take the sum invested at 3% rate of interest to be x and that at 5% rate of interest to be y and that at 6% be ₹4200 – $(x + y)$.

(ii) $\frac{x\times 3}{100}=\frac{y\times 5}{100}=\frac{[4200-(x+y)]\times 6}{100}$.

(iii) Solve the above and find y.

16. (i) Let the two parts be ₹x and ₹$(300-x)$, respectively.

(ii) In $\frac{x\times 10}{100}+\frac{(300-x)20}{100}=500$, find x.

17. (i) Let the sum be ₹P.

(ii) $P\left(1+\frac{20}{100}\right)^n>2P$, solve the inequality for n.

18. (i) Let the sum be ₹P.

(ii) $P\left(1+\frac{20}{100}\right)^n>2P$, solve the inequality for n.

19. (i) Find the interests in each case and then equate their difference to the given value.

(ii) Let the total sum be ₹$2x$.

(iii) $\frac{x\times 20\times 6}{100}-\frac{x\times 40\times 2}{100}=72.$

(iv) Solve for x and find $2x$.

20. (i) Let the total sum be ₹$5x$.

(ii) 25% of $5x$ = ₹$\frac{5x}{4}$.

(iii) Find the interest on each sum of ₹x using given data, and check which is greater than $\frac{5x}{4}$.

21. (i) Let the sum lent at CI be x and the sum lent at SI be $(2500 - x)$.

(ii) $\frac{(2500-x)4\times 2}{100}+x\left[\left(1+\frac{4}{100}\right)^2-1\right]=201.$

(iii) Solve for x.

22. (i) Let the total sum be ₹$5x$.

(ii) Find the interest on each part.

(iii) Find the sum of the interest of 5 parts. Check which is greater than 20% of the sum of interests.

23. (i) Let the total sum be ₹$2x$.

(ii) $SI=\frac{x\times 4x}{100}$.

(iii) Total CI $=\frac{x}{3}\left[\left(1+\frac{10}{100}\right)^4-1\right]+\frac{x}{3}\left[\left(1+\frac{20}{100}\right)^2-1\right]+\frac{x}{3}\left[\left(1+\frac{40}{100}\right)^1-1\right]$.

(iv) Use, CI – SI = 1184.10.

24. (i) Find the SI for 1 year and CI for the last two years.

(ii) Find SI and CI as per the given data.

(iii) Find the difference between CI and SI.

25. (i) Find SI on ₹4000 at 10% per annum for 4 years.

(ii) Find sum of SI on ₹4000 at 10% per annum for 2 years and CI on 4000 at 10% per annum for 2 years.

(iii) Find the difference of the above two.

27. (i) Let A borrowed ₹x from B. Therefore, the money lent to C is ₹$\frac{x}{4}$ and the money lent to D is ₹$\frac{3x}{4}$.

(ii) $\left[\frac{x}{4}\times\frac{8\times 15}{100}+\frac{3x\times 15\times 15}{4\times 100}\right]-\frac{x\times 10\times 15}{100}=5850.$

(iii) Solve the above equation and find x, and then $\frac{3x}{4}$.

28. (i) Let the sums lent be ₹x, ₹y and ₹22,000 – $(x + y)$.

(ii) $\frac{x\times 3\times 5}{100}=\frac{y\times 5\times 4}{100}=\frac{[22{,}000-(x+y)]6\times 3}{100}$.

(iii) Solve the above equation and find the least of the sums.

29. (i) Let the rate of interest be r%.

(ii) $4000\left[\left(1+\frac{r}{100}\right)^2-1\right]-\frac{4000\times r\times 2}{100}=250.$

(iii) Solve the above to find r.

30. (i) Let ₹x be lent at simple interest and ₹$(4000-x)$ at compound interest.

(ii) $\frac{x\times 20\times 2}{100}+(4000-x)\left[\left(1+\frac{20}{100}\right)^2-1\right]=1720,$

solve for x.

Level 2

31. Find the interest for 1 year and the rate of interest.

32. Assume that ₹P becomes ₹$2P$ in 3 years and proceed.

33. Find the interests in each case and the difference.

34. Apply the formula for finding the simple interest.

35. Use depreciation concept or let $A=P\left(1-\frac{r}{100}\right)^n$, when n = 3.

46. P = ₹40,000, R = 12%, n = 2.

$$\text{CI}=P\left[\left(1+\frac{R}{100}\right)^n-1\right]$$
$$=P\left[\left(1+\frac{12}{100}\right)^2-1\right]$$
$$=40,000\left[\left(\frac{28}{25}\right)^2-1\right]$$
$$=40,000\left[\frac{28^2-25^2}{(25)^2}\right]$$
$$=40,000\,\frac{(28+25)(28-25)}{25\times 25}$$
$$=64\times 53\times 3=₹10,176.$$

47. Let the sum borrowed be ₹P.

R = 25% per annum

n = 4

A = 10,000

$$A=P\left(1+\frac{R}{100}\right)^n$$
$$10,000=P\left(1+\frac{25}{100}\right)^4$$
$$\Rightarrow 10,000=P\left(1+\frac{1}{4}\right)^4$$
$$\Rightarrow 10,000=P\left(\frac{5}{4}\right)^4$$
$$\Rightarrow P=\frac{4\times 4\times 4\times 4}{5\times 5\times 5\times 5}\times 10,000$$
$$\Rightarrow P=₹4096.$$

48. $A=P\left(1+\frac{R}{100}\right)^n$

$$R=\frac{16}{2}=8\%$$
$$\Rightarrow A=15,625\left[1+\frac{8}{100}\right]^3$$
$$\Rightarrow A=15,625\left(1+\frac{2}{25}\right)^3$$
$$\Rightarrow A=15,625\left(\frac{27}{25}\right)^3$$
$$\Rightarrow A=15,625\left(\frac{27\times 27\times 27}{25\times 25\times 25}\right)$$
$$A=₹19,683.$$

Level 3

51. $A=P\left(1+\frac{R}{100}\right)^N$

A = ₹2560, R = $2\frac{1}{2}$%, n = 2 years, find P.

52. Use formula for amount and then use laws of indices.

$28,900=25,600\left(1+\frac{25}{400}\right)^n$, n is the number of quarters.

HINTS AND EXPLANATION

53. Let P be the original value.

$$P\left(1-\frac{4}{100}\right)^3 = 13,824$$

$$\Rightarrow P\left(1-\frac{1}{25}\right)^3 = 13,824$$

$$\Rightarrow P\left(\frac{25-1}{25}\right) = 13,824$$

$$\Rightarrow P\left(\frac{24\times24\times24}{25\times25\times25}\right) = 13,824$$

$$\Rightarrow P = 25\times25\times25 = ₹15,625.$$

54. 1st year's interest = 8%

Principal = P

$$I_1 = \frac{P\times1\times8}{100} = \frac{2P}{25}.$$

2nd year:

$$R = 10\%,\ \text{Principal} = P+\frac{2P}{25} = \frac{27P}{25}$$

$$I_2 = \frac{\frac{27P}{25}\times10}{100}$$

$$I_2 = \frac{27P}{250}.$$

3rd year:

R = 12%

$$\text{Principal} = \frac{27P}{25}+\frac{27P}{250} = \frac{297P}{250}$$

$$I_3 = \frac{\frac{297P}{250}\times12}{100}$$

$$= \frac{297P\times3}{6250}$$

$$I_3 = \frac{891P}{6250}.$$

Given that interest earned in the 3rd year = ₹891.

$$\therefore I_3 = \frac{891P}{6250} = 891$$

$$\Rightarrow P = ₹6250.$$

55. Time period = 4 years

Principal = P

Amount = $2P$

$$2P = P\left(1+\frac{R}{100}\right)^4$$

$$2 = \left(1+\frac{R}{100}\right)^4$$

$$\Rightarrow 2^{1/4} = \left(1+\frac{R}{100}\right)$$

Let the amount becomes $4P$ in 'n' years.

$$4P = P\left(1+\frac{R}{100}\right)^n$$

$$4P = \left(2^{\frac{1}{4}}\right)^n$$

$$4 = 2^{n/4}$$

$$2^2 = 2^{n/4}$$

$$\Rightarrow \frac{n}{4} = 2$$

$n = 8$ years.

56. Let the two parts be $2P$ and $3P$

$$\frac{(3P)\times3\times10}{100} - \frac{(2P)\times2\times10}{100} = 6000$$

$$\frac{90P-40P}{100} = 6000$$

$$\frac{50P}{100} = 6000 \Rightarrow 5P = ₹60,000.$$

57. $$\frac{PTR}{100} = P\left[\left(1+\frac{R}{100}\right)^2-1\right]\frac{10,000\times2\times R_s}{100}$$

$$= 10,000\left[\left(1+\frac{R_c}{100}\right)^2-1\right]$$

$$200R_s = 10,000\left[\frac{R_c^2}{100^2}+\frac{2R_c}{100}\right]$$

$$200R_s = R_c^2 + 200R_c$$

$$200R_s - 200R_c = R_c^2$$

$$200(R_s - R_c) = R_c^2$$

$$R_s - R_c = \frac{R_c^2}{200}$$

$$R_s = \frac{R_c^2}{200} + R_c.$$

Since R_s and R_c are integers and R_c^2 must be the smallest integer, it should be a multiple of 200 and a perfect square.

$\therefore R_c^2 = 400$

$\Rightarrow R_c = 20\%$

$$R_s = \frac{400}{200} + 20$$

$$R_s = 22\%$$

$\therefore$ Minimum difference between R_s and R_c is 2.

58. Amount after two years $= P\left(1+\frac{R}{100}\right)^2$

$$= 9900\left(1+\frac{20}{100}\right)^2$$

$$= 9900\left(\frac{36}{25}\right) = 396 \times 36 = 14,256.$$

59. $P = ₹9999$

$R = 10\%$

$T = 3$ years and 4 months

$= 3$ years $+ \frac{4}{12}$ years

$= 3\frac{1}{3}$ years $= \frac{10}{3}$ years.

$$A = P + \frac{PTR}{100}$$

$$A = 9999 + \frac{9999 \times \frac{10}{3} \times 10}{100}$$

$$= 9999 + 3333 = 13,332.$$

60. $SI_1 = SI_2 = \frac{840}{2} = 420.$

Difference between compound interest and simple interest for 2 years is the interest on the first year's simple interest.

$882 - 840 = R\%(420)$

$$42 = \frac{R}{100}(420)$$

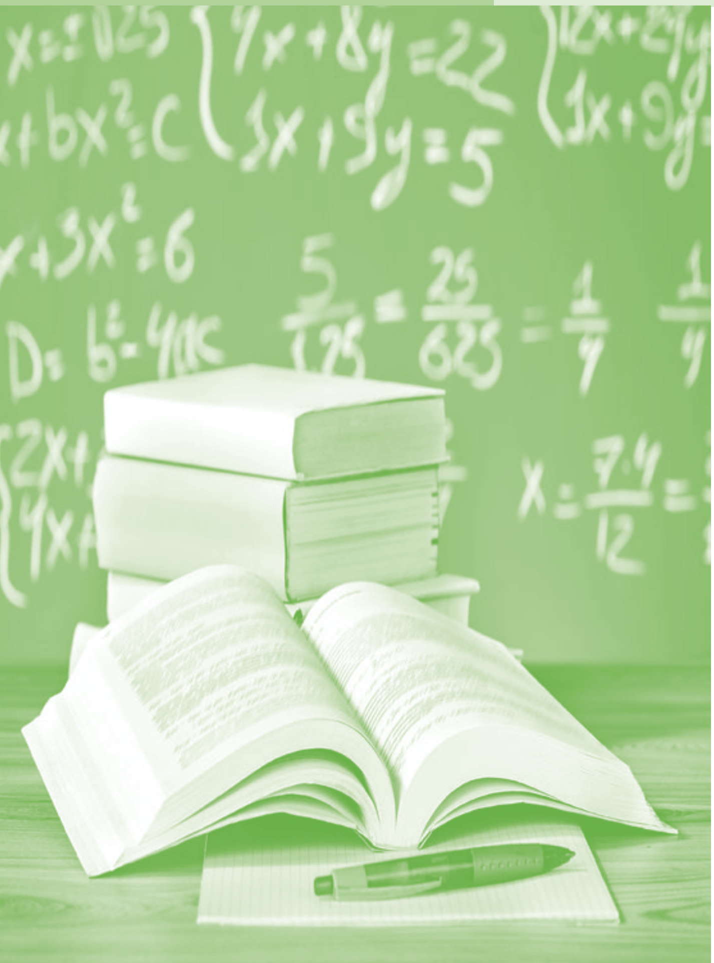
$\Rightarrow R = 10\%.$

HINTS AND EXPLANATION

Chapter

20

Ratio, Proportion and Variation



REMEMBER

Before beginning this chapter, you should be able to:

- Calculate the ratio of quantities given
- Understand the concepts of proportion and variation
- Apply unitary method in solving problems

KEY IDEAS

After completing this chapter, you should be able to:

- Learn about ratios and terms related
- Know the properties and types of ratios
- Study about proportion and variations
- Know properties and types of a proportion

RATIO

The value obtained when two similar quantities are compared by dividing one quantity with the other is called ratio. The ratio of two quantities is that value which gives us how many times one quantity is of the other. Only quantities of the same kind, i.e., the quantities with the same units can be compared.

Notes

1. The ratio of a and b is written as $a : b$ and is measured by the fraction $\frac{a}{b}$.

   Example: The ratio of 5 and 8 is written as 5 : 8 and measured by the fraction $\frac{5}{8}$.

2. If two quantities are in the ratio $a : b$, then the first and the second quantities will be $\frac{a}{a+b}$ times and $\frac{b}{a+b}$ times the sum of the two quantities respectively.

   Example: If $x : y = 3 : 5$, then $x = \frac{3}{8}(x + y)$ and $y = \frac{5}{8}(x + y)$.

3. The ratio of two quantities can be found, only when both the quantities are of the same kind.

   Example: Ratio between 1 meter and 5 seconds cannot be found, as the quantities given are not of the same kind.

4. A ratio is an abstract quantity and a ratio does not have any units.

   Example: The ratio of 30 seconds and one minute is 30 seconds : 60 seconds or 1 : 2.

Terms of a Ratio

For a given ratio $a : b$, we say that a is the **first term** or **antecedent** and b is the **second term** or **consequent**.

In the ratio 3 : 4, 3 is the antecedent while 4 is the consequent.

Properties of a Ratio

The value of a ratio remains the same, if both the terms of the ratio are multiplied or divided by the same non-zero quantity. If a, b and m are non-zero real numbers.

1. $\frac{a}{b} = \frac{ma}{mb} \Rightarrow a : b = am : bm$

2. $\frac{a}{b} = \frac{\frac{a}{m}}{\frac{b}{m}} \Rightarrow a : b = \frac{a}{m} : \frac{b}{m}$

Simplest Form of a Ratio

The ratio of two or more quantities is said to be in the simplest form, if the highest common factor (HCF) of the quantities is 1. If the HCF of the quantities is not 1, then each quantity of the ratio is divided by the HCF to convert the ratio into its simplest form.

For Example, suppose there are three numbers 6, 9 and 12. The HCF of the numbers is 3. Dividing each of 6, 9 and 12 by 3, the results obtained are 2, 3 and 4. The ratio of 6, 9 and 12, in the simplest form is 2 : 3 : 4.

EXAMPLE 20.1

Express 81 : 93 in its simplest form.

SOLUTION

The HCF of 81 and 93 is 3.

We divide each term by 3.

Then, $81:93 = \frac{81}{3}:\frac{93}{3} = 27:31$

∴ The ratio 81 : 93 in its simplest form is 27 : 31.

Comparison of Ratios

Two ratios $a : b$ and $c : d$ can be compared in the following way: If $\frac{a}{b} > \frac{c}{d}$ then $a : b > c : d$.

EXAMPLE 20.2

Compare the ratios 4 : 5 and 18 : 25.

SOLUTION

$4:5 = \frac{4}{5} = 0.8$

$18:25 = \frac{18}{25} = 0.72$

As, 0.8 > 0.72,

∴ 4 : 5 > 18 : 25.

Alternate method:

Ratios can also be compared by reducing them to equivalent fractions of a common denominator.

$4:5 = \frac{4}{5}$ and $18:25 = \frac{18}{25}$

$\frac{4}{5} = \frac{4\times 5}{5\times 5} = \frac{20}{25}$

As, $\frac{20}{25} > \frac{18}{25}; \frac{4}{5} > \frac{18}{25}$

∴ 4 : 5 > 18 : 25.

Types of Ratios

1. A ratio $a : b$, where $a > b$, is called a ratio of **greater inequality**. For a ratio of greater inequality, if a positive quantity is added to both terms, then the obtained ratio is lesser than $a : b$. In other words if a positive quantity x is added to the two terms in the ratio $a : b$ (where $a > b$), then $a + x : b + x < a : b$.

   Example: The ratio 4 : 3 is a ratio of greater inequality, because the antecedent (4) is greater than the consequent (3). It can be seen that for the ratio $4:3, \frac{4+1}{3+1} < \frac{4}{3}$.

2. A ratio $a : b$, where $a < b$, is called a ratio of **lesser inequality**. For a ratio of lesser inequality, if a positive quantity is added to both terms, the obtained ratio is greater than $a : b$. In other words, if a positive quantity x is added to the two terms of the ratio $a : b$ (where $a < b$), then $(a + x) : (b + x) > a : b$.

   Example: The ratio 3 : 4 is a ratio of lesser inequality, because the antecedent 3 is less than the consequent (4). It can be seen that for the ratio $3:4, \dfrac{3+1}{4+1} > \dfrac{3}{4}$.

3. A ratio $a : b$, where $a = b$, is called a ratio of equality. If a positive quantity is added to both the terms, the value of the ratio remains unchanged. In other words, if a positive quantity x is added to two terms in the ratio $a : b$, then $a + x : b + x = a : b$.
4. The duplicate ratio of $a : b$ is $a^2 : b^2$.
5. The sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$.
6. The triplicate ratio of $a : b$ is $a^3 : b^3$.
7. The sub-triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$.
8. The inverse ratio of $a : b$ is $b : a$.
9. The compound ratio of two ratios $a : b$ and $c : d$ is $ac : bd$.

Note If three quantities a, b and c are in the ratio 1 : 2 : 3, we write $a : b : c = 1 : 2 : 3$. It follows that $a : b = 1 : 2$, $a : c = 1 : 3$ and $b : c = 2 : 3$. The ratio of two quantities can be found similarly when there are more than 3 quantities given and the ratio of all quantities is known.

EXAMPLE 20.3

Divide ₹560 in the ratio 3 : 4.

SOLUTION

The sum of the terms of the ratio is 3 + 4 = 7.

$$\therefore \text{ The first part } = ₹\left(560 \times \frac{3}{7}\right) = ₹240$$

$$\text{and, the second part } = ₹\left(560 \times \frac{4}{7}\right) = ₹320.$$

EXAMPLE 20.4

Divide ₹780 among three friends P, Q and R in the ratio $\dfrac{1}{4} : \dfrac{1}{3} : \dfrac{1}{2}$.

SOLUTION

Given ratio is $\dfrac{1}{4} : \dfrac{1}{3} : \dfrac{1}{2}$.

The LCM of the denominators is 12

$$\Rightarrow \frac{1}{4} : \frac{1}{3} : \frac{1}{2} = \frac{1}{4} \times 12 : \frac{1}{3} \times 12 : \frac{1}{2} \times 12 = 3 : 4 : 6$$

The sum of the terms of the ratio = (3 + 4 + 6) = 13.

$$\text{P's share } = ₹\left(780 \times \frac{3}{13}\right) = ₹180.$$

$$\therefore \text{Q's share } = ₹\left(780 \times \frac{4}{13}\right) = ₹240$$

$$\text{and, R's share } = ₹\left(780 \times \frac{6}{13}\right) = ₹360.$$

EXAMPLE 20.5

If $x : y = 3 : 4$ and $y : z = 6 : 11$, then find $x : y : z$.

SOLUTION

Given $x : y = 3 : 4$ and $y : z = 6 : 11$.

In both the ratios y is common. Try to make the value of y equal in both the ratios.

Multiply each term of the ratio 6 : 11 by $\frac{2}{3}$ to make its first term (y) equal to 4.

$$\Rightarrow y : z = 6 \times \frac{2}{3} : 11 \times \frac{2}{3} = 4 : \frac{22}{3}$$

$$\therefore x : y : z = 3 : 4 : \frac{22}{3} = 9 : 12 : 22$$

EXAMPLE 20.6

If $B = 3A$ and $A = 4C$, then find $B : C$.

SOLUTION

$B = 3A = 3(4C) = 12C$

$\Rightarrow B = 12C$

$\Rightarrow \frac{B}{C} = \frac{12}{1}$

$\therefore B : C = 12 : 1.$

EXAMPLE 20.7

In a bag there are coins in the denominations ₹1, ₹2 and ₹5 in the ratio 3 : 5 : 7, respectively. If the total value of the coins in the bag is ₹144, find the number of coins in each denomination and also the total value of ₹2 coins.

SOLUTION

Let the number of coins of denominations of ₹1, ₹2 and ₹5 be $3x$, $5x$ and $7x$, respectively. The total value of the coins in the bag = ₹$[3x + 5x(2) + 7x(5)]$ = ₹$(3x + 10x + 35x)$ = ₹$48x$.

Given, the total amount in the bag = ₹144

$$\Rightarrow 48x = 144$$
$$\Rightarrow x = 3.$$

The number of ₹1 coins in the bag = $3x = 9$.

$\therefore$ The number of ₹2 coins in the bag = $5x = 5(3) = 15$

The number of ₹5 coins in the bag = $7x = 21$.

$\therefore$ The total value of ₹2 coins in the bag = ₹15 × 2 = ₹30.

EXAMPLE 20.8

The ratio of marks obtained by Amal, Bimal and Komal in an examination is 12 : 8 : 15. Find the marks obtained by Bimal and Komal, if Amal scored 15 marks less than that of Komal.

SOLUTION

Let the marks scored by Amal, Bimal and Komal be $12x$, $8x$ and $15x$, respectively.

Given that Amal scored 15 marks less than that of Komal.

$$\Rightarrow 12x = 15x - 15$$
$$\Rightarrow 3x = 15$$
$$\Rightarrow x = 5.$$

$\therefore$ Marks scored by Bimal = $8x = 8(5) = 40$.

Marks scored by Komal = $15x = 15(5) = 75$.

EXAMPLE 20.9

If $\dfrac{x+y}{p^3-q^3} = \dfrac{y+z}{q^3-r^3} = \dfrac{z+x}{r^3-p^3}$, then prove that $x + y + z = 0$.

SOLUTION

Let $\dfrac{x+y}{p^3-q^3} = \dfrac{y+z}{q^3-r^3} = \dfrac{z+x}{r^3-p^3} = k$

$\Rightarrow x + y = k(p^3 - q^3)$; $y + z = k(q^3 - r^3)$; $z + x = k(r^3 - p^3)$

$\Rightarrow x + y + y + z + z + x = k(p^3 - q^3 + q^3 - r^3 + r^3 - p^3)$

$\Rightarrow 2(x + y + z) = k(0)$

$\therefore x + y + z = 0$.

EXAMPLE 20.10

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$; then prove that each of these ratio's is equal to $\left(\dfrac{4a^2+3c^2-7e^2}{4b^2+3d^2-7f^2}\right)^{1/2}$.

SOLUTION

Let $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k$.

$\Rightarrow a = kb$; $c = kd$; $e = kf$.

Now, $\left(\dfrac{4a^2+3c^2-7e^2}{(4b^2+3d^2-7f^2)}\right)^{1/2} = \left(\dfrac{(4(kb)^2+3(kd)^2-7(kf)^2)}{4b^2+3d^2-7f^2}\right)^{1/2}$

$= \left[\dfrac{k^2(4b^2+3d^2-7f^2)}{(4b^2+3d^2-7f^2)}\right]^{1/2} = k.$

Hence, proved.

PROPORTION

The equality of two ratios is called proportion.

Note If $a : b = c : d$, then a, b, c and d are said to be in proportion and the same can be represented as $a : b :: c : d$. It is read as 'a is to b is as c is to d.'

Properties of Proportion

1. a, b, c and d are respectively known as the first, second, third and the fourth proportional.
2. The first and the fourth terms are called extremes while the second and the third terms are called means.
3. Product of extremes = Product of means
4. The fourth term can also be referred to as the fourth proportional of a, b and c.
5. If $a : b = c : d$, i.e., $\frac{a}{b} = \frac{c}{d}$, then the given proportion can be written as $b : a :: d : c$, i.e., $\frac{b}{a} = \frac{d}{c}$, by taking the reciprocals of terms on both sides. This relationship is known as *invertendo*.
6. If $a : b :: c : d$, then multiplying both sides of the proportion by $\frac{b}{c}$, we get $a : c = b : d$. This relationship is known as *alternendo*.
7. Adding 1 to both sides of the proportion $a : b :: c : d$, we get $\frac{a}{b} + 1 = \frac{c}{d} + 1$

   $$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d} \tag{1}$$

   That is, $(a + b) : b = (c + d) : d$

   This relationship is known as *componendo*.
8. Subtracting 1 from both sides of the proportion $a : b :: c : d$, we get

   $$\frac{a}{b} - 1 = \frac{c}{d} - 1 \Rightarrow \frac{a-b}{b} = \frac{c-d}{d} \tag{2}$$

   $$(a - b) : b = (c - d) : d$$

   This relationship is known as *dividendo*
9. Dividing the Eq. (1) by Eq. (2), we get,

   $$\frac{\frac{a+b}{b}}{\frac{a-b}{b}} = \frac{\frac{c+d}{d}}{\frac{c-d}{d}}$$

   $$\Rightarrow (a + b) : (a - b) = (c + d) : (c - d)$$

   This relationship is known as *componendo and dividendo*.
10. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ and l, m and n are any three non-zero numbers, then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{la + mc + ne}{lb + md + nf}$.

Continued Proportion

Three quantities a, b and c are said to be in continued proportion if $a : b :: b : c$.
If $a : b :: b : c$, then c is called the third proportional of a and b.

Mean Proportional of *a* and *c*

If $a : b :: b : c$, then b is called the mean proportional of a and c.

We have already learnt that, Product of means = Product of extremes.

$$\Rightarrow b \times b = a \times c$$
$$\Rightarrow b^2 = ac$$
$$\Rightarrow b = \pm\sqrt{ac}$$

$\therefore$ The mean proportional of a and c is $\sqrt{ac}$.

TYPES OF VARIATION

Direct Variation

If two quantities are related to each other such that an increase (or decrease) in the first quantity results in a corresponding proportionate increase (or decrease) in the second quantity, then the two quantities are said to vary directly with each other.

Example: At constant speed, distance covered varies directly as time.

This is expressed as, distance travelled $\propto$ time.

Indirect Variation

If two quantities are related to each other such that an increase (or decrease) in the first quantity results in a corresponding proportionate decrease (or increase) in the second quantity, then the two quantities are said to vary inversely with each other.

Example: Number of men working together to complete a job is inversely proportional to the time taken by them to finish the job. When the number of men increases, the time taken to finish the same job decreases.

$\therefore$ The number of men (n) working together to complete a job is inversely proportional to the time taken (t) by them to finish it.

This is expressed as, $n \propto \dfrac{1}{t}$.

Joint Variation

When a change in a quantity depends on the changes in two or more quantities, it is said to vary jointly with those quantities.

When a quantity A varies directly as B, when C is constant and varies directly as C, when B is constant, then A varies directly as the product of B and C.

It is represented as $A \propto B$ (C is constant) and $A \propto C$ (B is constant)

$\Rightarrow A \propto BC$ or $A = k \cdot BC$, where k is a constant.

When a quantity A varies directly as B, when C is constant and varies inversely as C, when B is constant, then A varies as $\dfrac{B}{C}$.

It is represented as $A \propto B$ (C is constant) and $A \propto \dfrac{1}{C}$ (B is constant).

$\Rightarrow A \propto \dfrac{1}{BC}$ or $A = k \cdot \dfrac{1}{BC}$, where k is a constant.

When a quantity A varies inversely as B, when C is constant and varies inversely as C, when B is constant, then A varies inversely as BC.

It is represented as $A \propto \dfrac{1}{B}$ (C is constant) and $A \propto \dfrac{1}{C}$ (B is constant).

$\Rightarrow A \propto \dfrac{1}{BC}$ or $A = k \cdot \dfrac{1}{BC}$, where k is a constant.

EXAMPLE 20.11

If $a : b = 4 : 5$, find $\frac{5a - b}{10a + 3b}$.

SOLUTION

Given, $\frac{a}{b} = \frac{4}{5}$

$$\Rightarrow a = \frac{4b}{5}$$

$$\frac{5a - b}{10a + 3b} = \frac{5\left(\frac{4b}{5}\right) - 5}{10\left(\frac{4b}{5}\right) + 3b} = \frac{3b}{11b} = \frac{3}{11}.$$

EXAMPLE 20.12

Verify whether 6, 7, 12 and 14 are in proportion or not.

SOLUTION

Ratio between 6 and 7 = 6 : 7.

Ratio between 12 and $14 = 12 : 14 = \frac{12}{14} = \frac{6}{7} = 6 : 7$.

$$\Rightarrow 6 : 7 = 12 : 14 \text{ or } 6 : 7 :: 12 : 14.$$

Hence, the numbers 6, 7, 12 and 14 are in proportion.

EXAMPLE 20.13

Find the fourth proportional to the numbers 8, 10 and 12.

SOLUTION

Let x be the fourth proportional to 8, 10 and 12.

$\Rightarrow 8 : 10 :: 12 : x$.

Product of means = Product of extremes

$$10 \times 12 = 8 \times x$$

$$x = \frac{10 \times 12}{8} = 15.$$

$\therefore$ The fourth proportional to 8, 10 and 12 is 15.

EXAMPLE 20.14

Find the mean proportional between 13 and 52.

SOLUTION

Mean proportional between 13 and $52 = \sqrt{13 \times 52} = \sqrt{676} = 26$.

EXAMPLE 20.15

If 4 taps can fill a tank in 10 hours, then in how many hours can 6 taps fill the same tank?

SOLUTION

Assume that 6 taps can do the same work in x hours. As more taps require less hours to do the same work, number of taps vary inversely as the number of hours

$\therefore$ (Inverse ratio of number of taps) :: (Ratio of number of hours)

$$\Rightarrow 6 : 4 :: 10 : x$$

$$\Rightarrow \frac{6}{4} = \frac{10}{x}$$

$$\Rightarrow x = \frac{40}{6} = 6\frac{2}{3} \text{ hours.}$$

$\therefore$ Six taps can fill the tank in $6\frac{2}{3}$ hours.

EXAMPLE 20.16

In a family, the consumption of power is 120 units for 18 days. Find how many units of power is consumed in 30 days.

SOLUTION

Let the consumption of power in 30 days be x units

The more the period of time, the more is the consumption of power. Hence the consumption of power varies directly as the number of days.

$\therefore$ (Ratio of number of days) :: (Ratio of consumption of power)

$$\Rightarrow 18 : 30 :: 120 : x$$

Product of means = Product of extremes

$$\Rightarrow 30 \times 120 = 18 \times x \Rightarrow x = \frac{30 \times 120}{18} = 200 \text{ units.}$$

$\therefore$ The consumption of power for 30 days is 200 units.

EXAMPLE 20.17

If $(7x + 4y) : (7x - 4y) :: (7p + 4q) : (7p - 4q)$, then show that x, y, p and q are in proportion.

SOLUTION

Given, $\dfrac{7x+4y}{7x-4y} = \dfrac{7p+4q}{7p-4q}$

Using componendo-dividendo rule,

$$\frac{(7x+4y)+(7x-4y)}{(7x+4y)-(7x-4y)} = \frac{(7p+4q)+(7p-4q)}{(7p+4q)-(7p-4q)} \Rightarrow \frac{14x}{8y} = \frac{14p}{8q} \Rightarrow \frac{x}{y} = \frac{p}{q}$$

$\therefore$ x, y, p and q are in proportion.

EXAMPLE 20.18

If $\frac{3k+4l+6m+7n}{3k+4l-6m-7n} = \frac{3k-4l+6m-7n}{3k-4l-6m+7n}$, then show that k, $2m$, $4l$ and $7n$ are in proportion.

SOLUTION

Given, $\frac{3k+4l+6m+7n}{3k+4l-6m-7n} = \frac{3k-4l+6m-7n}{3k-4l-6m+7n}$

$$\Rightarrow \frac{(3k+4l)+(6m+7n)}{(3k+4l)-(6m+7n)} = \frac{(3k-4l)+(6m-7n)}{(3k-4l)-(6m-7n)}$$

$$\Rightarrow \frac{3k+4l}{6m+7n} = \frac{3k-4l}{6m-7n} \Rightarrow \frac{3k+4l}{3k-4l} = \frac{6m+7n}{6m-7n}$$

$$\Rightarrow \frac{3k}{4l} = \frac{6m}{7n} \Rightarrow \frac{k}{4l} = \frac{2m}{7n}$$

$$\Rightarrow \frac{k}{2m} = \frac{4l}{7n}$$

$\therefore$ k, $2m$, $4l$ and $7n$ are in proportion.

EXAMPLE 20.19

If a, b, c and d are in proportion, then show that $\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{c^3+3cd^2}{3c^2d+d^3}$.

SOLUTION

Given a, b, c and d are in proportion

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Applying componendo and dividendo,

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Cubing on both sides, we have

$$\frac{(a+b)^3}{(a-b)^3} = \left[\frac{c+d}{c-d}\right]^3$$

$$\Rightarrow \frac{a^3+3a^2b+3ab^2+b^3}{a^3-3a^2b+3ab^2-b^3} = \frac{c^3+3c^2d+3cd^2+d^3}{c^3-3c^2d+3cd^2-d^3}$$

$$\frac{(a^3+3ab^2)+(3a^2b+b^3)}{(a^3+3ab^2)-(3a^2b+b^3)} = \frac{(c^3+3cd^2)+(3c^2d+d^3)}{(c^3+3cd^2)-(3c^2d+d^3)}$$

$$\Rightarrow \frac{a^3+3ab^2}{3a^2b+b^3} = \frac{c^3+3cd^2}{3c^2d+d^3}$$

Hence, proved.

EXAMPLE 20.20

4 men, each working 6 hours per day can build a wall in 9 days. How long will 6 men, each working 3 hours per day take to finish the same work?

SOLUTION

Clearly, $M \propto \frac{1}{D}$ and $M \propto \frac{1}{H}$

$$\Rightarrow MDH = \text{Constant.}$$
$$\Rightarrow M_1D_1H_1 = M_2D_2H_2$$
$$\Rightarrow 4 \times 9 \times 6 = 6 \times D_2 \times 3$$
$$\Rightarrow D_2 = 12.$$

$\therefore$ Six men each working 3 hours per day can finish the job in 12 days.

EXAMPLE 20.21

The area of a circle varies with the square of its radius. The area of a circle having radius 3 cm is C cm^2. Find the area of the circle having radius 9 cm.

SOLUTION

Let the area of the circle be denoted by C and its radius be denoted by r.

$$C \propto r^2 \Rightarrow \frac{C_1}{C_2} = \frac{r_1^2}{r_2^2}.$$

Taking $r_1 = 3$ cm, $r_2 = 9$ cm and $C_1 = C$ cm^2,

$$C_2 = C_1\left(\frac{r_2^2}{r_1^2}\right) = C\left(\frac{9}{3}\right)^2 = 9C\text{ cm}^2.$$

EXAMPLE 20.22

The pressure of a gas varies directly as the temperature when volume is kept constant and varies inversely as the volume when temperature is kept constant. The gas occupies volume of 400 ml when the temperature is 160 K and the pressure is 640 Pa. What is the temperature of gas whose volume and pressure are 200 ml and 320 Pa, respectively?

SOLUTION

Given $P \propto T$ and $P \propto \frac{1}{V}$

$$\Rightarrow P \propto \frac{T}{V} \Rightarrow PV \propto T$$
$$\Rightarrow \frac{PV}{T} = \text{Constant}$$
$$\Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$\Rightarrow \frac{640 \times 400}{160} = \frac{320 \times 200}{T_2} \Rightarrow T_2 = \frac{160}{4} = 40\text{ K.}$$

EXAMPLE 20.23

A garrison of 700 men is provisioned for 30 days at the rate of $2\frac{1}{2}$ kg per day per man. If 100 men had left the garrison, then the provisions would have lasted for x days at the rate of $3\frac{1}{2}$ kg per day per man. Find the value of x. Choose the correct answer from the following options:

(a) 18 **(b)** 20 **(c)** 25 **(d)** 15

SOLUTION

In both cases, the provisions are the same.

$$700 \times 30 \times 2\frac{1}{2}$$
$$= (700 - 100) \times x \times 3\frac{1}{2}.$$
$$\Rightarrow 700 \times 30 \times \frac{5}{2} = 600 \times x \times \frac{7}{2}$$
$$\therefore x = 25.$$

EXAMPLE 20.24

Three positive numbers are in the ratio 1 : 3 : 5. The sum of their squares is 875. Find the sum of the numbers. Choose the correct answer from the following options:

(a) 45 **(b)** 90 **(c)** 75 **(d)** 150

SOLUTION

Given that the ratio of the three numbers is 1 : 3 : 5.

Let the numbers be x, $3x$ and $5x$

And also given,

$x^2 + (3x)^2 + (5x)^2 = 875$

$\Rightarrow 35x^2 = 875$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$ (the numbers are positive)

$\therefore$ The required sum $= x + 3x + 5x = 9x = 9(5) = 45$.

EXAMPLE 20.25

Two positive numbers x and y satisfy the condition $4x^2 + 25y^2 = 20xy$. Find the value of $x : y$.

(a) 5 : 2 **(b)** 2 : 5 **(c)** 3 : 2 **(d)** 2 : 3

SOLUTION

Given,

$$4x^2 + 25y^2 = 20xy$$
$$\Rightarrow (2x - 5y)^2 = 0 \Rightarrow 2x - 5y = 0$$
$$\Rightarrow \frac{x}{y} = \frac{5}{2} \Rightarrow x : y = 5 : 2.$$

EXAMPLE 20.26

There are three numbers such that twice the first number is equal to thrice the second number and four times the third number. Find the ratio of the first, the second and the third numbers. Choose the correct answer from the following options:

(a) 2 : 3 : 4 **(b)** 6 : 4 : 3 **(c)** 4 : 3 : 4 **(d)** 3 : 4 : 3

SOLUTION

Let the three numbers be x, y and z.

Given, $2x = 3y = 4z$

Let $2x = 3y = 4z = k$

$$\Rightarrow x = \frac{k}{2}, y = \frac{k}{3} \text{ and } z = \frac{k}{4}$$

$$\Rightarrow x : y : z = \frac{k}{2} : \frac{k}{3} : \frac{k}{4}$$

$$= 6 : 4 : 3.$$

EXAMPLE 20.27

₹585 is to be divided among A, B and C in the ratio 3 : 4 : 6. By mistake, it is divided in the ratio $\frac{1}{6} : \frac{1}{4} : \frac{1}{3}$. Find the loss incurred to C due to this mistake (in ₹). Choose the correct answer from the following options:

(a) 10 **(b)** 15 **(c)** 20 **(d)** 25

SOLUTION

Total money = ₹585

(a) Ratio of actual shares is 3 : 4 : 6

Actual share of C (in ₹) $= \frac{6}{13}(585)$

$= 6(45) = 270.$

(b) Ratio of shares given $= \frac{1}{6} : \frac{1}{4} : \frac{1}{3}$

$= 2 : 3 : 4$

Now, share of C (in ₹) $= \frac{4}{9}(585) = 4(65) = 260$

Loss incurred by C (in ₹) = 270 – 260 = 10.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 200 metres is ______ of a kilometre.
2. The result obtained after applying componendo to $b : (a - b) :: d : (c - d)$ is ______.
3. The simplest form of 57 : 72 is ______.
4. If $X = 3Y = 4Z$, then $X : Y : Z =$ ______.
5. Which of the two ratios 4 : 9 and 5 : 8 is greater?
6. The inverse ratio of $\frac{1}{8}$: 4 is ______.
7. The result obtained after applying dividendo to $(a + b) : b :: (c + d) : d$ is ______.
8. The compound ratio of $p : r$ and $r : q$ is ______.
9. The numbers 4, 7, 8 and 14 are in proportion. (True/False)
10. Express the ratio 1.5 litres : 250 ml in the lowest terms.
11. If P and Q^2 are in direct proportion. If Q doubles then P becomes ______.
12. If 10 men can do a certain amount of work in 6 days, 15 men can do the same amount of work in ______ days.
13. x varies directly as the cube of y. x is 32 when y is 4, then what is the value of y when x is 108?
14. A varies inversely as the square of B and A is 1, when B is 3. What is the value of B, when A is 3?
15. Express the ratio $\frac{3}{5} : \frac{7}{9}$ in the lowest terms.
16. The mean proportional between 4 and 8 is 6. (True/False)
17. If $7\frac{3}{4} : 11\frac{2}{3} = 93 : (2x + 10)$, then what is the value of x?
18. If $x : y = 5 : 6$ and $y : z = 3 : 7$, then find $x : z$.
19. If $a : b = 2 : 3$, then find $(3a - b) : (2a + 3b)$.
20. Two numbers are in the ratio 3 : 4. When 7 is added to each, the ratio becomes 4 : 5. Find the numbers.
21. What is the compounded ratio of $6\frac{1}{2} : 8\frac{2}{3}$ and $3\frac{3}{4} : 5\frac{1}{3}$?
22. If $(x + 2)$, 3, $(3x - 4)$ and 6 are in proportion, Find x.
23. Find the fourth proportional of 4, 9 and 12.
24. Write a ratio equal to 7 : 9 in which antecedent is 63.
25. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{2}{5}$, then $\frac{3a + 4c + 9e}{3b + 4d + 9f} =$ ______.
26. If $(3a + 2b) : (5a + 7b) = 1 : 2$, then find $a : b$.
27. Arrange the ratios 4 : 5, 6 : 7, 2 : 3 and 10 : 11 in the increasing order of their magnitude.
28. A certain sum of money is distributed among A, B and C in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and B gets ₹120. Find the shares of A and C.
29. When a = 3, 5, 8, 10, …, b = 9, 15, 24, 30, …, then a and b are in ______.
30. What number must be subtracted from each of the numbers 32, 38, 17 and 20, so that the terms formed will be in proportion?

Short Answer Type Questions

31. The number of boys and girls in a class are in the ratio of 3 : 4. Find the strength of the class if there are 10 more girls than there are boys.
32. The numbers A, $A + 2$, $A + 12$ and $A + 22$ are in proportion. Find A.
33. If $\frac{a^3 + 2ab^2}{2a^2b + b^3} = \frac{123}{136}$, then find the value of $a : b$.
34. P, Q and R have to share 52 apples among themselves such that P gets thrice as many apples as Q gets and Q gets thrice as many as R gets. Find the number of apples R must get.
35. The ratio of the present ages of a father and his son is 3 : 1. The sum of their ages after five years is 58. Find the ratio of their ages 3 years ago.
36. The ratio of the present ages of Raju and his wife is 5 : 4. Which of the following cannot be the ratio of their ages 20 years hence?

    (a) 11 : 10 (b) 6 : 5
    (c) 23 : 20 (d) 13 : 10
37. The mass of a liquid (in grams) varies directly with its volume (in cm^3). A liquid would have a mass of

20 grams if its volume is 10 cm^3. Find the mass of the liquid in grams, if its volume is 8 cm^3.

38. The ratio of the present ages of Swarupa and her daughter is 7 : 3. When Swarupa was 26 years old, her daughter was 6 years old. Find the present age of Swarupa. (in years)

39. Solve: $\dfrac{\sqrt{10+3x}+\sqrt{10-3x}}{\sqrt{10+3x}-\sqrt{10-3x}} = 3.$

40. Find $\dfrac{x+2p}{x-2p}+\dfrac{x+2q}{x-2q}$, when $x=\dfrac{4pq}{p+q}$.

41. The volume (V) of a mass of gas varies directly as its absolute temperature (T) and inversely as the pressure (P) applied to it. The gas occupies a volume of 300 ml, when the temperature is 210 K and pressure is 150 Pa. What is the temperature of gas whose volume and pressure are 200 ml and 250 Pa respectively?

42. The volume of a sphere (V) varies directly as the cube of its radius. The volume of the sphere of radius 3 cm is 36π cm^3. What is the volume of a sphere of radius 15 cm?

43. The ratio of the tens digit and the units digit of a two digit number is 1 : 2. How many two digit numbers satisfy this condition?

44. The ratio of the monthly earnings of A and B is 3 : 2. The ratio of the monthly expenditures of A and B is 4 : 3. A saves ₹300 each month. Which of the following cannot be the monthly savings of B? (in ₹)

(a) 180 (b) 195
(c) 165 (d) 205

45. If $\dfrac{2p+5q}{2r+5s}=\dfrac{4p-3q}{4r-3s}$, then find the relation between p, q, r and s.

Essay Type Questions

46. There are two boxes, red and white in colour. The ratio of the number of chocolates in the white box to the number of biscuits in the red box is 3 : 2 and the ratio of the number of biscuits in the white box to the number of chocolates in the red box is 3 : 4. If the ratio of the total number of chocolates and biscuits in the white box to the total number of chocolates and biscuits in the red box is 15 : 16. Find the ratio of the total number of chocolates to the total number of biscuits in the two boxes.

47. A father distributed ₹12250 among his four sons A, B, C and D such that $\dfrac{\text{A's share}}{\text{B's share}}=\dfrac{\text{B's share}}{\text{C's share}}=\dfrac{\text{C's share}}{\text{D's share}}=\dfrac{3}{4}$. Then find the share of C.

48. Some amount is to be divided between X and Y in the ratio of 3 : 4. But due to wrong calculation, it was found that X got one-seventh of the total amount more than his expected share. Find the share of Y, when X got an amount of ₹480.

49. If $\dfrac{x}{y+z}=\dfrac{y}{z+x}=\dfrac{z}{x+y}$, then find the value of each fraction.

50. The amount collected per month by the Hyderabad Metro Water Works consists of two parts, a fixed charge for providing the service and a variable charge which is directly proportional to the number of kilolitres of water consumed. An amount of ₹300 is charged for consuming 100 kilolitres in a particular month. It is also noticed that when the consumption increases from 300 kilolitres per month to 400 kilolitres per month, the bill amount increases to $\dfrac{6}{5}$ times that of the former. How much is the fixed charge per month?

CONCEPT APPLICATION

Level 1

1. If $a:b=5:4$ and $b:c=16:25$, then find $a:b:c$.

(a) 20 : 25 : 16 (b) 25 : 20 : 16
(c) 25 : 16 : 20 (d) 20 : 16 : 25

2. P and Q have some coins with them. The ratio of the numbers of coins with P and Q is 7 : 8. Q has 5 more coins than P has. Find the number of coins with Q.

PRACTICE QUESTIONS

(a) 72 (b) 40
(c) 48 (d) 80

3. The numbers $x - 4$, $x - 2$, $x + 2$ and $x + 10$ are in proportion. Find x.

(a) 8 (b) 10
(c) 6 (d) 12

4. If $qr : pr : pq = 1 : 4 : 7$, then find $\frac{p}{qr} : \frac{q}{pr}$.

(a) 4 : 1 (b) 1 : 4
(c) 1 : 16 (d) 16 : 1

5. What number must be added to each term of the ratio 9 : 11, so that it becomes 4 : 5.

(a) 1 (b) 2
(c) –2 (d) –1

6. A purse contains ₹50, ₹10 and ₹5 notes in the ratio 6 : 3 : 7 and the total amount in the purse is ₹730. Find the number of ₹5 notes in the purse.

(a) 7 (b) 14
(c) 21 (d) 28

7. If $p : q : r : s = 3 : 4 : 7 : 8$ and $p + s = 55$, then find $q + r$.

(a) 33 (b) 55
(c) 44 (d) 66

8. A metal X is 15 times as dense as metal Z and a metal Y is 8 times as dense as metal Z. In what ratio should these two metals be mixed to get an alloy which is 13 times as dense as metal Z?

(a) 2 : 5 (b) 2 : 3
(c) 3 : 2 (d) 5 : 2

9. What must be added to each of the numbers 3, 7, 8 and 16 so that the resulting numbers are in proportion?

(a) 4 (b) 3
(c) 2 (d) 1

10. The cost of 570 bags of rice is ₹17,100. Find the number of bags of rice which can be bought for ₹19,200.

(a) 720 (b) 800
(c) 640 (d) 600

11. If x, y and z are in continued proportion, then $(x + y + z)(x - y + z) =$ ______.

(a) $x^2 - y^2 + z^2$ (b) $x^2 - y^2 - z^2$
(c) $x^2 + y^2 + z^2$ (d) None of these

12. Seven friends planned a tea party. The expenses per boy in rupees is numerically 1 less than the number of girls and the expenses per girl in rupees is numerically 1 less than the number of boys. If the ratio of the total expenses of the boys and the girls is 8 : 9, then what is the expenditure of each boy?

(a) ₹2 (b) ₹3
(c) ₹1 (d) ₹4

13. If a, b and c are in continued proportion, then $a^2 : b^2$ is

(a) $a^2 : c^2$ (b) $a : b$
(c) $a : c$ (d) $b : c$

14. A metal X is 16 times as dense as metal Z and metal Y is 7 times as dense as metal Z. In what ratio should X and Y be mixed to get an alloy 12 times as dense as metal Z?

(a) 2 : 3 (b) 5 : 2
(c) 5 : 4 (d) 6 : 5

15. The velocity of a freely falling body when it strikes the ground is directly proportional to the square of the time taken by it to strike the ground. It takes 5 seconds to strike the ground, with a velocity of 10 m/sec. If it strikes the ground with the velocity of 40 m/sec, then for how long would it have travelled?

(a) 7.5 sec (b) 2.5 sec
(c) 10 sec (d) 20 sec

16. The ratio of the number of students in two classrooms, C_1 and C_2, is 2 : 3. It is observed that after shifting ten students from C_1 to C_2, the ratio is 3 : 7. Further, how many students have to be shifted from C_2 to C_1 for the new ratio to become 9 : 11?

(a) 10 (b) 15
(c) 20 (d) 8

17. Rakesh attended a party, where a lady asked him his age. He said that ten years ago the sum of his and his daughter's ages was 40 years, and the ratio of his present age to that of his daughter is 7 : 3. What is the present age of Rakesh in years?

(a) 42 (b) 28
(c) 25 (d) 30

18. The mean proportional between two numbers is 56. If the third proportional of the same two numbers is 448, then find the sum of the two numbers.

(a) 180 (b) 440
(c) 240 (d) 140

19. The speed at which a person runs is directly proportional to the blood pressure in his body. If the speed of a person is 2 m/sec his blood pressure would be 120 units. Find the blood pressure in units of a person running at 8.5 m/sec.

(a) 510 (b) 425
(c) 340 (d) 5950

20. A father wants to divide ₹22,515 among his four sons P, Q, R and S such that $\frac{\text{P's share}}{\text{Q's share}} = \frac{\text{Q's share}}{\text{R's share}} = \frac{\text{R's share}}{\text{S's share}} = \frac{1}{2}$. Find the share of Q.

(a) ₹1507 (b) ₹3002
(c) ₹6004 (d) ₹12,008

21. The mean proportion of two numbers is 24 and their third proportion is 72. Find the sum of the two numbers.

(a) 11 (b) 24
(c) 32 (d) 80

22. If 40 men can complete a job in 6 days, then find the number of days taken by 60 men to complete it.

(a) 3 (b) 10
(c) 12 (d) 4

23. The cost of a precious stone varies as the cube of its weight. The stone broke into 3 pieces whose weights are in the ratio 1 : 2 : 3. As a result, its cost reduces. If the cost of the unbroken stone is ₹96,336, then find the loss incurred due to breakage.

(a) ₹80,280 (b) ₹16,056
(c) ₹40,140 (d) ₹8028

24. A diamond falls and breaks into pieces whose weights are in the ratio 2 : 3 : 5. The value of the diamond is directly proportional to the square of its weight. Find the loss incurred, if the actual cost of the diamond is ₹96,000. (in ₹)

(a) 36,480 (b) 59,520
(c) 72,960 (d) None of these

25. In a monthly unit test, the marks scored by Bunny and Sunny are in the ratio of 10 : 7 and those scored by Bharat and Sunny are in the ratio of 8 : 9. If Bharat scores 112 marks, then find the marks scored by Bunny.

(a) 170 (b) 190
(c) 200 (d) 180

26. If $\frac{x^3 + 4xy^2}{4x^2y + y^3} = \frac{15}{16}$, then find $x : y$.

(a) 2 : 3 (b) 3 : 2
(c) 4 : 3 (d) 3 : 4

27. The ratio of the tens digit and the units digit of a two digits number is 2 : 3. How many possible values can it assume?

(a) 3 (b) 4
(c) 2 (d) 5

28. The least integer which when subtracted from the antecedent and added to the consequent of the ratio 9 : 8 gives a ratio less than the ratio 15 : 26 is

(a) 2 (b) 3
(c) 4 (d) 1

29. Ram wanted to distribute a certain amount between his two children Lava and Kusha in the ratio 5 : 7. But it was found that due to incorrect calculations Lava got one-sixth of the total amount more than what he should get. Find the share of Kusha, in rupees, if Lava got ₹560 in all.

(a) 300 (b) 350
(c) 400 (d) 450

30. A bag contains one rupee, 50 paise and 25 paise coins. The ratio of the number of 1 rupee coins to that of 50 paise coins is 5 : 9 and the ratio of the number of 50 paise coins to that of 25 paise coins is 2 : 1. Find the value of the 50 paise coins in the bag if the total value of the coins in the bag is ₹425.

(a) ₹254 (b) ₹180
(c) ₹78 (d) Cannot be determined

Level 2

31. Seven years ago the ratio of the ages of P and Q (in years) was 7 : 6. Which of the following cannot be the ratio of their ages 6 years from now?

(a) 13 : 11 (b) 15 : 14
(c) 13 : 12 (d) 16 : 15

32. The ratio of the monthly incomes of Ram and Shyam is 3 : 4 and the ratio of their monthly expenditures is 4 : 5. If Shyam saves ₹400 per month, which of the following cannot be the savings of Ram? (In ₹/month)

(a) 290 (b) 280
(c) 270 (d) 310

33. The ratio of the present ages of Anand and Bala is 8 : 3. When Anand was 30 years old, Bala was 5 years old. Find the present age of Bala. (In years)

(a) 10 (b) 12
(c) 15 (d) 20

34. There are three sections A, B and C in class VIII of a school. The ratio of the number of students in A, B and C is 2 : 3 : 4. The section which has neither the maximum number of students nor the minimum number of students has a strength of 30. Find the total strength of the three sections.

(a) 63 (b) 81
(c) 72 (d) 90

35. In a solution of 45 litres of milk and water, 40% is water. How many litres of milk must be added to become the ratio of milk and water 5 : 3?

(a) 8 (b) 3
(c) 7 (d) 6

36. Find the triplicate ratio of $(2y - x) : (2x - y)$, if $x : y = 4 : 3$.

(a) 64 : 125 (b) 27 : 64
(c) 8 : 125 (d) 27 : 125

37. Nine years ago A's age and B's age were in the ratio 5 : 7. Which of the following cannot be the ratio of their ages 5 years from now?

(a) 11 : 13 (b) 13 : 19
(c) 21 : 25 (d) 15 : 16

38. A varies directly with x^2. If $x = 2$ or 4, then $A = 20$. Find A if $x = 8$.

(a) 48 (b) 120
(c) 64 (d) 320

39. The force applied on a stationary body varies directly with the acceleration with which it starts to move. If a force of 10 N is applied on a stationary body, it starts to move with an acceleration of 2 m/sec^2. Find the force (in N) to be applied on the body at which it starts to move with an acceleration of 4 m/sec^2.

(a) 30 (b) 20
(c) 40 (d) 50

40. The ratio of the present ages of Ram and Shyam is 3 : 2. Which of the following cannot be the ratio of their ages 20 years ago?

(a) 8 : 5 (b) 17 : 10
(c) 9 : 5 (d) 7 : 5

41. Ninety coins are to be distributed among P, Q and R such that P gets twice as many coins as Q gets and Q gets thrice as many coins as R gets. Find the number of coins R gets.

(a) 6 (b) 3
(c) 10 (d) 9

42. P varies inversely with $\sqrt{y}$. If $y = 2$, then $P = 40$. If $P = 20$ then find y.

(a) $\sqrt{8}$ (b) 8
(c) 4 (d) $\sqrt{2}$

43. The monthly electricity bill raised by the municipal corporation consists of two parts–a fixed charge for providing the service and a variable charge which is directly proportional to the number of watts of power consumed. An amount of ₹500 is charged for consuming 125 watts in a particular month. The ratio of the amount charged for 400 watts to that of 500 watts is 21 : 25. How much is the fixed charge per month?

(a) ₹125 (b) ₹200
(c) ₹250 (d) ₹500

44. A solution of 30 litres of milk and water, has 70% milk. How many litres of water must be added so that the volumes of milk and water will be in the ratio 3 : 2?

(a) 6 (b) 3
(c) 5 (d) 4

45. If $\dfrac{a+b}{xa+yb} = \dfrac{b+c}{xb+yc} = \dfrac{c+a}{xc+ya}$ where $x + y \neq 0$ and $a + b + c \neq 0$, then each of these ratios is equal to

(a) 1 (b) $\dfrac{1}{x+y}$

(c) $\dfrac{2}{x+y}$ (d) $\dfrac{2}{a+b}$

46. The ratio of sugar and other ingredients in the biscuits of three bakeries—Mongunies, Karachi and Baker's Inn are 5 : 4, 13 : 12 and 29 : 24 respectively. The biscuits of which bakery are the sweetest?

(a) Mongunies

(b) Karachi

(c) Baker's inn

(d) All the three types of biscuits are equally sweet.

47. ₹x is divided among three persons Mr Vaayu, Mr Jala and Mr Agni in the ratio 3 : 4 : 7. If the share of Mr Jala is ₹5600, then find the value of x.

(a) 15,400 (b) 16,800

(c) 18,200 (d) 19,600

48. The ratio of the monthly incomes of Mr Anand and Mr Milind is 9 : 10 and the ratio of their monthly savings is 9 : 10. If the monthly expenditure of Mr Milind is ₹15,000, then find the monthly expenditure of Mr Anand (in ₹).

(a) ₹12,000 (b) ₹9000

(c) ₹8500 (d) ₹13,500

49. The ratio of the present ages of Mr Dhuryodhana and Mr Dhushyasana is 5 : 4. Which of the following cannot be the ratio of their ages 5 years ago?

(a) 4 : 3 (b) 3 : 2

(c) 9 : 7 (d) 8 : 7

50. What must be added to both the terms of $x : y$ so that the resultant ratio is inverse of the given ratio $(x \neq y)$?

(a) $y - x$ (b) $x - y$

(c) $-(x + y)$ (d) $x + y$

Level 3

51. Nine friends had a tea party. All boys took only coffee and all girls took only tea. The cost per cup of coffee in rupees is numerically 2 less than the number of girls and the cost per cup of tea in rupees is numerically 2 less than the number of boys. If the ratio of the total expenses of the boys and the girls is 5 : 6, then what is the cost of each coffee? (In ₹)

(a) 2 (b) 5

(c) 7 (d) 3

52. If $x : y = 3 : 5$, find the duplicate ratio of $(3x + y) : (5x - y)$.

(a) 49 : 25 (b) 25 : 9

(c) 36 : 25 (d) 49 : 36

53. The ratio of the number of students in two classrooms A and B is 3 : 2. If ten students shift from A to B, the ratio becomes 7 : 8. Now how many students must shift from A to B in order for the ratio to become 8 : 7?

(a) 5 (b) 10

(c) 15 (d) 20

54. There are some students in an auditorium, some of them are dressed in white and the others are dressed in blue. The ratio of the number of boys dressed in white to the number of girls dressed in blue is 4 : 3 and the ratio of the total number of girls dressed in white to the number of boys dressed in blue is 4 : 5. The ratio of the total number of boys and girls dressed in white to the total number of boys and girls dressed in blue is 12 : 13. Find the ratio of the total number of boys to that of girls in the auditorium.

(a) 13 : 15 (b) 19 : 17

(c) 14 : 11 (d) Cannot be determined

55. Eleven friends had a tea party. All boys took only coffee and all girls took only tea. The cost per cup of coffee in rupees is numerically one more than the number of girls and the cost per cup of tea in rupees is numerically equal to the number of boys. If the ratio of the total expenses of the boys and the girls is 7 : 6, then what is the cost of each coffee? (In ₹)

(a) 4 (b) 5

(c) 6 (d) 7

PRACTICE QUESTIONS

56. The ratio of the present ages of Mrs. Anoukika and Mrs. Mythili is 5 : 6. After 6 years, Mrs. Anoukika would reach the present age of Mrs. Mythili. Find the sum of their present ages (in years).

(a) 66 (b) 44
(c) 33 (d) 15

57. The incomes of A and B are in the ratio 4 : 3. The expenditures of A and B are in the ratio5 : 2. If B saves ₹3000, then which of the following cannot be the savings of A?

(a) ₹1500 (b) ₹2500
(c) ₹3500 (d) ₹4500

58. If $a + b$ varies directly with $a - b$, then which of the following vary directly?

(a) a and b
(b) $a^2 + b^2$ and $a^2 - b^2$
(c) both (a) and (b)
(d) None of these

59. If 20 men take 15 days to complete a certain amount of work working at 10 hours per day, then how many more men are required to complete twice the previous work in 10 days working at 12 hours a day?

(a) 10 (b) 20
(c) 30 (d) 40

60. The ratio of the amounts with Mr Umar and Mr Gumar is 3 : 4. If Mr Gumar gives ₹5 to Mr Umar, then the ratio of the amounts with Umar and Gumar is 4 : 3. Mr Umar gives ₹5 to Mr Gumar. Find the ratio of the amounts with them.

(a) 3 : 5 (b) 2 : 5
(c) 4 : 5 (d) 1 : 5

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\frac{1}{5} = 0.2$
2. $\frac{a}{a-b} = \frac{c}{c-d}$
3. 19 : 24
4. 12 : 4 : 3
5. 5 : 8
6. 32 : 1
7. $\frac{a}{b} = \frac{c}{d}$
8. $p : q$
9. True
10. 6 : 1
11. 4 times itself
12. 4
13. 6
14. $\sqrt{3}$
15. 27 : 35
16. False
17. 65
18. 5 : 14
19. 3 : 13
20. 21 and 28
21. 135 : 256
22. 8
23. 27
24. 63 : 81
25. $\frac{2}{5}$
26. 3 : 1
27. $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}$ and $\frac{10}{11}$
28. ₹90, ₹180
29. direct proportion
30. 2

Shot Answer Type Questions

31. 70
32. 3
33. 3 : 4
34. 4
35. 11 : 3
36. 13 : 10
37. 16
38. 35
39. 2
40. 2
41. $\frac{700}{3}$ K
42. 4500π cm^3
43. 4
44. 205
45. $ps = qr$

Essay Type Questions

46. 18 : 13
47. ₹3360
48. ₹360
49. –1
50. ₹200

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (c)
11. (c)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (b)
21. (d)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (b)

Level 2

31. (a)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (d)
41. (d)	**42.** (c)	**43.** (c)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (c)

Level 3

51. (a)	**52.** (a)	**53.** (a)	**54.** (c)	**55.** (d)	**56.** (a)	**57.** (d)	**58.** (c)	**59.** (c)	**60.** (b)

CONCEPT APPLICATION

Level 1

1. Compare both the ratios by making b equal in both of them.
2. Let P = $7x$ and Q = $8x$. The difference in the number of coins with P and Q is 5.
3. Product of extremes = Product of means.
4. Simplify the required ratio.
5. Assume 'x' as the number to be added and apply Product of extremes = Product of means.
6. Let the number of notes be $6x$, $3x$ and $7x$.
7. Let p, q, r and s be $3x$, $4x$, $7x$ and $8x$.
8. (i) Let the required ratio be $x : 1$.
   (ii) Let the density of metal z be k.
   $\therefore$ Density of metal $x = 15k$
   Density of metal $y = 8k$.
   (iii) Let required ratio be $m : 1$.
   (iv) $(15k)m + (8k)1 = (m + 1)(13k)$.
   Solve this equation to get m.
9. Let the number added be x and use, Product of extremes = Product of means.
10. Use the unitary method.
11. (i) If x, y, z are in continued proportion, then $y^2 = xz$.
    (ii) First of all multiply the given expressions and simplify.
    (iii) Substitute $xz = y^2$, since x, y and z are in continued proportion.
12. (i) Consider the expenses of boys and girls as x and y respectively and proceed.
    (ii)

	Number of Boys	Number of Girls
	b	g
Expense per student	$g - 1$	$b - 1$

    (iii) $\dfrac{bg - b}{gb - g} = \dfrac{8}{9}$.
    (iv) Solve $b + g = 7$ and the above equation and get $(g - 1)$ in rupees.
13. (i) If a, b and c are in continued proportion, then $b^2 = ac$.
    (ii) Substitute $b^2 = ac$ in $a^2 : b^2$.
14. (i) Let the required ratio be $x : 1$.
    (ii) Let the density of metal z be k.
    $\therefore$ Density of metal $x = 16k$ and that of $y = 7k$.
    (iii) Let the required ratio be $m : 1$.
    (iv) $(16k)m + (7k)1 = (m + 1)12k$, solve this for m.
15. (i) $\dfrac{V_1}{V_2} = \dfrac{t_1^2}{t_2^2}$. (Apply the concept of variation).
    (ii) Given that $\dfrac{V}{t^2} = k$ (where k is constant).
    (iii) $\dfrac{v_1}{t_1^2} = \dfrac{v_2}{t_2^2}$.
16. (i) Consider the number of students in C_1 and C_2 as $2x$ and $3x$ respectively.
    (ii) Let the number of students in the two classes C_1 and C_2 be $2x$ and $3x$ respectively.
    (iii) Given $(2x - 10) : (3x + 10) = 3 : 7$.
    Find x from this equation.
    (iv) Let y be the number of students to be shifted from C_2 to C_1.
    (v) Substitute value of x in $(2x - 10 + y) : (3x + 10 - y) = 9 : 11$ and find y.
17. (i) Frame the linear equations and solve them.
    (ii) The sum of present ages of Rakesh and his daughter is (40 + 20) years.
    (iii) Divide 60 in the ratio 7 : 3.
18. (i) If $b^2 = ac$, then b is the mean proportional and c is the third proportional.
    (ii) Let the two numbers be a and b.
    (iii) $ab = (56)^2$.
    (iv) $b^2 = 448a$.
19. (i) $\dfrac{S_1}{S_2} = \dfrac{BP_1}{BP_2}$. (Apply the concept of variation).
    $\Rightarrow \dfrac{2}{8.5} = \dfrac{120}{BP_2}$
    $\therefore BP_2 = 510$

20. (i) Find the ratio of P : Q : R : S and proceed.

(ii) Given P : Q = Q : R = R : S = 1 : 2, find P : Q : R : S.

(iii) R's share

$$= \frac{\text{term corresponding to R}}{\text{Sum of the terms of the ratio}} \times 22{,}515.$$

21. (i) In $b^2 = ac$, b is the mean proportion and c is the third proportional.

(ii) Let the two numbers be a and b.

(iii) $(ab) = (24)^2$, $b^2 = 72a$, solve for a and b.

22. Use the unitary method.

23. (i) Let the weights of the three pieces be x, $2x$ and $3x$ and their costs be C_1, C_2 and C_3 respectively.

(ii) $\dfrac{C_1}{x^3} = \dfrac{C_2}{(2x)^3} = \dfrac{C_3}{(3x)^3} = \dfrac{96{,}336}{(6x)^3}$.

(iii) Find C_1, C_2 and C_3 and then $96{,}336 - (C_1 + C_2 + C_3)$.

24. Let the weights of the three pieces be $2x$, $3x$ and $5x$ and their costs be v_1, v_2 and v_3 respectively.

$v_1 = 4x^2k$, $v_2 = 9x^2k$, $v_3 = 25x^2k$,

$96{,}000 = 100x^2k$

$x^2k = 960$

$v_1 + v_2 + v_3 = 38x^2k$

$= 38 \times 960 = 36{,}480$,

$\therefore$ loss $= 96{,}000 - 36{,}480 = 59{,}520$.

25. (i) Find the ratio marks of Bunny, Sunny and Bharat.

(ii) Marks scored by sunny $= \left(\dfrac{9}{8}\right)$ (marks scored by Bharat).

(iii) Marks scored by Bunny $= \left(\dfrac{10}{7}\right)$ (marks scored by Sunny).

26. Divide both the numerator and the denominator by y^3.

27. Let the number be $10a + b$. Given $\dfrac{a}{b} = \dfrac{2}{3}$.

28. (i) Let the required number be x.

(ii) $\dfrac{9-x}{8+x} < \dfrac{15}{26}$.

(iii) Solve the inequation to get least integer value of x.

29. (i) Frame the linear equation and solve it.

(ii) Let the actual shares of Lava and Kusha be ₹$5x$ and ₹$7x$ respectively.

(iii) As the amount is $(5x + 7x)$, one sixth of total amount $= 2x$.

(iv) The amount is divided in the ratio $(5x + 2x) : (7x - 2x)$.

30. (i) Find the ratio of number of 1 rupee, 50 paise and 25 paise coins.

(ii) The ratio of number of one rupee coins, number of 50 paise coins and number 25 paise coins is $(5 \times 2) : (9 \times 2) : (1 \times 9)$, i.e., $10 : 18 : 9$.

(iii) Let the number of one rupee coins, 50 paise coins and 25 paise coins be $10x$, $18x$ and $9x$ respectively.

(iv) Given $(10x) + (18x)\dfrac{1}{2} + (9x)\dfrac{1}{4} = 425$,

Find $\left(\dfrac{18x}{2}\right)$, i.e., $9x$.

Level 2

32. Let the monthly incomes of Ram and Shyam be $3x$ and $4x$ and their expenditures be $4y$ and $5y$.

33. Let Anand's age k years ago, be 30 years.

34. Let the number of students in A, B and C be $2x$, $3x$ and $4x$.

39. Apply the concept of variation.

40. Let their present ages be $3x$ and $2x$ years.

43. (i) Let x be the fixed charge and y be the variable charge.

(ii) Given data can be expressed as $x + 100y$.

$$= 300 \cdot \frac{x + 300y}{x + 400y} = \frac{5}{6}.$$

(iii) Solve the above two equations and get the value of x.

45. (i) Each ratio $= \dfrac{\text{Sum of all the antecedents}}{\text{Sum of all the consequents}}$.

(ii) Each fraction is equal to the ratio of sum of the numerators and the sum of the denominators of all the equivalent fractions.

46.

Name of the Bakery	Ratio of Sugar and Other Ingredients
Mongunies	5 : 4 = 30 : 24
Karachi	13 : 12 = 26 : 24
Bakers Inn	29 : 24

From the above table, when the other ingredients are 24 units, their sugar quantities are 30 units, 26 units and 29 units.

∴ Mongunies biscuits are the sweetest.

47. Given that the ratio of shares of Mr Vaayu, Mr Jala and Mr Agni is 3 : 4 : 7.

Share of Mr Jala = ₹5600.

∴ The total sum (in ₹) $= \dfrac{(3+4+7)}{4}$

(5600) = (14)(1400) = 19,600.

48. Let the incomes of Mr Anand and Milind be $9x$ and $10x$, and their savings be $9y$ and $10y$.

∴ The ratio of expenditures of Mr Anand and Mr Milind $= \dfrac{9x-9y}{10x-10y}$

$= \dfrac{9(x-y)}{10(x-y)} = \dfrac{9}{10}$.

Given that the expenditure of Mr Milind is ₹15,000.

∴ The expenditure of Mr Anand (in ₹) $= \dfrac{9}{10}(15,000) = 13,500$.

49. The ratio of the present ages of Mr Dhuryodhana and Mr Dhushyasana is 5 : 4.

The given ratio is the ratio of greater inequality.

∴ The ratio of their ages k years ago is greater than the ratio of their ages now.

But 8 : 7 is less than 5 : 4.

50. Let $\dfrac{x+k}{y+k} = \dfrac{y}{x} \Rightarrow x^2 + kx = y^2 + ky$

$\Rightarrow k(x-y) = y^2 - x^2 \Rightarrow k = -(y+x)$.

Level 3

51. (i) Number of boys be 'b' and number of girls be 'g'.

(ii) Expenses of each boy = $g - 2$

expenses of each girl = $b - 2$

(iii) Solve $b + g = 9$ and the above equation and get $(g - 2)$ in rupees.

53. (i) Let the number of students in the two classes A and B be $2x$ and $3x$ respectively.

(ii) Given $(3x - 10) : (2x + 10) = 7 : 8$.

Find x from this equation.

(iii) Let y be the number of students to be shifted from A to B.

(iv) Substitute value of x in $(2x - 10 + k) : (3x + 10 - k) = 8 : 7$ and find k.

54. (i) Frame the linear equations and solve them.

(ii) Given data can be expressed as

the number of boys dressed in white = $4p$

the number of girls dressed in blue = $3p$

the number of girls dressed in white = $4q$

the number of boys dressed in blue = $5q$.

(iii) From the above data

$\dfrac{4p+4q}{3p+5q} = \dfrac{12}{13}$, find $\left(\dfrac{p}{q}\right)$ from this equation.

(iv) Required ratio is $(4p + 5q) : (3p + 4q)$,

i.e., $4\left(\dfrac{p}{q}\right) + 5 : 3\left(\dfrac{p}{q}\right) + 4$.

55. (i) Let the number of boys be 'b' and number of girls be 'g'.

(ii) Expenses of each boy = $g + 1$,

expenses of each girl = $b + 1$.

(iii) $\dfrac{b(g+1)}{g(b)} = \dfrac{7}{6}$.

(iv) Solve $b + g = 11$ and the above equation and get $(g + 1)$ in ₹.

56. Let the ages of Mrs. Anoukika and Mrs. Mythili be $5x$ years and $6x$ years.

From the given data.

$5x + 6 = 6x \Rightarrow x = 6$

$\therefore$ The sum of their present ages $= 5x + 6x = 11x = 11(6) = 66$ years.

57.

	A	B
Income	$4x$	$3x$
Expenditure	$5y$	$2y$
Savings	$4x - 5y$	$3x - 2y$

Given, $3x - 2y = 3000 \Rightarrow x = 1000 + \frac{2y}{3}$

Now, saving of $A = 4x - 5y = 4\left(1000 + \frac{2y}{3}\right)$

$-5y = 4000 + \frac{8y}{3} - 5y = 4000 - \frac{7y}{3}$.

$\therefore$ The saving of A cannot be more than ₹4000.

$\therefore$ Hence, the correct answer is option(d).

58. Given that $a + b$ vary directly with $a - b$.

$\frac{a+b}{a-b} = k$ (constant)

Applying *componendo–dividendo* theorem,

$\frac{a}{b} = \frac{k+1}{k-1}$ as k is constant, $\frac{k+1}{k-1}$ is also a constant.

$\therefore$ a and b vary directly.

Let $\frac{k+1}{k-1}$ be p

$\Rightarrow \frac{a}{b} = p \Rightarrow \frac{a^2}{b^2} = p^2$.

By applying *componendo–dividendo* theorem,

$\frac{a^2+b^2}{a^2-b^2} = \frac{p^2+1}{p^2-1}$.

As p is a constant, $\frac{p^2+1}{p^2-1}$ is also a consonant.

$\therefore$ $a^2 + b^2$ and $a^2 - b^2$ vary directly.

59. Let the required number of men be x.

We have, $\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$

$\frac{(20)(15)(10)}{1} = \frac{(20+x)(10)(12)}{2}$

$\Rightarrow 20 + x = 50 \Rightarrow x = 30$.

60. Let the amounts with Mr Umar and Mr Gumar be ₹$3x$ and ₹$4x$ respectively.

Given, $\frac{3x+5}{4x-5} = \frac{4}{3}$

$\Rightarrow 9x + 15 = 16x - 20$

$\Rightarrow x = 5$.

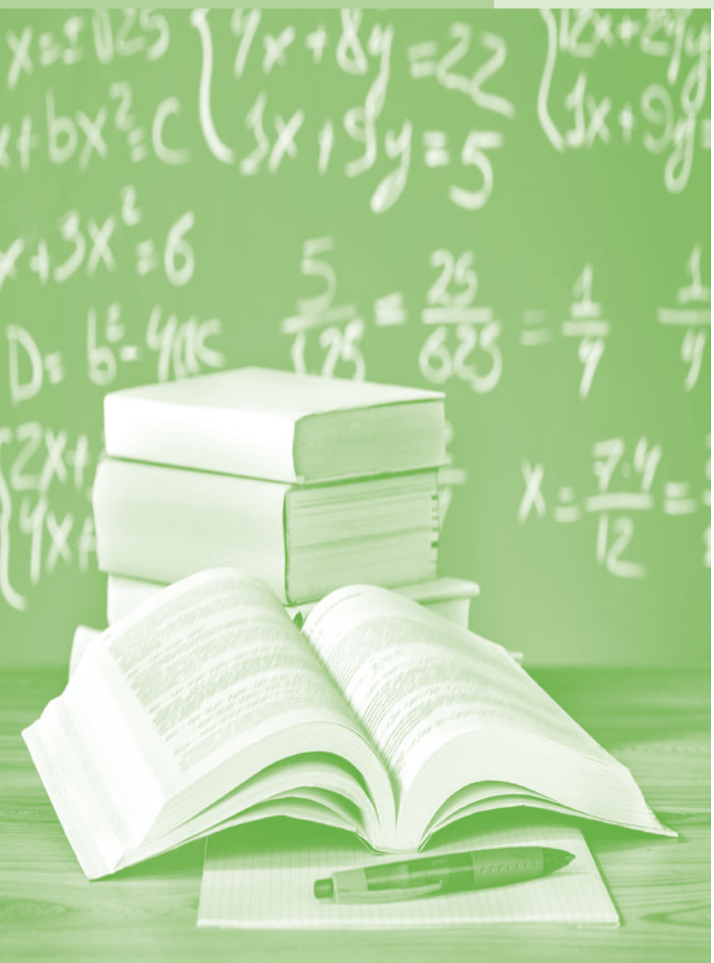
$\therefore$ The amounts with Mr Umar and Mr Gumar are ₹15 and ₹20 respectively.

The required ratio $= \frac{15-5}{20+5} = \frac{10}{25} = \frac{2}{5}$.

Ratio of the amount with them is 2 : 5.

Chapter 21

Shares and Dividends



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the terms related to banking
- Outline ideas on financial market
- Calculate the percentage values

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the terms such as shares and dividends
- Know the nominal value of a share (NV) and market value of a share (MV)
- Solve basic numerical problems on shares and dividends

INTRODUCTION

The total amount of money required to start a business or a company is called 'capital'. Generally, huge amounts of capital are required to start a company. It may be even hundreds of crores of rupees. It is not possible for a single person or two to arrange such large amount of money.

So, the total amount of capital is divided into several small and equal units called 'shares'. The management of the company invites investors to subscribe for these shares.

Example: Suppose, the capital requirement of the company is ₹15 crores. The whole amount of capital can be divided into:

1. 15 lakh shares of ₹100 each, or
2. 30 lakh shares of ₹50 each, or
3. 60 lakh shares of ₹25 each.

For every investment made, the management issues a share-certificate showing the value of each share and the number of shares held by each person.

A person who purchases and owns the shares of a company is called 'a shareholder'.

Nominal Value of a Share (NV)

Nominal value of a share is the value printed on the share certificate. It is also known as 'face value' (FV) or 'par value'.

Market Value of a Share (MV)

Market value of a share is the value of the share at which it is bought or sold in the market. A share is said to be issued

1. at premium or above par, if MV > FV.
2. at discount or below par, if MV < FV.
3. at par, if MV = FV.

Dividend

Dividend is referred to as the annual profit distributed among the share holders.

Dividend is always reckoned on the face value of a share.

Notes

1. The face value of a share always remains the same.
2. The market value of a share changes from time to time.
3. Dividend is always paid on the face value of a share.
4. The number of shares held by a person

$$= \frac{\text{Total investment}}{\text{Market value of each share}} \text{(or)}$$

$$= \frac{\text{Total income}}{\text{Income from each share}} \text{(or)}$$

$$= \frac{\text{Total face value}}{\text{Face value of each share}}.$$

Examples Based on Basic Concepts

EXAMPLE 21.1

Find the market value of a ₹300 share bought at a discount of ₹60.

SOLUTION

Face value of the share = ₹300

The share is bought at a discount of ₹60.

∴ Market value of the share = ₹(300 − 60) = ₹240.

EXAMPLE 21.2

If a share of ₹150 is available at a premium of ₹50, then find the market value of 250 such shares.

SOLUTION

Face value of the share = ₹150

The share is available at a premium of ₹50

Market value of each share = ₹(150 + 50) = ₹200

∴ Market value of 250 such shares = ₹(250 × 200) = ₹50,000.

EXAMPLE 21.3

Dhanik invests a certain amount in 300, ₹75 shares of a company paying 10% dividend. Find his annual income from this investment.

SOLUTION

Face value of the share = ₹75

Number of shares bought = 300

Dividend paid = 10%

Annual income from each share $= ₹\left[\frac{10}{100}(75)\right] = ₹7.50.$

∴ Annual income from 300 shares = ₹(7.50 × 300) = ₹2250.

EXAMPLE 21.4

Ameer invests ₹24,200 in buying ₹100 shares of a company available at a premium of 10%. If the company pays a dividend of 15%, then find the number of shares bought by Ameer, and the rate of return on his investment.

SOLUTION

Face value of each share = ₹100

The shares are available at a premium of 10%, i.e., 10% of 100 = ₹10.

Market value of each share = ₹(100 + 10) = ₹110

Money invested = ₹24,200

$$\text{Number of shares bought} = \frac{\text{Total investment}}{\text{Market value of each share}} = \frac{24200}{110} = 220$$

Dividend paid = 15%

Annual income from each share = 15% of 100 = ₹15

⇒ Annual income from 220 shares = ₹(220 × 15) = ₹3300

$$\text{The rate of return on his investment} = \frac{\text{Annual income}}{\text{Investment}} \times 100\% = \frac{3300}{24200} \times 100\% = \frac{150}{11}\% = 13\frac{7}{11}\%$$

∴ Ameer gets a return of $13\frac{7}{11}\%$ per annum on his investment.

EXAMPLE 21.5

Which is better investment: ₹300 shares at ₹320 that pays a dividend of 10%, or ₹200 shares at ₹215 that pays a dividend of 10%?

SOLUTION

Let the investment made in each case be ₹(320 × 215)

Case 1: ₹300 shares at ₹320 paying a dividend of 10%

Annual income from each share = 10% of 300 = $\frac{10}{100} \times 100$ = ₹30

$$\text{Number of shares bought} = \frac{\text{Total investment}}{\text{Market value of each share}} = \frac{320 \times 215}{320} = 215$$

⇒ Annual income from 215 shares = ₹(215 × 30) = ₹6450.

Case 2: ₹200 shares at ₹215 paying a dividend of 10%

Annual income from each share = 10% of 200 = $\frac{10}{100} \times 200$ = ₹20

$$\text{Number of shares bought} = \frac{\text{Total investment}}{\text{Market value of each share}} = \frac{320 \times 215}{215} = 320$$

∴ Annual income from 320 shares = ₹(320 × 20) = ₹6400.

So, it can be observed that the same amount of investment is made in each case, but the income is more in the first case.

∴ 10%, ₹300 shares at ₹320 is better investment.

Note Sometimes, in case of start-up companies, the entire amount of authorized capital is not required. In such a situation a company may collect a part of the capital from the shares. It is collected equally from all share-holders. This amount is called, 'paid-up capital'.

When more money is required, the company has the right to collect the remaining amount from the share holders. The paid-up amount of the shares is the paid-up value of the shares.

EXAMPLE 21.6

The capital of a company is ₹250,000. If this capital is raised by issuing shares of ₹8 each, then how many shares are there?

SOLUTION

$$\text{Number of shares} = \frac{\text{Authorized capital}}{\text{Face value of a share}} = \frac{250,000}{8} = 31,250.$$

EXAMPLE 21.7

A company has 500 shares of ₹25 each. If ₹15 is paid-up for each share, then what is the paid-up capital? Also, find the amount to be paid in the second investment.

SOLUTION

Paid-up amount per share = ₹15

Number of shares = 500

∴ Paid-up capital = ₹(500 × 15) = ₹7500

Total capital of the company = ₹(500 × 25) = ₹12,500

∴ Amount to be paid in the second instalment = ₹(12,500 − 7500) = ₹5000.

EXAMPLE 21.8

The authorized capital of a company is ₹400,000, and the number of shares is 800. What is the face value of each share? If the paid-up capital is ₹260,000, then what is the paid-up value of each share?

SOLUTION

Authorized capital = ₹400,000

Number of shares = 800

$$\therefore \text{ Face value per share} = ₹\frac{400,000}{800} = ₹500$$

Paid-up capital = ₹260,000

Number of shares = 800

$$\therefore \text{ Paid-up amount per share} = ₹\frac{260,000}{800} = ₹325.$$

EXAMPLE 21.9

Which of the following is the least attractive scheme?

(a) 5% of ₹120 shares at ₹150.

(b) 6% ₹105 shares at ₹140.

(c) 7% ₹90 shares at ₹125.

(d) 8% ₹80 shares at ₹108.

SOLUTION

Choice 1: Face value/share = ₹120

Dividend rate = 5%

Annual income/share $= \frac{5}{100}$ (₹120) = ₹6

Market value/share = ₹150

Rate of return $= \frac{6}{150}(100) = 4\%$

In a similar manner, the rates of return for the remaining choices can be worked out.

Choice 2: Annual income/share = ₹6.30

Rate of return $= \frac{6.3}{140}(100) = \frac{9}{2}\%.$

Choice 3: Annual income/share = ₹6.30

Rate of return $= \frac{6.3}{125}(100) = 5.04\%.$

Choice 4: Annual income/share = ₹6.40

Rate of return $= \frac{6.4}{108}(100) = 5\frac{25}{27}\%.$

The least attractive scheme is the scheme giving minimum rate of return.

∴ Choice (a) gives the minimum rate of return among all the choices.

EXAMPLE 21.10

Mohan invested ₹24,000 in 8% ₹400 shares at ₹320. He sold them at ₹360 after one year. Find the total profit he earned. (in ₹)

(a) 6540 **(b)** 5400 **(c)** 6600 **(d)** 7200

SOLUTION

Face value/share = ₹400

Dividend rate = 8%

Annual income/share = ₹32

Market value/share = ₹320

Number of shares bought $= \frac{24,000}{320} = 75$

∴ Profit earned (in ₹) = 75(360 – 320 + 32) = 5400.

EXAMPLE 21.11

Rahul bought 30 shares at ₹160 each. The par value of each share was ₹150. The dividend paid to him was at 6% per annum. By how much did his total investment exceed his total annual income? (in ₹).

(a) 4530 **(b)** 4430 **(c)** 4630 **(d)** 4330

SOLUTION

Face value/share = ₹150

Annual income/share = 6% (₹150) = ₹9

Total annual income = (30) (₹9) = ₹270

Total investment = (30) (₹160) = ₹4800

∴ The required value = ₹4800 – ₹270 = ₹4530.

EXAMPLE 21.12

Alex invested in the shares of a company. If the price of each share was ₹20 more, then his investment would be ₹6000 more. How many shares did he buy?

(a) 300 **(b)** 400 **(c)** 360 **(d)** 450

SOLUTION

Let us assume that Alex bought 'x' shares at a value of 'n'.

∴ The total investment on shares = ₹$x \cdot n$. The increased cost of a share = ₹$x + 20$.

∴ From the given problem, $(x + 20) \times n = xn + 6000$

$\Rightarrow 20n = 6000 \Rightarrow n = 300$.

∴ Total number of shares = 300.

EXAMPLE 21.13

Imran invested equal amounts in the shares of two companies A and B. A offered him a 4% return while B offered him a 6% return. Which of the following can be the effective rate of return he receives on his entire investment?

(a) 4% **(b)** 5% **(c)** 7% **(d)** 6%

SOLUTION

The effective rate of return must lie between 4% and 6%.

∴ Choice (b) can be the effective rate of return.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. A company has 5 lakh shares of ₹15 each. The capital of the company is _______.
2. If ₹150 share is quoted at a premium of ₹10, then the market value of the share is ₹_______.
3. The annual income derived from each ₹200 share at a premium of ₹30, paying 10% dividend is ₹_______.
4. A man invests ₹13,200 in a company to buy ₹55 shares. The number of shares he bought is ______.
5. A company is selling shares at ₹120 each. Each has a par value of ₹100. The premium percentage is _______.
6. A company is selling shares at ₹96 each. Each has a par value of ₹120. The discount percentage is _______.
7. Mukesh invests ₹12,000 in a company to buy ₹100 shares, paying 10% dividend. His annual income from the shares is _______.
8. Ratan had ₹2500 worth of shares. If he had 500 shares, the face value of each share is _______.
9. Pardhiv bought two types of shares A and B worth ₹5000 each. If he bought 25 type A shares and 20 type B shares, then the difference between their face value is _______.
10. The person who subscribes in shares is called a _______.
11. Face value and market value of a share are ₹100 and ₹120 respectively, and the rate of return is 10%. Then, dividend is _______ (in %).
12. The paid-up capital of a company is ₹50,000. If the company has shares of face value of ₹25 in which ₹20 is paid-up, then how many shares does it have? What is the authorized capital of the company?
13. A company has 500 shares of face value ₹50 in which ₹30 is paid-up. The company collects ₹12,000 as the second instalment. What is the paid-up value of each share now?
14. The authorized capital of a company is ₹150,000 and the number of shares is 500. What is the face value of each share? If the paid-up capital is ₹60,000, then what is the paid-up value of each share?
15. The authorized capital of a company is ₹50,000 and the number of shares is 1000. If the paid-up capital is ₹25,000, then the paid-up value of each share is ₹_______.

Short Answer Type Questions

16. When shares of face value ₹50 each are sold above par at ₹10 premium, then the money required to buy 30 such shares is _______.
17. A company has shares of face value ₹50 in which ₹25 is paid-up and the paid-up capital is ₹10,000. Then, the authorized capital is _______.
18. Ramesh bought 30 ₹200 shares of a company available at a premium of ₹25. Find the investment made by Ramesh, and also the rate of interest (return) he gets on his money if the company pays a dividend of 8% per annum.
19. Rakesh invested ₹27,000 in ₹27 shares of a company which pays a dividend of 10%. Find the market value of each share if he derives an annual income of ₹540 from this investment, and also find the number of shares he bought.
20. A ₹100 share is bought at a premium of ₹25. If the investment is worth 9% per annum, then find the rate at which the company pays the dividend.

Essay Type Questions

21. Ranvir gets 8% per annum, on his investment made in buying ₹80 shares of a company for ₹100 each. What is the rate of dividend, and what is his annual dividend if he purchases 500 such shares?
22. Determine the amount that is to be invested in ₹200 shares available at a premium of ₹40, so that the annual income earned is ₹4800 from the investment if the dividend offered is 12%.

23. ₹33,000 is to be divided into two parts, such that the income from one part invested in ₹200 shares at ₹250 for 10% dividend is the same as that from the other investment which was invested in ₹120 shares at ₹150 for 12% dividend. Determine the amounts invested in the two kinds of shares.

24. An investment in buying 400 shares of a company at a premium of ₹2.50 earns an income of 9.6% per annum. If the rate of dividend paid by the company is 12%, then find the face value of each share.

25. If the investment made in buying 150 shares of a company at ₹6 above par is ₹99,000, then find the face value of each share.

CONCEPT APPLICATION

Level 1

1. Amit bought 150 shares of ₹100 each. The paid-up value of each share is ₹60. Find the amount to be paid as a second instalment. (in ₹)

(a) 5000 (b) 6000
(c) 4500 (d) 7500

2. Lavan had two types of shares. He had 800 shares of type A which gave him an annual dividend of ₹4000. If he had 200 more shares of type B than A, which gave him the same annual dividend, then find his annual income from each share of B. (in ₹)

(a) 4 (b) 3.50
(c) 2 (d) 2.50

3. Prakash had ₹120 shares of worth ₹7200. If he sold each of them for ₹30 more than its face value, then find his revenue from sales. (in ₹)

(a) 7200 (b) 8000
(c) 9000 (d) 6400

4. Bhaskar bought ₹130 shares at a discount of ₹10. Find the number of shares he bought for ₹15,600.

(a) 120 (b) 130
(c) 140 (d) 150

5. Ashok bought ₹135 shares of a company at a premium. If the company provides the same premium amount as discount, then its market value will decrease by 50%. Find the premium. (in ₹)

(a) 40 (b) 45
(c) 50 (d) 55

6. The total annual dividend obtained by Ajay from the shares of a company was 25% of the total investment on them. Find the ratio of his investment on each share, and the dividend from each share.

(a) 3 : 1 (b) 4 : 1
(c) 5 : 1 (d) 6 : 1

7. Kalyan bought 600, ₹x shares at 20% premium from a company. If these had been bought at 20% discount, then his investment would have been ₹2400 less. How many ₹x shares have a total face value of ₹20,000?

(a) 2000 (b) 1600
(c) 1000 (d) 800

8. Aswin bought some shares of ₹75 from a company. He paid ₹50 as a paid-up value of each share, ₹32,000 as a paid-up capital. Find the total authorized capital of Aswin. (in ₹)

(a) 40,000 (b) 48,000
(c) 56,000 (d) 60,000

9. Shyam had some shares. He sold them at ₹12 discount. He realized ₹800 less than what he would have realized if he had sold them at ₹8 premium. Find the number of shares he sold.

(a) 40 (b) 50
(c) 60 (d) 80

10. Sashi bought ₹16 shares of a company at 25% dividend. Find the premium he paid for the shares (in ₹), if he received 20% rate of return.

(a) 4 (b) 3
(c) 6 (d) 5

11. Kiran bought ₹10 shares of a company. He received a rate of return which was one-third of

the dividend rate he received. Market value of each share is ______. (in ₹)

(a) 10 (b) $\frac{10}{3}$

(c) 30 (d) 15

12. A company sells two types of shares A and B. The market value of A is 50% more than that of B and is 25% less than the face value of A. The market value of A and face value of B are equal. By what percentage is the sum of the face values of both more than the sum of the market values of both?

(a) 30% (b) 45%

(c) 35% (d) 40%

13. Prasad invested ₹4000 to buy type A shares of a company. He invested in the same company to buy ₹4000 worth type B shares at face value. Market value of A equals face value of B. If he bought a total 64 shares of both, how many type A shares did he buy?

(a) 6 (b) 24

(c) 32 (d) 40

14. Kishore invested a certain sum of money in two types of shares A and B of a company. The market value of A is ₹30 more than that of B. He bought 400 shares of A and 300 shares of B. If he spent ₹18,000 more on A than B, then find the market value of each share of B. (in ₹)

(a) 30 (b) 40

(c) 50 (d) 60

15. Raju bought ₹40 shares of a company which had a market value of ₹60. He received a dividend of 30%. How much more (or) less would have been his annual dividend per share, if his dividend rate and rate of return were interchanged?

(a) ₹2 more per share.

(b) ₹3 more per share.

(c) ₹4 less per share.

(d) ₹6 less per share.

Level 2

16. Gopal bought two types of shares P and Q, of a company at their face values. The dividend rates provided by P and Q are 9% and 12%, respectively. Gopal received an annual dividend of ₹4500 more from P than from Q. Which of the following can be the ratio of his investments in P and Q?

(a) 6 : 5 (b) 5 : 4

(c) 4 : 3 (d) 2 : 1

17. If the rate of return and dividend from a share are r% and d% respectively, find the premium/discount %, given $d > r$.

(a) $\frac{(r-d)}{r}$% premium

(b) $\frac{(r-d)}{d}$% discount

(c) $\frac{(d-r)}{r}$% premium

(d) $\frac{(d-r)}{d}$% discount

18. If the rate of return from share P was 4% more, then the annual income from it would be ₹5 more. Find its market value. (in ₹)

(a) 125 (b) 100

(c) 120 (d) 150

19. If ₹4000 more was invested in share A, 20 more shares can be purchased. If ₹4000 less was invested in A, 20 less shares can be purchased. Find the market value of A. (in ₹)

(a) 100 (b) 150

(c) 200 (d) 300

20. If the price of share A was ₹10 more, 50 less shares can be purchased by investing ₹x. If the price of A was ₹20 less, 25 more shares can be purchased by investing ₹x. Find the ratio of the price of each share and the number of shares purchased.

(a) 3 : 5 (b) 5 : 3

(c) 2 : 5 (d) 5 : 2

21. Which of the following can be concluded from the information given below?

'Ajay bought, 12% ₹100 shares at ₹120'.

(a) Dividend per share = ₹12

(b) Rate of return = 10%

PRACTICE QUESTIONS

(c) Both (a) and (b)

(d) Neither (a) nor (b)

22. Hari invested ₹24,000 in buying shares worth ₹300 each. The dividend paid to him was ₹20 per share. Find his total annual income from the shares. (in ₹)

(a) 1200 (b) 2000

(c) 2400 (d) 1600

23. Shares worth ₹300 each were bought at a discount of ₹60. The dividend paid was 6% per annum. Find the rate of return.

(a) 5% per annum (b) 6% per annum

(c) 7.5% per annum (d) 8% per annum

24. Which of the following is the least attractive scheme?

(a) 4% ₹40 shares at ₹50.

(b) 5% ₹50 shares at ₹60.

(c) 6% ₹60 shares at ₹70.

(d) 7% ₹70 shares at ₹80.

25. Shares worth ₹200 each were bought at a premium of ₹40. The dividend paid was 8% per annum. Find the annual income per share. (in ₹)

(a) 16 (b) 12.6

(c) 8 (d) 19.2

Level 3

26. Rohit invested in two types of shares, P and Q, of a company. He purchased P at x% discount and Q at x% premium. If the total market value of each is equal, find the rate of effective discount (in per cent).

(a) 0.1% (b) $\dfrac{x^2}{400}$%

(c) $\dfrac{x^2}{200}$% (d) $\dfrac{x^2}{100}$%

27. Shares A and B have face values of ₹100 each. A is sold at ₹x premium, and B is sold at ₹x discount. The rate of return from each is x%. The sum of the annual dividends from both is ₹10. Find x.

(a) 4 (b) 8

(c) 5 (d) 10

28. Mahendar invested in three types of shares, P, Q and R, of a company. The total annual dividend he received from them was ₹1000. His investments in P, Q and R were ₹1000, ₹1000 and ₹3000, respectively. The rate of return he received from R was the average of the rates of return of the other two. Find the rate of return he received from R.

(a) 10% (b) 20%

(c) 15% (d) 25%

29. Goutham invested ₹4000 to buy ₹125 shares of type A from a company and invested ₹4800 to buy ₹120 shares of type B from it. He obtains 8% dividend from A and 10% dividend from B. Find the total annual dividend he carried if the market value of each share equals its face value. (in ₹)

(a) 600 (b) 750

(c) 800 (d) 1000

30. Ashok invested in three types of shares, P, Q and R, of a company. He invested ₹5000 and ₹7500 in P and Q, respectively. He obtained rates of returns of 10%, 8% and 15% from P, Q and R, respectively. His annual income from the three types was a total of ₹1550. How much did he invest in R? (in ₹)

(a) 2500 (b) 2000

(c) 3500 (d) 3000

31. If Lokesh's investment was ₹4800 more, his annual income would be ₹240 more. Find the rate of return.

(a) 4.5% (b) 4%

(c) 5.5% (d) 5%

32. In the previous question, the rate of return is

(a) equal to the rate of dividend.

(b) greater than the rate of dividend.

(c) less than the rate of dividend.

(d) Either (a) or (b)

33. Kiran invests in the shares of a company. If he buys 48 shares more, he will invest ₹1200 more. If he buys 40 shares less, the total face value of his shares will be ₹800 less. He must buy each share _______.

(a) at par (b) below par

(c) above par (d) Cannot be determined

34. Rohit invested ₹39,600 in buying shares of nominal value of ₹100 at a 20% premium. The dividend paid to him was at 6% per annum. Find his annual income. (in ₹)
(a) 1950 (b) 1980
(c) 1920 (d) 2010

35. In the previous question, if Rohit bought all shares at par, then find the rate at which the dividend was paid to him.
(a) 4.5% (b) 4%
(c) 5.5% (d) 5%

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. ₹7,500,000
2. ₹160
3. ₹20
4. 240
5. 20%
6. 20%
7. ₹1200
8. 5
9. ₹50
10. shareholder
11. 12%
12. ₹62,500
13. ₹54
14. ₹120
15. 25

Short Answer Type Questions

16. ₹1800
17. ₹20,000
18. ₹6750; 7.11%
19. ₹135; 200
20. 11.25%

Essay Type Questions

21. 10%; ₹4000
22. ₹48,000
23. ₹18,000; ₹15,000
24. ₹10
25. ₹654

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (a)
11. (c)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (c)					

Level 2

16. (d)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (a)

Level 3

26. (d)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (d)	**31.** (d)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (d)

CONCEPT APPLICATION

Level 1

1. Find the paid-up capital.

2. (i) Annual dividend from each share $= \dfrac{\text{Total annual dividend}}{\text{Total number of shares}}$.

   (ii) Number of shares of type B = (Number of shares of type A) + 1200.

   (iii) Total dividend on shares of type B = Total dividend on shares of type A = ₹4000.

   (iv) Annual income from each share of type $B = \dfrac{\text{Total dividend for type B}}{\text{Number of shares of type B}}$.

3. Find the number of shares.

4. Find the market value of each share.

5. Let premium be ₹x. Find the market value of a share.

6. $\dfrac{\text{Investment on each share}}{\text{Dividend from each share}} = \dfrac{\text{Total investment}}{\text{Total dividend}}$

7. Find the market of a share in each case.

8. (i) Number of shares $= \dfrac{\text{Paid-up capital}}{\text{Paid-up value of each share}}$.

   (ii) Number of shares $= \dfrac{\text{Total paid-up value}}{\text{Paid-up value per share}}$.

   (iii) Total capital = (Number of shares) (₹75).

9. Change in the price of a share is ₹20.

10. (i) Find the annual dividend from each share.

    (ii) First of all, find the market value of each share.

    (iii) Use, FV × Rate of dividend (RO) = MV × Rate of return (RR).

    (v) Premium = MV − FV.

11. (i) Rate of dividend × Face value = Rate of return × Market value.

    (ii) Let the rate of dividend be x%.

    (iii) Now, find the rate of return in terms of x.

    (iv) Then, find the MV by using the formula, FV × RD = MV × RR.

12. (i) Find the face values of A and B.

    (ii) Let the MV of each share of type B be ₹x.

    (iii) Now, MV of each share of type A be 150% of x.

    (iv) FV of each share of type B = MV of each share of type A.

    (v) Similarly find FV of each share of type A.

14. Difference of the investments is ₹18,000.

15. (i) Find the dividend from each share and rate of return.

    (ii) First of all, find rate of return by using the following formula: FV × Rate of dividend = MV × Rate of return.

    (iii) Let x be the rate of return. Now, find 30% of 40 − x% of 40.

Level 2

16. (i) The ratio of rate of dividends on P and Q is 3 : 4.

    (ii) If the ratio of investments on P and Q is 4 : 3, dividend is same on both P and Q.

    (iii) Therefore, required investment must be more than 4 : 3 for the given condition.

    (iv) Check the options to get the ratio which is more than 4 : 3.

17. (i) Annual dividend $= \dfrac{d}{100} \times$ share value $= \dfrac{r}{100} \times$ market value.

    (ii) As $d > r$, shares are brought at premium.

    (iii) Let the income from each share be $d = \dfrac{i}{\text{FV}} \times 100$ and $r = \dfrac{i}{\text{MV}} \times 100$.

    (iii) Premium = MV − FV. Percentage of premium $= \dfrac{\text{MV} - \text{FV}}{\text{FV}} \times 100$.

18. (i) Rate of dividend × Face value = Rate of return × Market value.

HINTS AND EXPLANATION

(ii) Let the market value be x and rate of return be y.

(iii) $x(y + 4)\% - xy\% = 5$.

19. Obtain the relation between the amount invested and the number of shares.

20. His investment in each case is same, i.e., ₹x.

21. From the given information, dividend rate = 12%, face value/share = ₹100 and market value/share = ₹120.

Dividend = 12% (₹100) = ₹12

∴ Choice (a) can be concluded.

Rate of return $= \frac{12}{120}(100)\% = 10\%$

Choice (b) can be concluded. Choice (c) follows.

22. Total investment = ₹24,000

Cost of each share = ₹300

Number of shares bought $= \frac{24000}{300} = 80$

Annual income/share = ₹20

Total annual income (in ₹) = (80)(20) = 1600.

23. Face value of each share = ₹300

Discount/share = ₹60

Market value of each share (in ₹) = 300 – 60 = 240

Dividend rate = 6% per annum

Dividend/share = 6% of (₹300) = ₹18

Let the rate of return be r%.

$r = \frac{18}{240}(100) = 7.5\%$.

24. Rate of return $= \frac{\text{Face value} \times \text{Rate of dividend}}{\text{Market value}}$

Choice (a): Rate of return $= \frac{40 \times 4\%}{50}$

$= \frac{16}{5}\%$.

Choice (b): Rate of return $= \frac{50 \times 5\%}{60}$

$= \frac{25}{6}\%$.

Choice (c): Rate of return $= \frac{60 \times 6\%}{70} = \frac{36}{7}\%$.

Choice (d): Rate of return $= \frac{70 \times 7\%}{80}$

$= \frac{49}{8}\%$.

∴ The least attractive scheme is choice (1), since it the yields the least rate of return.

25. Face value of each share = ₹200

Dividend rate = 8% per annum

∴ Annual income per share

= 8% of (₹200) = ₹16.

Level 3

26. (i) Effective discount $= \frac{\text{Total discount}}{\text{Total market value}} \times 100$.

(ii) Let the market value of each share be m.

(iii) $FV_1 = \frac{100m}{100 + x}$ and $FV_2 = \frac{100m}{100 - x}$

(iv) Find the total face value which is less than the total market value.

(v) Effective discount

$= \frac{(\text{Total FV} - \text{Total MV})}{\text{Total FV}} \times 100$.

27. (i) Find the annual dividend from A and B.

(ii) MV of share A = 100 + x.

MV of share B = 100 − x

(iii) Dividend from share A = x% of (100 + x), where x% is the rate of return and (100 + x) is the MV.

(iv) Similarly, find dividend from share B.

(v) Equate total dividend in x to 10.

HINTS AND EXPLANATION

28. Let the rates of returns from P, Q and R be $x\%$, $y\%$ and $z\%$ and $\frac{x+y}{2} = z$.

29. (i) Find the dividend received per share in each case.

(ii) Total dividend from share A is 8% of ₹4000.

(iii) Total dividend from share B is 10% of ₹4800.

30. (i) Let the investment of Ashok be ₹x in R and equate the total dividend to ₹1550.

(ii) Let the investment on share R be ₹x.

(iii) 10% of 5000 + 8% of 7500 + 15% of x = 1550.

31. Let the rate of return that Lokesh gets be $r\%$. The extra annual income of Lokesh = $r\%$ of (his extra investment).

$\therefore 240 = \frac{r}{100}(4800)$

$\therefore r = 5\%$.

32. Since he has to spend more than the face value to purchase each share, $r\% < d\%$.

33. Market value of each share (in ₹) $= \frac{1200}{48} = 25$

$\therefore\ m = 25$

Face value of each share (in ₹) $= \frac{800}{40} = 20$

$\therefore f = 20$

$\therefore\ m > f$

∴ Kiran bought each share above par.

34. Face value/share = ₹100

Premium/share = 20% of ₹100 = ₹20

Market value/share (in ₹) = 100 + 20 = 120

Total market value = ₹39,600

Number of shares bought $= \frac{39,600}{120} = 330$

Dividend rate = 6% per annum

Annual income/share $= \frac{6}{100}$ (₹100)

= ₹6

Total annual income (in ₹) = (330)(6)

= 1980.

35. Whenever a person buys all shares at par, the rate at which the dividend is paid to him is the rate of return that he receives.

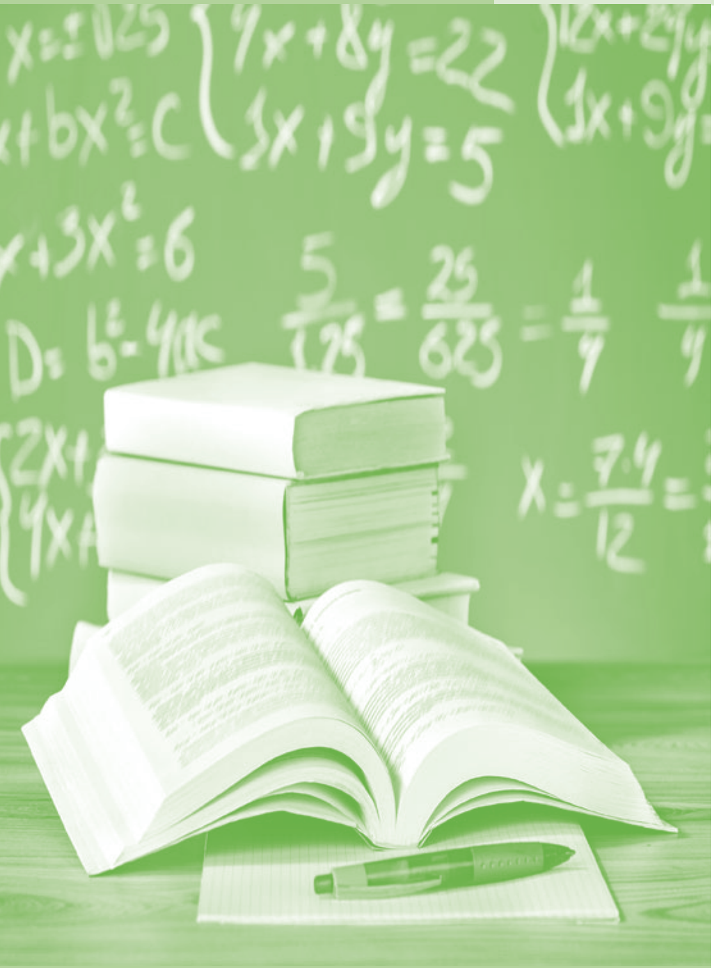
∴ Rohit would be paid a dividend at 5%.

HINTS AND EXPLANATION

Chapter

22

Time and Work



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the time taken for each work we do
- Know basic concepts of work
- Explore the idea of increase in work-load in shorter time durations

KEY IDEAS

After completing this chapter, you should be able to:

- Know the fundamental assumptions that are made while solving the problems on time and work
- Understand the concept of man-days
- Know about the sharing of the money earned
- Find the time taken by pipes and cisterns to fill a tank

INTRODUCTION

In this chapter, we will look at the relation between the amount of work done, the time taken to do that work and the number of people and their rates of doing work. The work may be constructing a wall or a road, filling up or emptying a tank (or a cistern) or eating a certain amount of food. This could be measured in any convenient unit. Quite often, the entire work mentioned can be taken as 1 unit. But this is not necessary. Time is measured in days, hours, etc.

There are some basic assumptions that are made while solving the problems on Time and Work. These are taken for granted and are not specified in every problem.

1. If a person (or one member of the workforce) does some work in a certain number of days, then we assume (unless otherwise explicitly stated in the problem) that he/she does the work uniformly, i.e., he/she does the same amount of work everyday. For example, if a person can complete a work in 15 days, we assume that he completes $\frac{1}{15}$ of the work in one day. If a person completes a piece of work in 4 days, we assume that he/she completes $\frac{1}{4}$ of the work on each day and conversely, if a person can complete $\frac{1}{4}$ of the work in one day, we assume that he can complete the total work in 4 days.
2. If there is more than one person (or members of 'workforce') carrying out the work, it is assumed that each person (or members of the workforce), unless otherwise specified, does the same amount of work each day. This means that they share the work equally. If two people together can do the work in 8 days, it means that each person can complete it in 16 days. This, in turn, means that each person can do $\frac{1}{16}$ of the work per day.

In a number of problems, we find people working at different rates. We consider the following two examples:

1. If two persons A and B can individually do some work in 20 days and 30 days respectively, we can find out how much work can be done by them together in one day. Since A can do $\frac{1}{20}$ part of the work in one day and B can do $\frac{1}{30}$ part of the work in one day, both of them together can do $\left[\frac{1}{20}+\frac{1}{30}\right]$ part of the work in one day.
2. A man works three times as fast as a boy does and the boy takes 12 days to complete a certain piece of work. If the boy takes 12 days to complete the work, then the man takes only 4 days to complete the same work. Per day the boy does $\frac{1}{12}$ of the work and the man does $\frac{1}{4}$, i.e., $\frac{3}{12}$ of the work. If they work together, each day, $\left(\frac{1}{12}+\frac{3}{12}\right)$ or $\frac{1}{3}$ of the work gets done or it takes 3 days for the work to be completed.

From the above two examples, we can arrive at the following conclusion/formula.

If A can do a piece of work in p days and B can do it in q days, then A and B together can complete the same in $\frac{pq}{p+q}$ days.

We should recollect the fundamentals of variation (direct and inverse) here.

1. If the number of days is constant, work and men are directly proportional to each other, i.e., if the work to be done increases, more men are required to complete the work in the same number of days.
2. If the number of men is constant, work and days are directly proportional, i.e., if the work increases, more days are required to complete the work.
3. If the work is constant, the number of men and days are inversely proportional, i.e., if the number of men increases, fewer days are required to complete the same work and vice-versa.

The concept of man-days is very important and useful here. The number of men multiplied by the number of days for which they work is a measure of the work in man-days. The total number of man-days representing a specific task is constant. So, if we change one of the variable—men or days, then the other will change accordingly, so that their product remains constant (remember from our knowledge of variation, two variables whose product is a constant, are said to be inversely proportional to each other). The two variables 'men' and 'days' are inversely proportional to each other, if the work to be done remains constant.

EXAMPLE 22.1

If 20 men take 30 days to complete a job, in how many days can 25 men complete the job?

SOLUTION

If 20 men can complete the job in 30 days, then the work is $20 \times 30 = 600$ man-days.

If this work is to be done by 25 men, then the number of days they take is $\frac{600}{25} = 24$.

EXAMPLE 22.2

Fifteen men take 10 days to complete a job working 12 hours a day. How many hours a day should 10 men work to complete the same job in 20 days?

SOLUTION

Since 15 men take 10 days working 12 hours per day, the total work done measured in terms of man-hours is $15 \times 10 \times 12$.

When 10 men are required to complete the same job in 20 days working h hours a day, work done $= 10 \times 20 \times h$. But $10 \times 20 \times h = 15 \times 10 \times 12$. Hence, the number of hours for which they should work per day is $\frac{15 \times 10 \times 12}{10 \times 20} = 9$.

$\therefore$ 10 men can complete the work in 20 days working for 9 hours per day.

Hence, in general we can say that:

If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day (where all men work at the same rate), then $\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$.

EXAMPLE 22.3

A piece of work can be done by 16 men in 8 days working 12 hours a day. How many men are needed to complete another work, which is three times the first one, in 24 days working 8 hours a day?

SOLUTION

Using the formula, $\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$.

Let $W_1 = x$ and $W_2 = 3x$.

$M_1 = 16,\ H_1 = 12,\ D_1 = 8$

$H_2 = 8,\ D_2 = 24.$

$$\therefore \frac{16 \times 8 \times 12}{x} = \frac{M_2 \times 24 \times 8}{3x}$$

$$\Rightarrow M_2 = 24.$$

$\therefore$ Required number of men is 24.

EXAMPLE 22.4

A can do a piece of work in 9 days and B can do the same in 12 days. In how many days can the work be completed if A and B work together?

SOLUTION

One day work of A and B $= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$.

So, they can complete the work in $\frac{36}{7}$, i.e., $5\frac{1}{7}$ days.

EXAMPLE 22.5

A and B working together can do a piece of work in 12 days and A alone can complete the work in 18 days. How long will B alone take to complete the job?

SOLUTION

In a day, A and B together can do $\frac{1}{12}$ of the work. In a day, A alone can do $\frac{1}{18}$ of the work.

$\therefore$ In one day, work done by B alone is $= \frac{1}{12} - \frac{1}{18} = \frac{1}{36}$.

$\therefore$ B alone can complete the work in 36 days.

EXAMPLE 22.6

A and B working together can do a piece of work in 12 days, B and C can do it in 15 days and C and A can do the same work in 20 days. How long would each of them take to complete the job?

SOLUTION

Work done by A and B in 1 day $= \dfrac{1}{12}$.

Work done by B and C in 1 day $= \dfrac{1}{15}$.

Work done by C and A in 1 day $= \dfrac{1}{20}$.

Adding all the three, we get the work done by

2(A + B + C) in 1 day $= \dfrac{1}{12} + \dfrac{1}{15} + \dfrac{1}{20} = \dfrac{1}{5}$.

$\therefore$ A, B and C can together finish $\dfrac{1}{10}$ of the work in 1 day.

$\therefore$ Work done by A in 1 day

= Work done by A, B and C in 1 day − Work done by B and C in 1 day.

$= \dfrac{1}{10} - \dfrac{1}{15} = \dfrac{1}{30}$.

$\therefore$ A alone can do it in 30 days.

Work done by B in 1 day $= \dfrac{1}{10} - \dfrac{1}{20} = \dfrac{1}{20}$.

$\therefore$ B alone can do it in 20 days.

Work done by C in 1 day $= \dfrac{1}{10} - \dfrac{1}{12} = \dfrac{1}{60}$.

$\therefore$ C alone can do it in 60 days.

EXAMPLE 22.7

To complete a certain work, C working alone takes twice as long as A and B working together. A working alone takes 3 times as long as B and C working together. All the three together can complete the work in 5 days. How long would each take to complete the work individually?

SOLUTION

Given that, three times A's daily work = (B + C)'s one day's work.

$\Rightarrow$ (A + B + C)'s daily work = four times A's daily work.

But (A + B + C)'s daily work $= \dfrac{1}{5}$.

$\therefore$ A's daily work $= \dfrac{1}{20}$; A takes 20 days to do the work.

Also, given two times C's daily work = (A + B)'s daily work

$\Rightarrow$ three times C's daily work = (A + B + C)'s daily work

But (A + B + C)'s daily work $= \dfrac{1}{5}$.

$\therefore$ C's daily work $= \frac{1}{15}$; C takes 15 days to do the work.

$\therefore$ B's daily work $= \frac{1}{5} - \left(\frac{1}{20} + \frac{1}{15}\right) = \frac{1}{12}$.

$\therefore$ A, B and C working alone can complete the work in 20 days, 12 days and 15 days respectively.

EXAMPLE 22.8

If 4 men or 5 women can construct a wall in 82 days, then how long will it take for 5 men and 4 women to do the same work?

SOLUTION

Given $4m = 5w$, where m is the work done by one man in one day and w is the work done by one woman in one day.

$$\Rightarrow 1m = \frac{5w}{4}.$$

Now, $5m + 4w = 5\left(\frac{5w}{4}\right) + 4w = \frac{41w}{4}$.

If $5w$ can do the work in 82 days, then $\frac{41w}{4}$ can do in $\left(5w \times 82 \times \frac{4}{41w}\right)$ days, i.e., 40 days.

EXAMPLE 22.9

While 4 men and 6 boys can do a piece of work in 2 days, 1 man and 3 boys can do the same work in 6 days. In how many days can 1 man and 1 boy complete the same work?

SOLUTION

Let the capacities of one man and one boy be m and b respectively.

Given,

$$4m + 6b = \frac{1}{2}$$

That is, $2m + 3b = \frac{1}{4}$ (1)

Also, $m + 3b = \frac{1}{6}$ (2)

Eqs. (1) − (2) gives, $m = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

On substituting 'm' in Eq. (2), we have $b = \frac{1}{36}$.

$$m + b = \frac{1}{12} + \frac{1}{36} = \frac{1}{9}.$$

$\therefore$ 1 man and 1 boy can complete the work in 9 days.

EXAMPLE 22.10

If X works 3 times as fast as Y and is able to complete a work in 40 days less than the number of days taken by Y, then find the time in which they can complete the work working together.

SOLUTION

If Y does the work in 3 days, X does it in 1 day, i.e., the difference is 2 days. But the actual difference is 40 days.

If difference is 2 days, X takes 1 day and Y takes 3 days. If difference is 40 days (i.e., 20 times), X takes 20 days and Y takes 60 days.

$\therefore$ Time taken together $= \dfrac{20 \times 60}{20 + 60} = 15$ days.

EXAMPLE 22.11

A and B can do a piece of work in 10 days and 15 days, respectively. They started the work together but B left after sometime and A finished the remaining work in 5 days. After how many days (from the start) did B leave?

SOLUTION

A's work for 5 days $= 5 \times \dfrac{1}{10} = \dfrac{1}{2}$ work.

The remaining half of the work was done by A and B together.

Work done by A and B in a day $= \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{1}{6}$.

$\therefore$ The number of days they worked together $= \dfrac{\frac{1}{2}}{\frac{1}{6}} = 3$ days.

Hence, B left after 3 days from the day the work started.

EXAMPLE 22.12

A and B can do a piece of work in 6 days and 9 days respectively. They work on alternate days starting with A on the first day. In how many days will the work be completed?

SOLUTION

Since they work on alternate days, let us consider a period of two days.

In a period of two days, work done by A and B $= \dfrac{1}{6} + \dfrac{1}{9} = \dfrac{5}{18}$.

If we consider three such time periods (we consider 3 periods because in the fraction $\dfrac{5}{18}$, the numerator 5 goes **three times** in the denominator 18).

Work done $= 3 \times \left(\dfrac{5}{18}\right) = \dfrac{15}{18}$.

Remaining work $1-\frac{15}{18}=\frac{3}{18}=\frac{1}{6}$.

Now it is A's turn, since 3 whole number of periods are over.

Time taken by A to finish $\frac{1}{6}$ of the work is one day.

So, total time taken = $(3 \times 2) + 1 = 7$ days.

EXAMPLE 22.13

P, Q and R can complete a job in 20 days, 30 days and 40 days respectively. They started working on it. Q and R left after working for certain number of days and P alone completed the remaining work 7 days. Find the time taken to complete the job (in days).

(a) 12 **(b)** 13 **(c)** 14 **(d)** 11

SOLUTION

Let the time taken to complete the job be t days.

Job = $(t-7)$ days of work of P, Q and R + 7 days of work of

$$P=(t-7)\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{40}\right)+7\left(\frac{1}{20}\right)$$

$$(t-7)\left(\frac{13}{120}\right)+\frac{7}{20}=1 \Rightarrow \frac{(t-7)(13)+42}{120}=1$$

$$13(t-7)=78 \Rightarrow t=13.$$

EXAMPLE 22.14

Y is thrice as efficient as Z who is half as efficient as X. X, Y and Z can complete a job in 25 days. Find the time in which X and Y can complete it (in days).

(a) 30 **(b)** 36 **(c)** 40 **(d)** 45

SOLUTION

Let the time in which X, Y and Z can complete the job be x days, y days and z days respectively.

$\frac{1}{y}=3\left(\frac{1}{z}\right)$ and $\frac{1}{z}=\frac{1}{2}\left(\frac{1}{x}\right)$.

$\therefore y=\frac{z}{3}$ and $x=\frac{z}{2}$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{25}$$

$$\frac{2}{z}+\frac{3}{z}+\frac{1}{z}=\frac{1}{25} \Rightarrow z=150.$$

$\therefore y=50$ and $x=75$.

X and Y can complete it in $\frac{(75)(50)}{75+50}$ days = 30 days.

Sharing of the Money Earned

When a group of people do some work together and earn some money together for doing that work, this money has to be shared by all these people.

In general, money earned is shared by people who worked together in the ratio of the **total work** done by each of them.

For example, let A and B complete a work. If A does $\frac{2}{5}$ of the work, then he should get $\frac{2}{5}$ of the total earnings due to the work. The remaining $\frac{3}{5}$ of the work is done by B, then the remaining $\frac{3}{5}$ of the earnings should be paid to B.

When people work for the same number of days each, then the ratio of the total work done will be the same as the work done by each of them per day. Hence, if all the people involved, work for the same number of days, then the earnings can directly be divided in the ratio of **work done per day** by each of them.

EXAMPLE 22.15

A, B and C can do a piece of work in 4 days, 5 days and 7 days respectively. They get ₹415 for completing the job. If A, B and C have worked together to complete the job, what is A's share?

SOLUTION

Since they work for the same number of days, the ratio in which they share the money is the ratio of the work done per day.

That is, $\frac{1}{4}:\frac{1}{5}:\frac{1}{7} = 35:28:20.$

Hence, A's share is $\left(\frac{35}{83}\right) \times 415 =$ ₹175.

EXAMPLE 22.16

The ratio of the efficiencies of X, Y and Z is 3 : 4 : 5. The total amount of wages of X, Y and Z working for 20 days, 10 days and 12 days respectively was ₹6400. Find the amount of their total wages, if X, Y and Z worked for 30 days, 15 days and 8 days respectively (in ₹).

(a) 5700 **(b)** 9500 **(c)** 7600 **(d)** 8550

SOLUTION

Ratio of daily wages of X, Y and Z = ratio of the efficiencies of x, y and $z = 3 : 4 : 5$.

Let the daily wages of X, Y and Z be ₹$3w$, ₹$4w$ and ₹$5w$ respectively.

$20(3w) + 10(4w) + 12(5w) = 6400$

$160w = 6400$

$w = 40.$

$\therefore$ Required total wage (in ₹) $= 30(3w) + 15(4w) + 8(5w) = 190w = 7600.$

PIPES AND CISTERNS

There can be pipes (or taps) filling or emptying tanks with water. The time taken by different taps (to fill or empty the tank) may be different. Problems related to these can also be dealt with in the same manner as the **problems on work** have been dealt with, so far in this chapter.

There is only one difference between the **problems on regular** work (of the type seen earlier on in the chapter) and those in **pipes and cisterns**. In pipes and cisterns, a filling pipe or tap does positive work and an emptying pipe or a 'leak' does negative work.

EXAMPLE 22.17

Two pipes A and B can fill a tank in 12 minutes and 18 minutes respectively. If both the pipes are opened simultaneously, how long will they take to fill the tank?

SOLUTION

The part of the tank filled by A in 1 minute $= \frac{1}{12}$.

The part of the tank filled by B in 1 minute $= \frac{1}{18}$.

The part of the tank filled by both the pipes in 1 minute $= \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$.

$\therefore$ The tank can be filled in $\frac{36}{5} = 7\frac{1}{5}$ minutes.

EXAMPLE 22.18

Pipe A can fill a tank in 12 minutes, pipe B in 18 minutes and pipe C can empty the full tank in 36 minutes. If all of them are opened simultaneously, find the time taken to fill the empty tank.

SOLUTION

The work done by the 3 pipes together in 1 minute $= \frac{1}{12} + \frac{1}{18} - \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$.

So, the empty tank will be filled in 9 minutes.

EXAMPLE 22.19

Two taps P and Q which can independently fill a tank in 15 hours and 30 hours respectively were opened simultaneously, when the tank was empty. When the tank was half-full, an emptying tap R was opened. If the emptying tap R can empty the full tank in 20 hours, then in how much time will the tank be completely filled?

(a) 12 hours **(b)** 15 hours **(c)** 18 hours **(d)** 20 hours

HINTS

(i) Half of the tank is filled in 5 hours
(ii) Let the time taken to fill the tank be x hours

Use, $\frac{x}{15} + \frac{x}{30} - \frac{(x-5)}{20} = 1$ and find x.

EXAMPLE 22.20

Ten pipes are fitted to a tank. Some of these are filling pipes while the others are emptying pipes. Each filling pipe can fill the tank in 8 hours. Each emptying pipe can empty it in 16 hours. All the pipes are opened simultaneously. The tank takes 2 hours to be filled. How many filling pipes are fitted to it? Choose the correct answer from the following option.

(a) 8 **(b)** 7 **(c)** 9 **(d)** 6

SOLUTION

Let the number of filling pipes be f.

Number of emptying pipes = $10 - f$

Part of the tank filled each hour $= \frac{1}{2}$.

$$\therefore \frac{1}{8}f - \frac{1}{16}(10 - f) = \frac{1}{2}$$

$$\frac{1}{8}f - \frac{5}{8} + \frac{f}{16} = \frac{1}{2}$$

$$\frac{3f}{16} = \frac{9}{8} \Rightarrow f = 6.$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. A can read a book in x minutes. What part of the book can he read in k minutes?
2. Work done by A in 3 days is equal to the work done by B in 4 days. What is the ratio of time taken by A and B to complete a work?
3. A can complete $\frac{2}{3}$ of work in 6 days. In what time can he complete $\frac{4}{9}$ of the work?
4. A can do a work in x days and B can do the same work in y days. If $x > y$, then who can do more work in 6 days?
5. If x men can do a work in y days, $2x$ men can do the same work in $2y$ days. (True/False)
6. A is twice as good workman as B. A takes 6 days to complete a work. In what time can B complete the work?
7. A and B together can do a work in 20 days, B and C can do the same work in 60 days. Who is the most efficient among A, B and C?
8. A and B completed a work and divided the earnings in the ratio of 2 : 3. What is the ratio of time taken to complete the work by A and B, respectively?
9. 8 men can knit 8 baskets in 8 days. In how many days can 3 men knit 3 baskets?
10. 1 man can load 1 box in a truck in 5 minutes. How many full trucks can 8 men load in 45 minutes given that the truck can hold 10 boxes?
11. Pipe A can fill a tank in 40 minutes. Pipe B can empty the tank in 30 minutes. If both the pipes are opened simultaneously when tank is half full, then will the tank become full or empty?
12. Pipe A can fill a tank in 2 hours at the rate of 6 litres per minute. What is the capacity of the tank?
13. If A and B can do a work in x and y days, respectively, then the time taken by A and B together to complete the work is $\frac{x+y}{2}$. (True/False)
14. A can do a work in 18 days. He worked for 10 days and left. The remaining work is completed by B and C. If they got ₹900 for completing the work, then who receives more share in the earnings?
15. 12 men or 18 women can do a work in 16 days. In how many days 12 men and 18 women can do the work?
16. A, B and C can do a work in x, $3x$ and $5x$ days, respectively. They worked together and earned ₹460. What is the share of B?
17. A and B can do a work in 20 and 30 days, respectively. They completed the work in 10 days with the help of C. In how many days can C alone do the work?
18. A can paint 6 walls in 5 days. B can paint 8 walls (of the same area) in 4 days. Working together, in how many days can they paint 48 walls (of the same area)?
19. The ratio of efficiencies of A, B and C is 2 : 3 : 4. Working together, they can complete a work in 20 days. In how many days can C alone finish the work?
20. On working 10 hours a day, 15 men can complete a piece of work in 20 days. In how many days can 30 men complete it if they work 5 hours per day?
21. A and B can do a work in 20 days, B and C in 40 days, and A and C in 30 days. Who is the most efficient among the three persons?
22. A and B together can do a work in 18 days. A worked for 12 days with B and left the work. B attended the work for 27 days and by that time the work gets completed. In how many days can B alone complete the work.
23. Work done by $(x + 4)$ men in $(x + 5)$ days is equal to the work done by $(x - 5)$ men in $(x + 20)$ days. What is the value of x?
24. Pipe A can fill a tank in 15 hours Pipe A and Pipe B together can fill it in 10 hours Pipe B can fill 8 litres per minute. Find the capacity of the tank.
25. 18 men can construct 200 m long wall in 16 days working 9 hours per day. In how many days can 36 men construct 500 m long wall working 6 hours per day?

PRACTICE QUESTIONS

Short Answer Type Questions

26. Vinay and Vikram can complete a work in 48 and 36 days, respectively. If they work on alternate days beginning with Vinay, in how many days will the work be completed?

27. A is twice as efficient as B. Time taken by B to complete $\frac{4}{7}$ part of the work is 6 days more than the time taken by A to complete $\frac{3}{7}$ part of the same work. In how many days can A alone do the work?

28. The ratio of work efficiency of a man and a woman is 3 : 2. A woman can do a work in 30 days. In how many days can 4 men and 4 women do the work?

29. 12 men or 15 women can do a work in 40 days. 8 men and 10 women can do the work in how many days?

30. 12 men and 15 women can do a work in 6 days, and 6 men and 12 women can do it in 10 days. In how many days can 8 men and 10 women do the same work?

31. Certain sum is sufficient to pay wage to A for 15 days or to B for 30 days. For how many days is the sum sufficient to pay both A and B?

32. A, B and C can do a work in 20, 25 and 30 days, respectively. They together started the work, but B and A left the work 7 days and 4 days, respectively before completion of the work. How many days did C work for?

33. Anand is 50% more efficient than Bharath and Bharath is 100% more efficient than Chandu. Working together, they can complete a work in 10 days. In how many days can Anand alone do the work?

34. Three hundred men can construct a 200 m long wall in 12 days working 8 hours a day. In how many days can 120 men construct 180 m long wall working 9 hours per day?

35. A, B and C can complete a piece of work in 4 days, 8 days and 12 days, respectively. They worked together and earned a total of ₹660. Find B's share (in rupees).

36. Thirty men can dig a well in 10 days on working 8 hours a day. They started the work but after 2 days, 10 men left. How many hours a day should the remaining men work to complete it on time?

37. Time taken by A to complete a work is 7 days more than the time taken by B to complete the same work. Working together they can complete the work in 12 days. In what time can A alone complete the work?

38. A is 20% more efficient than B. B can do a piece of work in 20 days. In how many days can A do the same work?

39. Three men or five women can complete a piece of work in 8 days. In how many days can 6 men and 6 women complete it?

40. A certain number of men completed a job in 10 days. If there were 5 more men, it could have been completed in 8 days. How many men will be required to complete it in 5 days?

41. Two pipes A and B can fill a tank in 20 and 24 minutes, respectively. Pipe C can empty the tank in 10 minutes. Pipes A and B are opened for 5 minutes and then pipe C is also opened. In what time can the tank become full or empty after opening pipe C?

42. A can complete a piece of work in 15 days. B can complete it in 10 days. With the help of C, they can complete it in 4 days. They all worked together and earned a total of ₹750. Find C's share.

43. Time taken by 8 men and 6 boys is 3 times the time taken by 15 men and 30 boys to complete a work. If 8 men and 12 boys can do a work in 34 days, then how many men can do it in 17 days?

44. Prakash, Rohit and Sameer can complete a job in 1 day, 20 days and 10 days, respectively. They started the job but Prakash was unwell on the first day, he could not work at his full capacity and Prakash left after a day. The other two completed the job. Rohit was paid ₹17,000 out of the total of ₹60,000 paid to them. At what percentage of his full capacity, did Prakash work on the first day?

45. Pipe A and pipe B can fill a tank in 24 minutes and 28 minutes, respectively. If both the pipes are opened simultaneously, then after how many minutes should pipe B be closed such that the tank becomes full in 18 minutes?

Essay Type Questions

46. In a group, there were M men. They started working on a job of 330 units. Each man could do 1 unit in 1 day. After each day of work, a man of the same efficiency as each man in the group join the group. The job was completed at the end of 11 days. Find M.

47. Ram and Shyam can complete a job in 20 days if they work together. If they complete it by working on alternate days, in how many days will it be completed?

48. A, B and C can complete a job in 20 days, 30 days and 40 days, respectively. They worked for 6 days and then A left. B and C completed the job. Find B's wages in the total wages of ₹20,000 paid to them. (in ₹)

49. 10 women can complete a job in 12 days. 4 of them started working on the job. After every 4 days, a woman joined the group. Every woman joining the group has equal capacity as any woman in the group. Find the time taken to complete the job. (In days).

50. Prakash, Ramesh and Suresh started a job. After 2 days, Prakash left. After another 2 days Ramesh left. In another 2 days, Suresh completed the remaining part of the job. If Prakash can complete the same work in more than 6 days and Ramesh can complete it in more than 12 days, who got the highest share of the wages?

CONCEPT APPLICATION

Level 1

1. A is thrice as efficient as B. A and B can complete a piece of work in 12 days. Find the number of days in which A alone can complete it.

   (a) 48 (b) 16
   (c) 20 (d) 32

2. A and B can complete a piece of work in 15 days and 10 days, respectively. They work together for 4 days and then B leaves. In how many days will A alone complete the remaining work?

   (a) 6 (b) 8
   (c) 5 (d) 10

3. Sunny can complete a piece of work in 30 days. He worked for 6 days and left. Bunny completed the remaining work in 16 days. In how many days can the entire work be completed if they work together?

   (a) 8 (b) 12
   (c) 16 (d) 20

4. B is twice as efficient as A, who works half as fast as C. If A, B and C can complete a piece of work in 20 days. In how many days can B and C together complete it?

   (a) 50 (b) 100
   (c) 25 (d) 30

5. Where 6 men and 9 women can complete a piece of work in 10 days. What is the time taken by 4 men and 6 women to complete it?

   (a) 10 days (b) 12 days
   (c) 15 days (d) 18 days

6. A and B can complete a piece of work in 12 days. B and C can complete it in 24 days. A and C can complete it in 16 days. In how many days can B alone complete it?

   (a) 16 (b) 32
   (c) 12 (d) 20

7. A can do a piece of work in 34 days. He worked for 14 days and then left. B completed the remaining work in 30 days. In how many days can B alone complete the work?

   (a) 38 (b) 51
   (c) 46 (d) 62

8. Certain men can do a piece of work in 22 days. If the number of men decreases by 55, then they will take 11 days more to complete the same work. Find the number of men present initially.

   (a) 165 (b) 155
   (c) 185 (d) 175

PRACTICE QUESTIONS

9. Pipe A can fill a tank in 20 minutes and pipe B can fill the tank in 30 minutes. Both the pipes can fill at the rate of 8 litres per second. What is the capacity of the tank (in litres)?

(a) 5670 (b) 6570
(c) 6750 (d) 5760

10. A, B and C can do a piece of work in 8, 16 and 24 days, respectively. They work together and earn ₹528. What is the share of B?

(a) ₹144 (b) ₹152
(c) ₹176 (d) ₹168

11. A works three times as fast as B. B takes 56 days more than A to complete a work. Working together in how many days can they complete the work?

(a) 24 (b) 21
(c) 27 (d) 29

12. A, B and C can complete a piece of work in 25, 30 and 50 days, respectively. They started the work together but A and C left 2 days before the completion of the work. In how many days will the work be completed?

(a) 14 (b) 12
(c) 18 (d) 10

13. When 6 men and 8 women can do a piece of work in 15 days, then 11 men and 16 women can do the same work in 8 days. In how many days can 4 men and 4 women do the work?

(a) 28 (b) 24
(c) 36 (d) 30

14. A and B can do a piece of work in 12 days, B and C in 15 days and A and C in 20 days. In how many days can each alone do the work?

(a) 30, 20, 50 (b) 30, 45, 60
(c) 30, 20, 60 (d) 20, 30, 50

15. P and Q can complete a certain work in 28 days and 56 days, respectively. P works for 7 days, and then Q joins P. In how many more days, can they complete the work?

(a) 7 (b) 14
(c) 21 (d) 28

16. A piece of work can be done by 64 men in 17 days working for 9 hours per day. In how many days can 34 men do a piece of work $\frac{8}{3}$ times of the previous one on working 8 hours per day?

(a) 92 (b) 98
(c) 96 (d) 94

17. P and Q can do a piece of work in 12 and 16 days, respectively. With the help of R, they completed the work in 5 days and earn ₹912. What is the share of R?

(a) ₹249 (b) ₹247
(c) ₹243 (d) ₹245

18. A piece of work can be done by 9 men and 15 women in 24 days. In how many days can 15 men and 15 women do the same work?

(a) 6 (b) 12
(c) 9 (d) 15

19. A piece of work can be done by 12 men in 24 days. After 4 days, they started the work and then 6 more men joined them. How many days will they all take to complete the remaining work?

(a) $12\frac{1}{3}$ (b) $13\frac{1}{3}$
(c) $11\frac{2}{3}$ (d) $13\frac{2}{3}$

20. A and B can complete a piece of work in 4 days and 8 days, respectively. They work on alternate days and A starts the work. In how many days will the work be completed?

(a) 3 (b) 4
(c) 5 (d) 6

21. X and Y can do a piece of work in 4 and 6 days, respectively. If Y works on the first day and they work on alternate days, in how many days will twice the amount of work be completed?

(a) $9\frac{2}{3}$ (b) $10\frac{2}{5}$
(c) $9\frac{1}{5}$ (d) $9\frac{1}{2}$

22. Three machines P, Q and R can do a piece of work in 7, 9 and 10 days, respectively. Due to a problem in P and Q, they are working only at 70% and 90%

of their efficiency, respectively. In how many days can P, Q and R together do the work?

(a) $1\frac{2}{3}$ days (b) $2\frac{1}{3}$ days

(c) $2\frac{2}{3}$ days (d) $3\frac{1}{3}$ days

23. A, B and C can complete a piece of work in 10 days, 20 days and 25 days, respectively. If they take 40 days to complete a piece of work, then in how many days can C alone complete the work?

(a) 65 days (b) 76 days

(c) 95 days (d) 190 days

24. A group of 5 people can do a piece of work in certain number of days. If 4 more people join the group, they take 12 days less to do the same work. In how many days can a group of 3 people do the work?

(a) 30 (b) 45

(c) 15 (d) 60

25. A and B can complete a piece of work in 10 days and 12 days, respectively. If they work on alternate days beginning with B, in how many days will the work be completed?

(a) $10\frac{5}{6}$ (b) 11

(c) $12\frac{1}{3}$ (d) 13

26. A, B and C can complete a piece of work in 27 days, 36 days and 45 days, respectively. B and C started the work. After 11 days, A joined them. If B left 12 days before its completion, in how many days will the work be completed?

(a) 10 (b) 20

(c) 15 (d) 25

27. If P can produce 60 cakes in 9 days and Q can produce 70 cakes in 21 days, how many days will they take to produce 100 cakes if they work together?

(a) 8 (b) 9

(c) 10 (d) 11

28. 15 men and 25 women can dig an area of 880 m^2 in 8 days. In how many days can 20 men and 12 women dig an area of 1040 m^2, if each man can dig twice the area that each woman can dig in the same amount of time?

(a) 6 days (b) 8 days

(c) 10 days (d) 2 days

29. P and Q can do a piece of work in 10 days and 35 days, respectively. If they work on alternate days beginning with Q, in how many days will the work be completed?

(a) $15\frac{4}{7}$ (b) $15\frac{5}{7}$

(c) $16\frac{4}{7}$ (d) $14\frac{5}{7}$

30. Ram, Shyam and Tarun are three people in a job. Each takes *m* times the time taken by other two, to complete a job. Find *m*.

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) None of these

Level 2

31. Anoop can complete a piece of work in 10 days working for 8 hours a day. Swaroop can complete it in 12 days working 10 hours a day. How many days will they take to complete it if they work 12 hours a day?

(a) 4 (b) 6

(c) 8 (d) 10

32. Two taps P and Q can fill a tank in 12 hours and 18 hours, respectively. Both taps were opened at 7:00 am and after some time, Q was closed. It was found that the tank was full at 3:00 pm. At what time was Q shut?

(a) 10:00 am (b) 12:00 noon

(c) 1:00 pm (d) 2:00 pm

33. Anand completed one-fifth of a piece of work in 4 days. He was then assisted by Bhargav and they completed the remaining work in 8 days. Bhargav can complete the work in ______ days.

(a) 12 (b) 16

(c) 20 (d) 24

34. P works 25% more efficiently than Q and Q works 50% more efficiently than R. To complete a certain project, P alone takes 50 days less than Q. If in this project P alone works for 60 days and then Q alone works for 125 days, then in how many days can R alone can complete the remaining work?

(a) 50 (b) 75

(c) 100 (d) 150

35. A man takes 80 days to complete a job. To complete this job, 4 men, 8 women and 4 machines take 5 days. Alternatively, 4 men, 1 woman and 2 machines take 10 days to complete the job. Find the time taken by a woman to complete the job (in days).

(a) 90 (b) 100

(c) 105 (d) 120

36. Two taps A and B, can fill a tank in 10 minutes and 15 minutes, respectively. In how many minutes will the tank be full if B was opened 3 minutes after A was opened?

(a) 4.2 (b) 6.2

(c) 7.2 (d) 8.2

37. Two men are as efficient as 3 women who are as efficient as 4 machines. The number of men, women and machines are in the ratio of 3 : 4 : 5 and they have completed a job. They are paid a total of ₹4900 for it. Find the total share of women. (in ₹)

(a) 1200 (b) 1800

(c) 2400 (d) 1600

38. There are ten taps fitted to a tank. The filling taps that are filling the tank are five in number and each tap takes 5 hours to fill the tank. The emptying taps are also five in number each tap takes 6 hours to empty the tank. In how many hours can the empty tank be filled, if all the taps are opened simultaneously?

(a) 6 (b) 5

(c) 10 (d) 12

39. P can complete a piece of work in 3 days. Q takes triple the time taken by P, R takes 4 times that taken by Q and S takes double the time taken by R to complete the same task. They are grouped into two pairs. One of the pairs takes $2\frac{1}{2}$ times the time taken by the other pair to complete the work. Which is the second pair?

(a) P, S (b) P, R

(c) Q, R (d) Q, S

40. There are 25 workers in a group. Each can do 1 unit/day. They start a job of 330 units. After each day, a worker of the same efficiency as each worker in the group joins the group. The job was completed in x days. Find x.

(a) 9 (b) 10

(c) 11 (d) 12

41. P, Q and R together can complete 50% of a work in 2 days. All three start the work but after two days Q left. P and R completes one-sixth of the work in the next day and then P leaves. The remaining work is done by R alone in 8 days. In how many days can P alone can complete the work?

(a) 6 (b) 8

(c) 10 (d) 12

42. A and B are two taps which can fill a tank in 6 hours and 9 hours, respectively. C is an emptying tap, which can empty the tank in 7.5 hours Tap B is opened 3 hours after tap A is opened. For how long does tap C to be kept open if the tank has to be filled in 6 hours?

(a) $1\frac{1}{2}$ hours (b) $2\frac{1}{2}$ hours

(c) 3 hours (d) 5 hours

43. Anand, Raju, Suresh and Venkat together produced 392 pieces of an item in 6 hours Suresh is four times as efficient as Anand and is one–third less efficient than Venkat. Raju is half as efficient as Venkat. How many pieces would Raju have produced, if he worked for 8 hours?

(a) 84 (b) 112

(c) 28 (d) 56

44. A worker has to complete a job of 175 units. On each day starting from the second, he does 75% of the part of the job he did on the previous day. Find the number of days in which the job is completed if he did 36 units of work on the 3rd day.

(a) 3 (b) 4

(c) 5 (d) 6

45. A is twice as good a workman as B. A takes 6 days to complete a task. What time does B take to complete the work (in days)?

(a) 3 (b) 6

(c) 9 (d) 12

46. 27 men can dig a well in 20 days working 5 hours a day. They start digging it. After 4 days, 12 men leave. How many hours a day should the remaining men work to complete digging it by the scheduled date?

(a) 6 (b) 8

(c) 10 (d) 9

47. One man can load 1 box in a truck in 5 minutes. How many full trucks can 8 men load in 45 minutes given that the truck can hold 10 boxes?

(a) 8 (b) 7

(c) 6 (d) 5

48. Five men or ten women can complete a job in 20 days. Find the time in which four men and four women can complete it (in days).

(a) 15 (b) 20

(c) $13\frac{1}{3}$ (d) $16\frac{2}{3}$

49. A can do a task in 18 days. He works for 10 days and leaves. The remaining work is completed by B and C. If they get ₹9000 for completing the work, then who receives more share in the earnings?

(a) C (b) A

(c) B (d) Cannot say

Level 3

50. There are 4 people who can complete a work in 19 days individually. The work is started by one of the people on the first day. Everyday one more person joins and starting from the 4th day, all of the 4 people work together. In how many days, will the work be completed?

(a) $6\frac{1}{4}$ days (b) $6\frac{1}{19}$ days

(c) 7 days (d) $7\frac{1}{19}$ days

51. The ratio of the efficiency of P, Q and R is 2 : 3 : 5. The total wages of P, Q and R working for 14, 24 and 20 days, respectively are ₹6000. Find the total wages of the three, if P works for 9 days, Q for 14 days and R for 8 days.

(a) ₹3000 (b) ₹2860

(c) ₹2450 (d) ₹3240

52. Amar, Bhavan and Chetan can make a total of 8 dosas in one minute. They have to make a total of 80 dosas. Amar started making dosas. After some time, Bhavan and Chetan took over and completed the job. If it took a total of 20 minutes to complete the job and Amar made atleast 5 dosas per minute, how long did Amar work alone (in minutes)?

(a) 9 (b) 10

(c) 11 (d) 12

53. Eswar and Harish take 12 days and 16 days, respectively to complete a job. Ganesh is atleast as efficient as Harish but almost as efficient as Eswar. Ganesh and Harish work on alternate days and completed the job in x days. Which of the following can be the value of x?

(a) 8 (b) 12

(c) 14 (d) 17

54. Pipes X, Y and Z are fitted to a tank. Each of Y and Z can fill the tank in 6 hours. The efficiency of X, which is an emptying pipe, is half of Y. If X is fitted at one fourth of the height of tank from the base and all the pipes are opened simultaneously, the tank would be filled in ________ hours.

(a) 4 (b) $3\frac{1}{2}$

(c) $3\frac{1}{2}$ (d) 3

55. X works four times as fast as Y. Y takes 60 days more than X to complete a job. Find the time in which X and Y working together can complete the job (in days)?

(a) 20 (b) 10

(c) 16 (d) 12

56. One man and 7 women can complete a job in 16 days. 19 men and 10 women can compete it

PRACTICE QUESTIONS

in 3 days. Find the time in which 6 men and 6 women can complete it (in days).

(a) 8 (b) 6

(c) 9 (d) 12

57. M and N can complete a job in 18 days and 20 days, respectively. They worked on it on alternative days starting with M. Find the time taken to complete it (in days).

(a) $18\frac{9}{10}$ (b) 19

(c) $18\frac{3}{5}$ (d) $18\frac{3}{10}$

58. Ganesh, Harish, Suresh and Mahesh worked together to produce 558 pieces of an item in 12 hours. Ganesh is twice as efficient as Harish and is five-sixth as efficient as Mahesh. Mahesh is thrice as efficient as Suresh. How many pieces can Mahesh produce in 9 hours?

(a) 144 (b) 162

(c) 126 (d) 180

59. The efficiency of A, B and C is the same. They started working on a certain job. After 5 days A leaves, after 5 more days B leaves and C completes the remaining work in 5 more days. Find the time taken by each of them alone to complete the job. (in days)

(a) 25 (b) 30

(c) 40 (d) 45

60. 18 men and 36 women can level an area of 810 sq. m. in 6 days. The ratio of the areas that each man and each woman can level in the same amount of time is 3 : 1. Find the number of days in which 22 men and 14 women can level an area of 1200 sq. m.

(a) 8 (b) 12

(c) 9 (d) 10

61. There are 15 workers in a group. Each can do 1 unit per day. All the workers started a job of 220 units. After each day, a worker who can do 1 unit per day joined the group. Find the time taken to days).

(a) 11 (b) 12

(c) 13 (d) 10

62. Amar, Bhavan and Chetan can complete a job in 24 days, 36 days and 48 days, respectively. Amar, Bhavan and Chetan started it. After 6 days, Amar left. The other two continued to work. Bhavan left 15 days before the completion of the job. Chetan completed the remaining work. Find the total time taken to complete the job. (in days)

(a) 21 (b) 24

(c) 27 (d) 30

63. There are 5 men in a group. Each man can complete a job in 35 days. One of them starts it. Starting from the second day, a man joins until the 5th day. Thereafter all the men work together. Find the total time taken to complete the job. (in days)

(a) 9 (b) 8

(c) 10 (d) 7

64. A job can be completed by 2 men, 3 women and 4 children in 15 days. The same work can be completed by 9 men and 6 children in 10 days. If 1 man and 1 woman can complete it in 48 days. Find the time in which one man can complete it. (in days)

(a) 60 (b) 75

(c) 80 (d) 90

65. Mohan can complete a job in 8 days working 9 hours a day. Sohan can complete it in 9 days working 10 hours a day. In how many days can Mohan and Sohan together complete it working 8 hours a day?

(a) 6 (b) 4

(c) 3 (d) 5

66. Kiran and Pavan can complete a job in 40 days and 50 days, respectively. They worked on alternative days to complete it. Find the minimum possible time in which they could have completed it. (in days)

(a) $44\frac{2}{5}$ (b) $44\frac{1}{2}$

(c) $44\frac{3}{5}$ (d) $44\frac{4}{5}$

67. Pipes A, B and C are fitted to a tank. Each of A and B can fill a tank in 9 hours. C is an emptying pipe which can empty it in 12 hours. It is fitted at

PRACTICE QUESTIONS

one-third the height of the tank above the base. All pipes are opened simultaneously. Find the time in which the tank will be filled (in hours).

(a) 5 (b) $5\frac{1}{2}$

(c) 6.3 (d) $6\frac{1}{2}$

68. P can complete a job in 60 days while Q can complete it in 90 days. With the help of R, they completed it in 20 days. If they earned a total of ₹3600, then find R's share. (in ₹)

(a) 1360 (b) 1600

(c) 1480 (d) 1540

69. Pipes X, Y and Z are fitted to a tank. X and Y can fill a tank in 18 hours and 24 hours, respectively. Z is an emptying pipe which can empty the tank in 9 hours. All the pipes are opened simultaneously when the tank is half full. The tank would be

(a) filled in 72 hours.

(b) emptied in 72 hours.

(c) filled in 36 hours.

(d) emptied in 36 hours.

70. P, Q and R can complete a job in 6 days, 9 days and 12 days, respectively. They worked together and completed it. They earned a total of ₹2600. Find P's share. (in ₹)

(a) 1350

(b) 1500

(c) 900

(d) 1200

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\frac{k}{x}$
2. 3 : 4
3. 4 days
4. B
5. False
6. 12 days
7. A
8. 3 : 2
9. 8 days
10. 7 trucks
11. empty
12. 720 litres
13. False
14. A
15. 8 days
16. ₹100
17. 60 days
18. 15
19. 45 days
20. 20
21. A
22. 45 days
23. 20
24. 14400 litres
25. 30

Shot Answer Type Questions

26. $41\frac{1}{4}$
27. $8\frac{2}{5}$ days
28. 3
29. 30
30. 9
31. 10
32. 12
33. 20
34. 21
35. 180
36. 12
37. 28
38. $16\frac{2}{3}$
39. 2.5
40. 40
41. 55 minutes
42. ₹250
43. 34
44. 15%
45. 7

Essay Type Questions

46. 25
47. 40
48. 8000
49. 20
50. Suresh

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (a)
11. (b)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (c)
21. (a)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (b)

Level 2

31. (a)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (c)
41. (b)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (b)	**48.** (d)	**49.** (b)	

Level 3

50. (a)	**51.** (a)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (b)	**59.** (b)
60. (d)	**61.** (a)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (d)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (d)
70. (d)									

CONCEPT APPLICATION

Level 1

1. A = 3B and (A + B)'s one day's work is equal to 4 days work by B.
2. Find the work done by A and B together and proceed.
3. Sunny's 6 day's work is $6\times\frac{1}{30}=\frac{1}{5}$. Remaining work is completed by Bunny in 16 days.

   So, Bunny's one day's work is $\frac{1}{20}$.
4. Given, B = 2A and A = $\frac{C}{2}$.
5. Work done by (2 men + 3 women) is 30 days is equal to the work done by (4 men + 6 women) in d days. Find d.
6. Find the work done by A + B + C in one day and proceed.
7. Find the work done by A in 14 days and then find the remaining work and proceed.
8. $M_1D_1 = M_2D_2$.
9. Find the part of the tank filled in one minute by A and B together and proceed.
10. Money is distributed according to their efficiencies.
11. Consider A = 3B and if A does the work in n days, then B will do it in $3n$ days. $3n - n = 56$ days.
12. Consider the number of days as x. A and C worked for $(x - 2)$ days.
13. Use $M_1D_1 = M_2D_2$.
14. Calculate the work done by (A + B + C) in one day and proceed.
15. (i) Frame the equations and solve.

    (ii) P works for 7 days, remaining work $=1-\frac{7}{28}=\frac{3}{4}$.

    (iii) Let the required number of days = x.

    $\therefore\ \frac{x}{28}+\frac{x}{56}=\frac{3}{4}$.
16. $\frac{M_1D_1H_1}{W_1}=\frac{M_2D_2H_2}{W_2}$.
17. Find what part of the work is done by P and Q in 5 days. The remaining work is done by R.
18. Convert $15M + 15W$ into one variable, i.e., M or W and proceed.
19. (i) The work of 12 men in 4 days $=\frac{1}{6}$.

    (ii) After four days, 12 men can do remaining work in (24 – 4) days.

    $\Rightarrow M_1 = 12$ and $D_1 = 20$.

    (iii) $M_2 = (12 + 6)$, find D_2 by using $M_1D_1 = M_2D_2$.
20. Find the work done by A and B in a period of 2 days and proceed.
21. Find the work done by X and Y in a period of two days.

    (i) The amount of work done by P and Q in one day is 70% of $\frac{1}{7}$ and 90% of $\frac{1}{9}$.

    (ii) The amount of work done by P and Q in one day is 70% of $\frac{1}{7}$ and 90% of $\frac{1}{9}$ respectively.

    (iii) Now find the part of work done by P, Q and R in one day and proceed.

23. (i) The ratio of capacities of A, B and C is 10 : 5 : 4.

    (ii) Find the time taken by A, B and C together to complete the work (say d).

    (iii) When they are taking d days, C alone takes 25 days.

    (iv) Find how many days C alone takes if they complete the work in 40 days.
24. (i) Use, $M_1D_1 = M_2D_2$.

    (ii) Let five people do the piece of work in x days. Find x.

    (iii) $5(x) = 9(x - 12)$

    (iv) Let three people do the same work in y days.

    (v) Use, $5 \times x = 3 \times y$ and find y.
25. Find the work done by A and B in a period of days and proceed.

26. (i) Consider the total number of days as 'x'. A, B and C works for $(x - 11)$, $(x - 12)$ and x days respectively.

(ii) Let A, B and C work for $(x - 11)$, $(x - 12)$ and x days respectively.

(iii) Add the total work done by each of them and equate it to 1 and find x.

27. (i) Find the number of cakes produced by both of them in a day and proceed.

(ii) From the given data find the number of days P and Q require to produce 100 cakes.

(iii) Consider 100 cakes as 1 unit and then proceed in general way.

28. (i) $\dfrac{(15M + 25W)8}{880} = \dfrac{(20M + 12W)d}{1040}$.

(ii) Find d using $1M = 2W$.

29. (i) Find the work done by P and Q in one day and proceed.

(ii) First of all find the part of the work done by Q and P in a period of 2 days.

(iii) Then find total number of such periods (integer) required.

(iv) Then find the time taken to complete remaining part starting with Q and then P.

30. (i) Frame equations and solve.

(ii) Let the capacities or work completed in one day by the three people be $\dfrac{1}{R}, \dfrac{1}{S}$ and $\dfrac{1}{T}$.

Take $\dfrac{1}{R} = \dfrac{1}{m}\left(\dfrac{1}{S} + \dfrac{1}{T}\right)$,

$\dfrac{1}{S} = \dfrac{1}{m}\left(\dfrac{1}{T} + \dfrac{1}{R}\right)$ and $\dfrac{1}{T} = \dfrac{1}{m}\left(\dfrac{1}{R} + \dfrac{1}{S}\right)$.

Add the above three equations and find m.

Level 2

31. (i) Anoop and Swaroop can complete a piece of work in 80 and 120 hours respectively.

(ii) Let the total number of man-hours they take, to complete the work be $12x$ hours, where x is the number of days for which they work together.

(iii) Use unitary method to equate the sum of their individual work done per day to the work done by both of them per day and proceed.

32. (i) Tank was filled in 8 hours P was opened all the time.

(ii) Tap P is opened for 8 hours. Let the tap Q be opened for x hours

(iii) $\dfrac{8}{12} + \dfrac{x}{18} = 1$.

(iv) Required time = 7 am + x.

33. (i) Find the capacity of Anand and proceed.

(ii) Anand can complete the work in (5×4) days.

(iii) Remaining work is four-fifth which is to be completed by both Anand and Bhargav.

34. (i) Let the amount of work done by R in a day be x units.

(ii) Let the efficiency of R be 100.

$\therefore$ Efficiency of Q = 150 and Efficiency of P = 187.5.

(iii) Now find the ratio of the efficiencies of P, Q and R.

(iv) Then find the ratio of the time taken by P, Q and R.

(iv) Now, find the time taken by P, Q and R to complete the work.

35. (i) Frame the equations as follows:

$4m + 8w + 4$ machine $= \dfrac{1}{5}$ and $4m + 1w + 2$ machine $= \dfrac{1}{10}$.

(ii) Since the work done by a man in one day is known, work done by a women can be found.

36. Remaining work after 3 minutes is $\dfrac{7}{10}$.

37. (i) Let the efficiencies of a man, a woman and a machine be x, y and z.

$\Rightarrow 2x = 3y = 4z = k$ say.

(iii) Find x, y, z in terms of k.

HINTS AND EXPLANATION

(iv) The ratio of shares of men, women and machines $= 3\left(\frac{k}{2}\right) : 4\left(\frac{k}{3}\right) : 5\left(\frac{k}{4}\right)$.

38. Take filling work as positive and emptying work as negative.

39. (i) Q and R take 9 days and 36 days respectively to complete a piece of work.

(ii) Let P take x days, Q take $3x$ days, R take $12x$ days and S take $24x$ days.

(iii) Find the part of the work done by each person, per day.

(iv) Then check from the options.

40. (i) On the first day 25 units of work is done. On the second day 26 units, third day 27 units the work is done in this way.

(ii) Let $25 + 26 + 27 + \cdots$ upto x terms = 330.

(iii) Find x.

41. (i) Remaining work = 1 – (P + Q + R)'s two day's work – (P + R)'s one day work.

(ii) Let p, q and r be the number of days taken by P, Q and R respectively.

(iii) $$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{1}{4} \quad (1)$$

$$\frac{1}{p}+\frac{1}{r}=\frac{1}{6} \quad (2)$$

$$\frac{3}{p}+\frac{2}{q}+\frac{11}{r}=1 \quad (3)$$

(iv) Solve the above equations to find P.

42. (i) Tap A is opened for 6 hours and Tap B for 3 hours.

(ii) Let C be opened for x hours .

(iii) Solve $\frac{6}{6}+\frac{3}{9}-\frac{x}{7.5}=1$, for x.

43. (i) The ratio of efficiency of Anand, Raju, Suresh and Venkat is 1 : 3 : 4 : 6.

(ii) Let the efficiency of Suresh be x.

$\therefore$ Efficiency of Anand $= \frac{x}{4}$.

Efficiency of Venkat $= \frac{3x}{2}$.

Efficiency of Raju $= \frac{3x}{4}$.

(iii) Now find the ratio of efficiencies of Anand, Raju, Suresh and Venkat and then divide 392 in the same ratio.

(iv) Now, find the number of pieces produced by Raju in 8 hours, by using the number of pieces produced by Raju in 6 hours

44. Let the work done on the first day be x.

Then the work done on the third day $= \frac{3}{4}\times\frac{3}{4}\times x$ which is equal to 36 units.

45. B can do the work in 12 days.

46. Let the required number of hours per day be h.

Let the work done by each man be 1 unit/hr.

Time for which 27 men worked in the first four days = (5)(4) hours = 20 hours.

Work completed in the first 4 days = (27)(1)(20) units = 540 units.

Remaining time = (20 – 4)(h) hours = $16h$ hours

Work completed in the remaining time = (27 – 12)(1)($16h$) units = $240h$ units

Total time taken = (5)(20) hours = 100 hours

Total work = (27)(1)(100) units = 2700 units

$2700 = 540 + 240h$

$h = 9$.

47. 1 man can load $\frac{45}{5} = 9$ boxes in 45 minutes.

So, 8 men can load $8 \times 9 = 72$ boxes in 45 minutes.

$\therefore$ They can load completely 7 trucks in 45 minutes.

48. Let the required time be t days.

Part of the job that five men can complete each day = part of the job that ten women can complete each day $= \frac{1}{20}$.

Part of the job that each man can complete each day $= \frac{\frac{1}{20}}{5} = \frac{1}{100}$.

Part of the job that each woman can complete each day $= \frac{\frac{1}{20}}{10} = \frac{1}{200}$.

HINTS AND EXPLANATION

Part of the job that four men and four women can complete each day

$= 4\left(\frac{1}{100}\right) + 4\left(\frac{1}{200}\right) = \frac{1}{25} + \frac{1}{50} = \frac{3}{50}$.

Time taken by them to complete the job $= \frac{50}{3}$ days $= 16\frac{2}{3}$ days.

49. A's 10 days work $= \frac{10}{18} = \frac{5}{9}$

A completed more than half the work.

Hence A gets more share.

Level 3

50. (i) Work done in the first three days is $\frac{1}{19} + \frac{2}{19} + \frac{3}{19}$ and from the 4th day onwards the work done is $\frac{4}{19}$.

(ii) Part of the work done in first three days $= \frac{1}{19} + \frac{2}{19} + \frac{3}{19} = \frac{6}{19}$.

$\therefore$ Remaining work $= \frac{13}{19}$.

(iii) Let time taken by the four together be x.

$\therefore (x)\left(\frac{4}{19}\right) = \frac{13}{19}$.

(iv) The required time = $(3 + x)$ days.

51. (i) The ratio of work done by P, Q and R is 7 : 18 : 25.

(ii) Let the efficiencies of P, Q and R be $2x$, $3x$ and $5x$.

(iii) Use, $(14 \times 2x) + (24 \times 3x) + (20 \times 5x) = 6000$ and find x.

(iv) Now, find $(9 \times 2x) + (14 \times 3x) + (8 \times 5x)$.

52. (i) Frame the equations and solve.

(ii) A or B or C can make $\frac{8}{3}$ dosas in one minute.

(iii) Let A alone work for x minutes, and B and C work for $(20 - x)$ minutes each.

(iv) $\frac{8}{3} \times x + (20 - x)\frac{8}{3} + (20 - x)\frac{8}{3} = 80$.

53. (i) Consider Ganesh to be as efficient as Eswar and proceed.

(ii) Let a be the time taken by Ganesh to complete the work. $\Rightarrow 12 \le a \le 16$. (1)

(iii) $\frac{\frac{x}{2}}{a} + \frac{\frac{x}{2}}{16} = 1$. (2)

(iv) Simplify the Eq. (2) and assume the value of x by using Eq. (1).

54. (i) Time taken by X to fill the tank is 12 hours.

If Y takes x hours to fill the tank, then X takes $2x$ hours to empty the full tank.

(ii) First $\frac{1}{4}$ of the tank will be filled by Y and Z in 3 hours.

(iii) Remaining $\left(\frac{3}{4}\right)$ of the tank will be filled by X, Y and Z where X will be emptying.

55. Let the time in which X and Y can complete the job be x days and y days respectively.

$$\frac{1}{x} = 4\left(\frac{1}{y}\right) \quad (1)$$

$$y = x + 60. \quad (2)$$

From Eq. (1), we get

$y = 4x$.

Substituting this value in Eq. (2), we get

$4x = x + 60$

$x = 20$

$y = 80$.

The time in which X and Y can complete the job (in days) $= \frac{xy}{x + y} = 16$.

56. Let the work which each man can complete each day be m units. Let the work which each women and complete each day be w units.

Work which one man and 7 women can complete each day = $(m + 7w)$ units

$\therefore$ Job = $(m + 7w)16$ units = $(19m + 10w)3$

$82w = 41m$

$2w = m.$

$$\text{Required time} = \frac{\text{Job}}{6m + 6w}\text{ days}$$
$$= \frac{16(2w + 7w)}{6(2w) + 6w}\text{ days}$$
$$= 8 \text{ days.}$$

57. Part of the job completed (by M) on the first day $= \frac{1}{18}.$

Part of the job completed (by N) on the second day $= \frac{1}{20}.$

Part of the job completed on the first two days $= \frac{1}{18} + \frac{1}{20} = \frac{19}{180}.$

Part of the job completed on the first 18 days $= \frac{18}{2}\left(\frac{19}{180}\right) = \frac{19}{20}.$

$$\text{Remaining part} = \frac{1}{20}.$$

M work on the 19th day. He can complete the remaining part in $\dfrac{\frac{1}{20}}{\frac{1}{18}}$ days $= \frac{9}{10}$ days.

$$\text{Total time taken} = 18\frac{9}{10}\text{ days.}$$

58. Let the number of units which Ganesh, Harish, Suresh and Mahesh can produce be g/hr, h/hr, s/hr and m/hr respectively.

$12(g + h + s + m) = 558$

$g + h + s + m = 46.5$

$$g = 2h = \frac{5}{6}m \quad (1)$$

$$m = 3s. \quad (2)$$

From Eq. (1), we get

$$h = \frac{5}{12}m.$$

And from Eq. (2), we get

$$s = \frac{1}{3}m.$$

$$\therefore \frac{5}{6}m + \frac{5}{12}m + \frac{1}{3}m + m = 46.5$$
$$\frac{31}{12}m = 46.5$$
$$m = 18.$$

In 9 hours Mahesh can produce $9m$ units, i.e., 162 units.

59. Let us assume that each of A, B and C alone can do in x days.

A works for 5 days, B works for 10 days and C works 15 days.

$$\therefore \frac{5}{x} + \frac{10}{x} + \frac{15}{x} = 1$$
$$\frac{30}{x} = 1 \Rightarrow x = 30.$$

60. Let the areas that each man and each woman can level (in sq. m.) be m/day and w/day respectively.

Area that 18 men and 36 women can level each day = $(18m + 36w)$ sq. m.

$\therefore 6(18m + 36w) = 810$

$18m + 36w = 135.$

Given: $\frac{m}{w} = \frac{3}{1}$, i.e., $m = 3w$

$\therefore 18(3w) + 36w = 135$

$90w = 135$

$$w = \frac{3}{2}$$

$$\therefore m = \frac{9}{2}.$$

Let the required number of days be d.

$d(22m + 14w) = 1200$

$$d\left(22\left(\frac{9}{2}\right) + 14\left(\frac{3}{2}\right)\right) = 1200$$

$\Rightarrow 120d = 1200 \Rightarrow d = 10.$

61. On the mth day, $(15 + m - 1)$ workers would work.

On the first day, 15 workers would work.

On the second day, 16 workers would work and so on.

Let the time taken to complete the job be N days.

$$\text{Job} = \sum_{m=1}^{N} (15 + m - 1)(1) = 220$$

$$\sum_{m=1}^{N} (14 + m) = 220.$$

$15 + 16 + 17 + \cdots + (14 + N) = 220.$

Expressing the RHS as a sum of consecutive natural numbers starting from 15, we have $15 + 16 + 17 + 18 + 19 + 20 + 21 + 22 + 23 + 24 + 25$.

$\therefore\ 14 + N = 25$

$\therefore\ N = 11.$

62. Amar worked for 6 days.

Bhavan worked for $(t - 15)$ days

Chetan worked for t days,

$$\text{Job} = 6\left(\frac{1}{24}\right) + (t-15)\left(\frac{1}{36}\right) + t\left(\frac{1}{48}\right) = 1$$

$$\frac{1}{4} + t\left(\frac{1}{36} + \frac{1}{48}\right) - \frac{5}{12} = 1$$

$$\frac{7t}{144} = \frac{7}{6}$$

$$t = 24.$$

63. Each man can complete $\frac{1}{35}$ of the job/day.

Part of the job completed in the first 5 days.

$$= \frac{1}{35} + \frac{2}{35} + \frac{3}{35} + \frac{4}{35} + \frac{5}{35} = \frac{15}{35} = \frac{3}{7}.$$

Remaining part $= \frac{4}{7}$

This will be completed by all the men working together.

Time taken to complete it

$$= \frac{\frac{4}{7}}{5\left(\frac{1}{35}\right)} \text{ days} = 4 \text{ days.}$$

Total time taken = 9 days.

64. Let the work which can be completed each day by each man, each woman and each child be m units, w units, and c units respectively.

Work which two men, three women and four children can complete $= (2m + 3w + 4c)$ units

$\therefore$ Job $= (2m + 3w + 4c)(15)$ units

$= (9w + 6c)(10)$ units $= (m + w)(48)$ units

$(2m + 3w + 4c)(15) = (9w + 6c)(10)$

$30m + 45w + 60c = 90w + 60c$

$30m = 45w$

$$m = \frac{3}{2}w.$$

$$\therefore \text{Job} = \left(\frac{3}{2}w + w\right)(48) \text{ units}$$

$$= 120w \text{ units.}$$

Time in which a man can complete it

$$= \frac{120w}{\frac{3}{2}w} \text{ days} = 80 \text{ days.}$$

65. Time in which Mohan can complete the job $= (8)(9)$ hours $= 72$ hours.

Time in which Sohan can complete it $= (9)(10)$ hours $= 90$ hours.

Time in which Mohan and Sohan can complete it

$$= \frac{(72)(90)}{72 + 90} \text{ hours} = 40 \text{ hours.}$$

Number of days in which they can complete it

$$40 = \frac{40}{8} \text{ days} = 5 \text{ days.}$$

66. One of Kiran and Pavan would work on the first day and the other person would work on the second day. Irrespective of who works on the first day, part of the job completed in the first two days

$$= \frac{1}{40} + \frac{1}{50} = \frac{9}{200}.$$

Part of the job completed in the first 44 days

$$= \frac{44}{2}\left(\frac{9}{200}\right) = \frac{198}{200} = \frac{99}{100}$$

Remaining part $= \frac{1}{100}$.

This will be completed in the minimum possible time if the person with a higher efficiency between the two completed.

$\therefore$ Total time taken will be minimum if Kiran completed.

Minimum possible total time = 44 days +

$$\frac{\frac{1}{100}}{\frac{1}{40}} \text{ days} = 44\frac{2}{5} \text{ days.}$$

HINTS AND EXPLANATION

67. Each of A and B can fill the bottom one-third of the tank in $\frac{1}{3}(9)$ hours = 3 hours.

Time in which it will be filled $= \frac{(3)(3)}{3+3}$ hours $= \frac{3}{2}$ hours $= 1\frac{1}{2}$ hours.

Part of the top two-third of the tank which A, B and C can fill each hour

$$= \frac{\frac{1}{2}}{3}\left(\frac{1}{9}+\frac{1}{9}-\frac{1}{12}\right) = \frac{1}{6}+\frac{1}{6}-\frac{1}{8} = \frac{5}{24}.$$

∴ Time in which it will be filled $= \frac{24}{5}$ hours.

Total time in which the tank will be filled $= \frac{3}{2}+\frac{24}{5} = 6.3$ hours.

68. Parts of the job completed by P and Q each day are $\frac{1}{60}$ and $\frac{1}{90}$ respectively.

Parts of the job completed by P and Q are

$20\left(\frac{1}{60}\right) = \frac{1}{3}$ and $20\left(\frac{1}{90}\right) = \frac{2}{9}$ respectively.

Part of the job completed by

$$R = 1 - \left(\frac{1}{3}+\frac{2}{9}\right) = \frac{4}{9}.$$

Ratio of the parts of the job completed by P, Q and R $= \frac{1}{3} : \frac{2}{9} : \frac{4}{9} = 3:2:4.$

∴ The ratio of the share of P, Q and R = 3 : 2 : 4.

∴ R's share $= \frac{4}{3+2+4} \times ₹3600$

= ₹1600.

69. Part of the tank which X and Y can fill in one hour $= \frac{1}{18}+\frac{1}{24} = \frac{7}{72}$.

Part of the tank that Z can empty in one hour $= \frac{1}{9} = \frac{8}{72}$.

Emptying rate of Z > filling rate of X and Y.

∴ The tank will be emptied.

Part of it emptied each hour $= \frac{8}{72} - \frac{7}{72} = \frac{1}{72}$.

Time taken to empty $\frac{1}{2}$ of the tank

$$= \frac{\frac{1}{2}}{\frac{1}{72}} \text{ hours}$$

= 36 hours.

70. Let the time taken to complete the job be x days.

Parts of the job complete by P, Q and R each day are $\frac{1}{6}, \frac{1}{9}$ and $\frac{1}{12}$ respectively.

Parts of the job completed by P, Q and R are $x\left(\frac{1}{6}\right), x\left(\frac{1}{9}\right)$ and $x\left(\frac{1}{12}\right)$ respectively.

Ratio of the parts of the job completed by P, Q and

$$R = \frac{x}{6} : \frac{x}{9} : \frac{x}{12} = \frac{1}{6} : \frac{1}{9} : \frac{1}{12} = 6:4:3.$$

∴ Ratio of the shares of P, Q and R = 6 : 4 : 3.

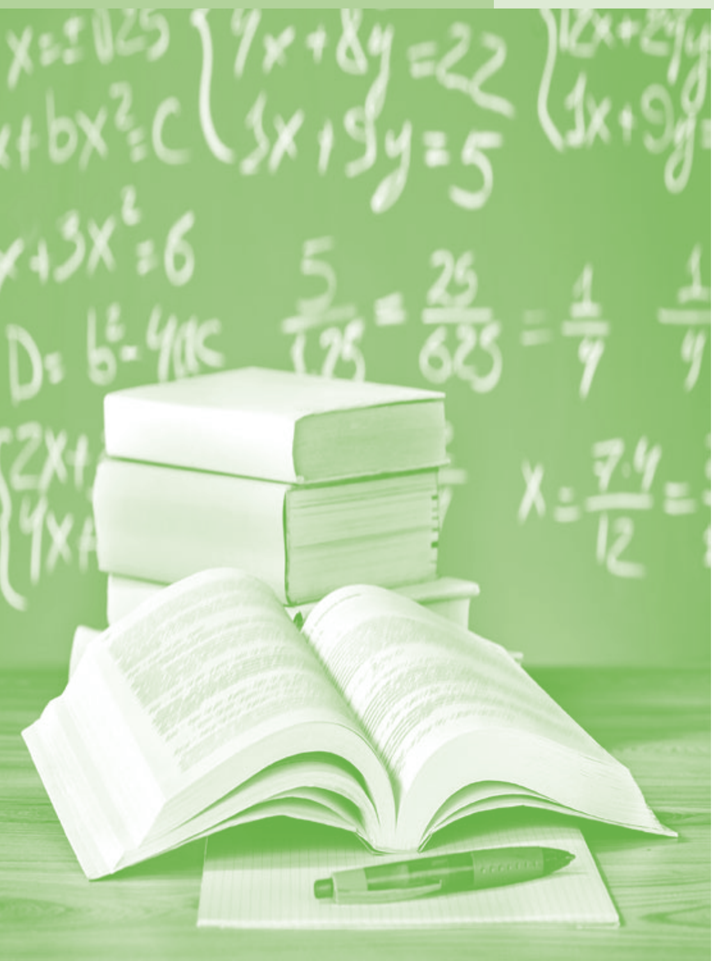
∴ P's share $= \frac{6}{6+4+3}(₹2600)$

= ₹1200.

Chapter

23

Time and Distance



REMEMBER

Before beginning this chapter, you should be able to:

- Know the definitions of speed, velocity, acceleration
- Understand the concepts of distance travelled by an object
- Relate motion of an object with time and distance

KEY IDEAS

After completing this chapter, you should be able to:

- Calculate the value of speed and velocity using formulae
- Understand average speed and relative speed
- Solve word-problems based on time and distance
- Calculate the speed of moving objects such as boats, streams, trains and cars

INTRODUCTION

Let us consider an object moving uniformly. It covers equal distances in equal intervals of time. For example, if in 1 second it covers 5 m, in the next second it covers another 5 m, i.e., in 2 seconds it covers 10 m, in 3 seconds it covers 15 m and so on. The distance in which the object covers in unit time, is called its speed. Hence, the speed of the object whose motion is described above is 5 m per second.

SPEED

The distance covered per unit time is called **speed**.

That is,
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The above relationship between the three quantities–distance, speed and time can also be expressed as follows:

$$\text{Distance} = \text{Speed} \times \text{Time (or) Time} = \frac{\text{Distance}}{\text{Speed}}$$

If two bodies travel with the same speed, the distance covered varies directly as time and it is written as Distance ∝ Time. Further, if two bodies travel for the same period of time, the distance covered varies directly as the speed, i.e., Distance ∝ Speed. If two bodies travel the same distance, time varies inversely as speed, i.e., $\text{Time} \propto \frac{1}{\text{Speed}}$.

Distance is usually measured in kilometres, metres or miles; time in hours, minutes or seconds and speed in kmph (also denoted by kmph) or miles/h (also denoted by mph) or metres/second (denoted by m/sec).

$$1 \text{ km per hour} = \frac{1 \times 1000 \text{ m}}{3600 \text{ sec}} = \frac{5}{18} \text{ m/s.}$$

Note To convert speed in kmph to m/sec, multiply it with $\frac{5}{18}$. And to convert speed in m/sec to kmph multiply it with $\frac{18}{5}$.

Average Speed

The average speed of a body travelling at different speeds for different time periods is defined as follows:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Note that the average speed of a moving body is not equal to the average of the speeds.

Consider a body travelling from point A to point B, (a distance of d units) with a speed of p units and back to point A (from point B) with a speed of q units.

Total distance covered = $2d$ units.

$$\text{Total time taken} = \frac{d}{p} + \frac{d}{q} = d\left(\frac{p+q}{pq}\right).$$

$$\therefore \text{Average speed} = \frac{2d}{d\left(\dfrac{p+q}{pq}\right)} = \frac{2pq}{p+q}.$$

Note that the average speed does not depend on the distance between A and B. If a body covers part of the journey at a speed p units and the remaining part of the journey at a speed q units and the distances of the two parts of the journey are in the ratio $m : n$, then the average speed for the entire journey is $\dfrac{(m+n)pq}{mq+np}$ units.

EXAMPLE 23.1

Express 54 kmph in m/sec.

SOLUTION

$$54\,\frac{\text{km}}{\text{hr}} = 54\,\frac{(1000)}{(3600)}\,\frac{\text{m}}{\text{s}} = 15 \text{ m/s} \quad \text{or}$$

$$\text{or, } 54\left(\frac{5}{18}\right) = 15 \text{ m/s.}$$

EXAMPLE 23.2

A car can cover 350 km in 4 hours. If its speed is decreased by $12\frac{1}{2}$ kmph, how much time does the car take to cover a distance of 450 km?

SOLUTION

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{350}{4} = 87\frac{1}{2} \text{ kmph}$$

Now this is reduced by $12\frac{1}{2}$ kmph. Hence, the speed is 75 kmph. Travelling at this speed, the time taken by the car $= \dfrac{450}{75} = 6$ hours.

EXAMPLE 23.3

A person covers a certain distance at a certain speed. If he increases his speed by 25%, then he takes 12 minutes less to cover the same distance. Find the time taken by him to cover the distance initially, travelling at the original speed.

SOLUTION

When the speed is increased by 25%, the increased speed is 125% of the original speed; it is $\frac{5}{4}$ times the original speed. Since speed and time vary inversely, if the increased speed is $\frac{5}{4}$ times the original speed, then the time taken decreases to $\frac{4}{5}$ times the original time.

This means that the decreased time is $\left(1-\frac{4}{5}\right)$ or $\frac{1}{5}$ part less than the original time.

But, we know that the reduced time is less by 12 minutes. This means $\frac{1}{5}$ of original time is 12 minutes; so, the original time = 5(12) = 60 minutes = 1 hour.

EXAMPLE 23.4

A car covers a certain distance travelling at a speed of 60 kmph and returns to the starting point at a speed of 40 kmph. Find the average speed for the entire journey.

SOLUTION

We know that the average speed is $\frac{2pq}{p+q}$, where p and q are the speeds in both directions, for equal distances.

$$\therefore \text{Average speed} = \frac{2(60)(40)}{(60+40)} = 48 \text{ kmph.}$$

EXAMPLE 23.5

A worker reaches his work place 15 minutes late when he walks at a speed of 4 kmph from his house. The next day he increases his speed by 2 kmph and reaches his work place on time. Find the distance from his house to workplace.

SOLUTION

Let the distance be x km.
Then the time taken on the 1st day = $\frac{x}{4}$ hours

Time taken on the 2nd day = $\frac{x}{6}$ hours

We are given,

$$\frac{x}{4} - \frac{x}{6} = \frac{15}{60}$$

$$\Rightarrow x = \frac{15}{60}(12) = 3\,\text{km}.$$

In general, if a person travelling between two points reaches p hours late travelling at a speed of u kmph and reaches q hours early, travelling at v kmph, then the distance between the two points is given by $\frac{vu}{(v-u)}(p+q)$.

EXAMPLE 23.6

A person leaves his house and travelling at 4 kmph, reaches his office 10 minutes late. Had he travelled at 6 kmph, he would have reached 20 minutes early. Find the distance from his house to the office.

SOLUTION

Let the distance be d km.

Time taken to travel at 6 kmph $= \dfrac{d}{6}$ hours

Time taken to travel at 4 kmph $= \dfrac{d}{4}$ hours

Given that, $\dfrac{d}{4} - \dfrac{d}{6} = \dfrac{30}{60}$

$$\Rightarrow d = 6 \text{ km.}$$

Alternate method:

As per the formula given,

$$\text{Distance} = \frac{4 \times 6}{6-4}\left(\frac{10}{60} + \frac{20}{60}\right)$$

$$= \frac{24}{2} \times \frac{30}{60}$$

$$= 6 \text{ km.}$$

EXAMPLE 23.7

When a person travelled at 25% faster than his usual speed, he reached his destination 48 minutes early. By how many minutes would he be late if he travelled at 20% less than his usual speed? Choose the correct answer from the following options:

(a) 48 **(b)** 54 **(c)** 60 **(d)** 72

HINTS

(i) Find the ratio of the speeds and corresponding times taken.

(ii) Let his usual speed be x kmph and distance travelled be d km.

(iii) Use $\dfrac{d}{x} - \dfrac{4d}{x} = 48$ and find $\dfrac{d}{x}$.

(iv) Speed $= x - \dfrac{20x}{100} = \dfrac{4x}{5}$, Time $= \dfrac{5d}{4x}$.

(v) Now, find the time taken using the value of $\dfrac{d}{x}$.

Relative Speed

The speed of one moving body in relation to another moving body is called the relative speed of these two bodies, i.e., it is the speed of one moving body as observed from the second moving body. If two bodies are moving in the same direction, then the relative speed is equal to the difference of the speeds of the two bodies.

If two bodies are moving in the opposite direction, then the relative speed is equal to the sum of the speeds of the two bodies.

Trains

In time and distance topic, we come across many problems on trains. While passing a stationary point or a telegraph/telephone pole completely, a train has to cover its entire length. Hence, the distance travelled by the train to pass a stationary point or a telegraph/telephone pole is equal to its own length.

While passing a platform, bridge or another stationary train, a train has to cover its own length as well as the length of the platform, the bridge or the other train. Hence, the distance travelled by the train to pass these objects is equal to the total length of the train and the length of the platform/bridge/the other train.

While overtaking another train (when the trains move in the same direction) or while crossing another moving train (when the trains move in the same or opposite directions) a train has to cover its own length as well as the length of the other train. Hence, in this case, the distance travelled by the train, is equal to the total length of the two trains, but the speed at which this distance is covered is the relative speed of the trains.

EXAMPLE 23.8

What is the time taken by a 180 m long train running at 54 kmph to cross a man standing on a platform?

SOLUTION

Speed of train = 54 kmph = $54\left(\frac{5}{18}\right)$ = 15 m/sec.

Distance = Length of the train = 180 m.

$\therefore$ Time taken to cross the man $= \frac{\text{Distance}}{\text{Speed}} = \frac{180}{15} = 12$ seconds.

EXAMPLE 23.9

How long will a train 100 m long and travelling at a speed of 45 kmph, take to cross a platform of length 150 m?

SOLUTION

Distance = Length of the train + Length of the platform = 100 + 150 = 250 m.

Speed of the train = 45 kmph = $45\left(\frac{5}{18}\right)$ = 12.5 m/sec

$\therefore$ Time taken $= \frac{250}{12.5} = 20$ second.

EXAMPLE 23.10

Find the length of a bridge on which a 120 m long train, travelling at 54 kmph, completely passes in 30 seconds.

SOLUTION

Speed of the train = 54 kmph = $54 \times \left(\frac{5}{18}\right)$ = 15 m/sec

Distance covered in 30 sec = $15 \times 30 = 450$ m

Length of the bridge = Distance covered – Length of the train = $450 - 120 = 330$ m.

EXAMPLE 23.11

Find the time taken by a train 150 m long, running at a speed of 63 kmph to cross another train of length 100 m running at a speed of 45 kmph in the same direction.

SOLUTION

Total distance covered = The sum of the lengths of the two trains = 100 + 150 = 250 m

The relative speed of the two trains = 63 − 45 = 18 kmph = $18\left(\frac{5}{18}\right) = 5$ m/sec.

(Since the trains are running in the same direction, the relative speed will be the difference in the speeds.)

$\therefore$ Time to cross each other = $\frac{250}{5} = 50$ seconds.

EXAMPLE 23.12

A train crosses two persons who are cycling in the same direction as the train, in 12 seconds and 18 seconds, respectively. If the speeds of the two cyclists are 9 kmph and 18 kmph, respectively, find the length and the speed of the train.

SOLUTION

The relative speed while overtaking the first cyclist = $(s - 9)$ kmph, where s kmph being the speed of the train. The time the train took to overtake the first cyclist = 12 seconds.

Hence, the length of the train $= (12)(s-9)\left(\frac{5}{18}\right)$ (1)

Similarly, considering the case of overtaking the second cyclist,

The length of the train $= (18)(s-18)\left(\frac{5}{18}\right)$ (2)

Equating Eqs. (1) and (2),

$$(12)(s-9)\left(\frac{5}{18}\right) = (18)(s-18)\left(\frac{5}{18}\right)$$

$$\Rightarrow 2s - 18 = 3s - 54 \Rightarrow s = 36.$$

That is, the speed of the train is 36 kmph.

$$\text{Length} = (12)(s-9)\left(\frac{5}{18}\right) = (12)(27)\left(\frac{5}{18}\right) = 90\,\text{m}.$$

EXAMPLE 23.13

Two trains running at 45 kmph and 54 kmph crosses each other in 12 seconds, when they run in opposite directions. When they run in the same direction, a person in the faster train observes that he crosses the other train in 32 seconds. Find the lengths of the two trains.

SOLUTION

Let p and q be the lengths of the slower and the faster trains respectively. When the trains are travelling in the opposite directions, their relative speed = 45 + 54 = 99 kmph = 27.5 m/sec.

The distance covered = The sum of the lengths of the two trains = $p + q$

Then we have, $p + q = 12(27.5) \Rightarrow p + q = 330$ m (1)

When the trains are travelling in the same direction, since we are given the time noted by a person in the faster train as 32 seconds, the distance covered is equal to the length of the slower train, i.e., distance covered = p.

The relative speed = 54 − 45 = 9 = 2.5 m/sec

$\therefore\ p = (2.5)\ 32 = 80$ m (2)

From Eqs. (1) and (2), we get $q = 250$ m.

EXAMPLE 23.14

Two trains of lengths 150 m and 250 m run on parallel tracks. When they run in the same direction, it takes 20 seconds to cross each other and when they run in the opposite direction, it takes 5 seconds. Find the speeds of the two trains.

SOLUTION

Let the speeds of the two trains be p m/sec and q m/sec.

The total distance covered = The sum of the lengths of the two trains = 150 + 250 = 400 m

When they run in the same direction, the relative speed $(p - q)$ is given by,

$$p - q = \frac{400}{20} = 20 \tag{1}$$

When they run in the opposite direction, their relative speed is given by

$$p + q = \frac{400}{5} = 80 \tag{2}$$

Solving Eqs. (1) and (2), we get,

$$p = 50 \text{ and } q = 30 \Rightarrow P = 50 \times \frac{18}{5} \text{ kmph and } q = 30 \times \frac{18}{5} \text{ kmph.}$$

$\therefore$ The speeds of the two trains are 180 kmph and 108 kmph.

EXAMPLE 23.15

Two trains take one minute to cross each other when travelling in opposite directions. The speeds of the trains are 54 kmph and 36 kmph. The length of the faster train is 50% more than that of the second. Find the length (in m) of the slower train from the following options:

(a) 450 **(b)** 750 **(c)** 600 **(d)** 500

HINTS

(i) Use relative speed concept.

(ii) Let the length of the slower train be L meters.

(iii) Solve $\dfrac{L + \dfrac{3L}{2}}{(54 + 36)\dfrac{5}{18}} = 60$, for L.

EXAMPLE 23.16

Two trains are travelling in the same direction at 70 kmph and 50 kmph. The faster train passes a man sitting in the slower train in 36 seconds. What is the length of the faster train? Choose the correct answer from the following options:

(a) 100 m **(b)** 150 m **(c)** 200 m **(d)** Cannot be determined

HINTS

(i) To pass a man in the slower train, the faster train has to travel its own length.

(ii) $\dfrac{\text{Length of the faster train}}{\text{Relative speed}} = 36$ seconds.

(iii) Relative speed = Difference of the speeds.

Boats and Streams

Problems related to boats and streams are different in the computation of relative speed from those of trains/cars.

When a boat is moving in the same direction as the stream or water current, the boat is said to be moving **with the stream or downstream**.

When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving **against the stream or upstream**.

If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then called the **speed of the boat in still water**.

When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.

When the boat is moving downstream, the speed of the water aids (and thus increases) the speed of the boat. Thus, we have

Speed of the boat against stream = Speed of the boat in still water − Speed of the stream

Speed of the boat with the stream = Speed of the boat in still water + Speed of the stream

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are speeds with respect to the bank.

If u is the speed of the boat downstream and v is the speed of the boat upstream, then we have the following two relationships.

$$\text{Speed of the boat in still water} = \frac{u+v}{2}$$

$$\text{Speed of the water current} = \frac{u-v}{2}$$

In some problems, instead of a boat, it may be a swimmer. But the approach is exactly the same.

EXAMPLE 23.17

A boat travels 24 km upstream in 6 hours and 20 km downstream in 4 hours. Find the speed of the boat in still water and the speed of the water current.

SOLUTION

$$\text{Upstream speed} = \frac{24}{6} = 4 \text{ kmph}$$

$$\text{Downstream speed} = \frac{20}{4} = 5 \text{ kmph}$$

$$\text{Speed in still water} = \frac{(4+5)}{2} = 4.5 \text{ kmph}$$

$$\text{Speed of the water current} = \frac{(5-4)}{2} = 0.5 \text{ kmph.}$$

EXAMPLE 23.18

A man can row 8 km in one hour in still water. The speed of the water current is 2 kmph and it takes 3 hours for him to go from point *P* to go to point *Q* and return to *P*. Find the distance *PQ*.

SOLUTION

Let the distance be x km.

$$\text{Upstream speed} = 8 - 2 = 6 \text{ kmph}$$

$$\text{Downstream speed} = 8 + 2 = 10 \text{ kmph}$$

Total time = Time taken travelling upstream + Time taken travelling downstream

$$= \frac{x}{6} + \frac{x}{10} = 3 \text{ hours (given)}$$

$$\therefore \frac{8x}{30} = 3$$

$$\Rightarrow x = \frac{90}{8} = 11\frac{1}{4} \text{ km.}$$

EXAMPLE 23.19

A man can row a distance of 6 km in 1 hour in still water and he can row the same distance in 45 minutes with the current. Find the total time taken by him to row 16 km with the current and return to the starting point.

SOLUTION

$$\text{Speed in still water} = \frac{6}{1} = 6 \text{ kmph}$$

$$\text{Upstream speed} = \frac{6}{\frac{45}{60}} = 6 \times \frac{60}{45} = 8 \text{ kmph}$$

$\therefore$ The speed of water current = 8 − 6 = 2 kmph

$\therefore$ The speed against the stream = 6 − 2 = 4 kmph

Hence, time taken to travel 16 km and back

$$= \frac{16}{8} + \frac{16}{4} = 2 + 4 = 6 \text{ hours.}$$

EXAMPLE 23.20

The distance travelled by a boat downstream is $1\frac{1}{2}$ times the distance travelled by it upstream in the same time. If the speed of the stream is 3 kmph, then find the speed of the boat in still water.

SOLUTION

If the distance covered downstream is $1\frac{1}{2}$ times that covered upstream, the speed downstream will also be $1\frac{1}{2}$ times the speed upstream.

Let the speeds of the boat in still water be u.
We get,

$$\frac{(u+3)}{(u-3)} = \frac{3}{2} \Rightarrow u = 15 \text{ kmph.}$$

EXAMPLE 23.21

A man can row $\frac{2}{3}$rd of a kilometre downstream in 5 minutes and return to the starting point in another 10 minutes. Find the speed of the man in still water.

SOLUTION

$$\text{Downstream speed} = \frac{2}{3} \times \frac{60}{5} = 8 \text{ kmph}$$

$$\text{Upstream speed} = \frac{2}{3} \times \frac{60}{10} = 4 \text{ kmph}$$

$$\text{Speed in still water} = \frac{8+4}{2} = 6 \text{ kmph.}$$

EXAMPLE 23.22

A boat would cover the journey from A to B in a river in 12 hours if the river was still. If the speed of the river is $\frac{1}{5}$ times the speed of the boat in still water, find the time (in hours) taken by it to cover around trip journey between A and B. Choose the correct answer from the following options:

(a) 20 **(b)** 30 **(c)** 25 **(d)** 24

HINTS

(i) Let distance be d km and speed of boat in still water be s kmph.

(ii) Given, $\frac{d}{s} = 12$

(iii) The required distance $= \frac{d}{\left(s + \frac{s}{5}\right)} + \frac{d}{\left(s - \frac{s}{5}\right)}$.

EXAMPLE 23.23

A man drove from town A to B. If he had driven 4 kmph faster, he would have reached his destination 2 hours earlier. If he had driven 6 kmph slower, he would have reached his destination 6 hours late. Find the distance between A and B (in kilometres) from the given options:

(a) 120 **(b)** 140 **(c)** 150 **(d)** 160

HINTS

(i) Distance = Speed × Time.

(ii) Let the distance, speed and time be d, s and t.

(iii) From the given data $d = st = (s + 4)(t - 2) = (s - 6)(t + 6) = d$.

(iv) Solve the above equation and find d.

Races

When two persons P and Q are running a race, they can start the race at the same time or one of them may start a little later than the other. In the second case, suppose P starts the race and after 5 seconds Q starts, then we say P has a 'start' of 5 seconds. In a race between P and Q, P starts first and then when P has covered a distance of 10 m, Q starts. Then we say that P has a 'start' or 'head start' of 10 m.

In a race between P and Q, where Q is the winner, by the time Q reaches the winning post, if P still has another 15 m to reach the winning post, then we say that Q has won the race by 15 m. Similarly, if P reaches the winning post 10 seconds after Q reaches it, then we say that Q has won the race by 10 seconds.

In problems on races, we normally consider a 100 m race or a 1 kilometre race. The length of the track need not necessarily be one of the two figures mentioned above but can be as given in the problem.

EXAMPLE 23.24

In a race of 1000 m, A beats B by 50 m or 5 seconds. Find

(a) B's speed.
(b) The time taken by A to complete the race.
(c) A's speed.

SOLUTION

(a) Since A beats B by 50 m, it means by the time A reaches the winning post, B is 50 m away and as A beats B by 5 seconds, it means B takes 5 seconds more than A to reach the winning post. This means B covers 50 m in 5 seconds, i.e., B's speed is 50/5 = 10 m/sec.

(b) Since A wins by 50 m, by the time A covers 1000 m, B covers 950 m at 10 m/sec, B can cover 950 m in 950/10, i.e., 95 seconds or 1 minute 35 seconds.

$\therefore$ A completes the race in 1 minute 35 seconds.

(c) $\therefore$ A's speed is $\dfrac{100}{95} = 10\dfrac{10}{19}$ m/s.

EXAMPLE 23.25

Rakesh runs $1\dfrac{1}{3}$ times as fast as Mukesh. In a race, Rakesh gives a head start of 60 m to Mukesh. After running for how many metres does Rakesh meet Mukesh?

SOLUTION

Since Rakesh runs $1\frac{1}{3}$ times as fast as Mukesh, in the time Mukesh runs 3 meters, Rakesh has run 4 meters, i.e., Rakesh gains 1 m for every 4 meters he runs.

Since he has given a lead of 60 m, he will gain this distance by covering $4 \times 60 = 240$ m.

Hence, they will meet at a point 240 m from the starting point.

EXAMPLE 23.26

In a 100 m race, Tina beats Mina by 20 m and in the same race Mina beats Rita by 10 m. By what distance does Tina beat Rita?

SOLUTION

$$\text{Tina} : \text{Mina} = 100 : 80$$
$$\text{Mina} : \text{Rita} = 100 : 90$$
$$\text{Tina} : \text{Rita} = \frac{100}{80} \times \frac{100}{90} = \frac{100}{72}.$$

$\therefore$ Tina beats Rita by $100 - 72$, i.e., 28 m.

EXAMPLE 23.27

In a 500 m race, the ratio of speeds of two runners, P and Q is 3 : 5. P has a start of 200 m. Who wins the race and by what distance does he win?

SOLUTION

Since the ratio of speeds of P and Q is 3 : 5, by the time P runs 300 m, Q runs 500 m.

Since, P has a start of 200 m, by the time Q starts at the starting point, P has already covered 200 m and he has another 300 m to cover. In the time P covers this 300 m, Q can cover 500 m, thus reaching the finishing point exactly at the same time as P.

$\therefore$ Both P and Q reach the finishing point at the same time.

EXAMPLE 23.28

In a 200 m race, Lokesh gave Rakesh a start of at most 20 m and was beaten by him by at least 20 m. Both have distinct speeds. Which of the following can be the ratio of the speeds of Lokesh and Rakesh?

(a) 2 : 1 **(b)** 4 : 3 **(c)** 5 : 4 **(d)** 5 : 6

SOLUTION

Lokesh gave Rakesh a headstart of atmost 20 m

$\therefore$ Rakesh would have run atleast $(200 - 20)$ m = 180 m

Lokesh was beaten by Rakesh by at least 20 m

$\therefore$ When Rakesh finished, Lokesh would have run atmost $(200 - 20) = 180$ m

Given that their speeds of both are distinct.

$\therefore$ Rakesh run more distance than Lokesh.

$\therefore$ Rakesh must be faster than Lokesh.

Only choice (d) satisfies this condition.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Speed of a person is 45 kmph. What is the distance covered by him in 8 minutes? (in km)

2. Some telegraphic poles are placed 50 m apart. How many such poles can a train cross in 12 minutes travelling at a speed of 36 kmph?

3. If the speed is increased by 20%, then the time taken to cover a certain distance decreases by 20%. [True/False]

4. If the ratio of speeds is $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$, then the time taken to cover a certain distance is in the ratio of $c:b:a$. [True/False]

5. A person travelled half the distance at 20 kmph and the remaining distance at 30 kmph in a total of 10 hours. Find the distance travelled.

6. A person covered two equal distances at 20 kmph and 30 kmph respectively. The average speed in covering the total distance is 25 kmph. [True/False]

7. A person travelled at a speed of 20 kmph for 3 hours and an equal distance at a speed of 15 kmph. What is the average speed of the person?

8. In what time does a train of length 300 m, running at 36 kmph cross an electric pole?

9. The time taken by two trains of lengths x m and y m running at p kmph and q kmph in opposite directions to cross each other is_______.

10. The ratio of speeds of A and B is 8 : 7. In a 1000 m race, by how many meters does A beat B?

11. In a race of 1 km A gives B a start of 100 m and still beats him by 80 m. What is the ratio of speeds of A and B?

12. Speed upstream is x kmph and speed of stream is y kmph. Speed downstream is__________.

13. For a boat, speed downstream and speed upstream are 18 kmph and 14 kmph respectively. Find the speed of the boat in still water.

14. A is $1\frac{2}{3}$ times as fast as B. In a race, A gives B a start of 200 m and still beats him by 120 m. What is the length of the race?

15. In a race of 1 km race A beats B by 40 m or 8 seconds. Is the speed of A 5 m/sec more than that of B?

16. Anand covers a certain distance in 10 hours. If he increases his speed by 5 kmph he takes 2 hours less to cover the same distance. Find the distance.

17. For a boat, ratio of speed downstream to speed upstream is 9 : 5. What is the ratio of speed of boat in still water to the speed of stream?

18. Speed of sound in air is 330 m/sec and in steel is 5960 m/sec. If sound travels 132 km in air, then how many km can it travel in steel in the same time?

19. Time taken by a train to cross a platform is more than the time taken to cross a person moving in opposite direction. [True/False]

20. A bus can cover 75 km in one hour without stoppages. If it covers 60 km in one hour with stoppages, then how many minutes does it stop per hour?

21. A is twice as fast as B and B is one-third as fast as C. If A and C run a race, C beats A by 500 meters. Find the distance of the race.

22. Pavan covered three equal distances at a speed of 20 kmph, 30 kmph and 50 kmph respectively. Find the average speed in covering the total distance.

23. A train crossed a 350 m long platform in 46 second and 725 m long platform in 58.5 second. Find the speed of the train.

24. A train takes 18 seconds to overtake a person travelling at 18 kmph and 27 seconds to overtake another person travelling at 36 kmph. Find the length of train.

25. Varun and Varma travelled from A to B at speeds of 6 kmph and 4 kmph respectively. Varun reaches B, returns immediately and meets Varma at point C. What is the distance between A and C, given that distance between A and B is 20 km?

26. The time taken by a man to row downstream is $\frac{5}{9}$ of the time taken to row upstream. If the product of speeds of the man and the stream, taken in kmph, is 224, find the speed of the man in still water.

27. Travelling at 80 kmph a person can reach his destination in a certain time. He covers $\frac{3}{4}$ of the journey in $\frac{4}{5}$ of the total time. At what speed should he travel the remaining distance to reach his destination on time?

28. Travelling at $\frac{3}{4}$ of his usual speed, a man is 20 minutes late. What is his usual time to cover the same distance?

29. A person travelled at a speed of 50 kmph and missed the bus by 40 minutes. Had he travelled at 60 kmph he would have still missed the bus by 20 minutes. At what minimum speed should he travel to catch the bus?

30. Travelling at 80% of his usual speed a man is late by 3 hours. In what time does he reach his destination travelling at 40% less than his usual speed?

Short Answer Type Questions

31. The poles on the road are 40 m apart. How many poles will be passed by a car in $2\frac{1}{4}$ hours if the speed of the car is 72 kmph?

32. Travelling at 25% less than his usual speed a person reaches his destination 14 minutes late. If he travels at 40% more than his usual speed, then in how many minutes can he reach his destination?

33. The speeds of two trains are in the ratio of 4 : 3. If the first train takes 15 seconds less to cover a distance, what is the time taken by the second train to cover the same distance?

34. Train A is 50% longer and twice as fast as train B, train B takes 40 seconds to cross a 200 m platform. Find the time taken by A to cross a 300 m platform. (in seconds)

35. A wooden log floating in a river took 48 hours to travel from P to Q. Find the time taken by a boat to make a round trip journey between P and Q. If its speed is seven times the speed of the river. (in hours)

36. In a 100-m race, Chetan gives Dinesh a start of atmost 10 m and is beaten by Dinesh by atmost 10 m. Which will be a possible ratio of the speeds of Chetan and Dinesh if their speeds are distinct?

37. In a race of 1 km A beats B by 20 seconds and B beats C by 10 seconds. If A beats C by 300 meters, then A beats B by how many meters?

38. A car travelling in fog crossed a person who is walking at a speed of 6 kmph in the same direction. The person can see the car only for 6 minutes and up to a distance of 150 meters. What is the speed of the car?

39. Car A started from P towards Q at 7 am at 30 kmph. Car B started from Q towards P at 9 am at 60 kmph. The distance between P and Q is 255 km. A did not stop anywhere on its journey but B stopped for 35 minutes at a point R after it had travelled 120 km from Q. Find the distance of the point where both cars meet from P. (in km)

40. A person covered 60 km travelling for 3 hours by foot and 2 hours by cycling. Had he travelled 2 hours on foot and 3 hours by cycling he would have travelled 5 km more. Find his speed when cycling.

41. In a race A beats B by 200 m and C by 300 m. In the same race B beats C by 125 m. What is the length of the race track?

42. A boat travelled from A to B and returned to A in 10 hours. The speed of boat in still water and speed of water is 10 kmph and 2 kmph respectively. Find the distance between A and B.

43. A person travelled from Hyderabad to Cuddapah. If he had travelled 15 kmph faster he would have reached Cuddapah 2 hours earlier. If he had travelled 10 kmph slower he would have reached Cuddapah 2 hours late. Find the distance between Hyderabad and Cuddapah.

44. A and B start at the same time from X and Y respectively and travel towards Y and X respectively. After they meet, they exchange their speeds and travel towards their respective destinations. If A takes 140 minutes to travel from X to Y, then find the time taken by B to travel from Y to X.

45. Two trains are 120 km apart and travelling in opposite directions at a speed of 20 kmph and 10 kmph respectively. A bird flew from one train engine and reaches other train engine at a speed of 25 kmph. This process repeated till the two trains crash. What is the total distance travelled by the bird?

Essay Type Questions

46. A and B started from P and Q towards each other. They met after 6 hours. After meeting A increased his speed by 2 kmph and B decreased his speed by 2 kmph. Both proceeded to their destinations at their new speeds and reached them simultaneously. Twice the initial speed of B was 18 kmph more than the initial speed of A. Find the initial speed of A. (in kmph)

47. The sum of the average of the speeds of Ram and Shyam and the average of the speeds of Ram and Tarun equals the sum of the average of the speeds of Shyam and Tarun and the average of the speeds of the three. If the three run a race and their speeds are distinct, who cannot be the winner of the race?

48. There are two points 10 km apart in a river. Ram took 8 hours more to cover the distance between these points upstream than to cover this distance downstream. If the boat's speed in still water had doubled, he would have taken $1\frac{1}{4}$ hours more to cover the same distance. Find the ratio of the speed of the boat in still water and the speed of the river.

49. Two trains are travelling at speeds of 20 kmph and 25 kmph on parallel tracks. And the distance between the trains is 45 km. A bird starts flying between the two trains at a speed of 30 kmph. What is the distance (in km) travelled by the bird when the two trains cross each other in opposite directions?

50. In a 100 m race, Ajay gives Bala a start of at least 10 m and is beaten by Bala by almost 10 m. Which of the following can be a possible ratio of the speeds of Ajay and Bala if their speeds are distinct?

    (a) 4 : 5 (b) 4 : 3

CONCEPT APPLICATION

Level 1

1. Travelling at 60 kmph, a person reaches his destination on time. If he travels half the distance in $\frac{3}{4}$ of the time, then at what speed should he travel the remaining distance to reach his destination on time?

   (a) 140 kmph (b) 120 kmph
   (c) 100 kmph (d) 160 kmph

2. A car covers a distance of 260 km at a constant speed. It would have taken 1 h 5 minutes less to travel the same distance, if its speed was 20 kmph more. Find the speed of the car. (in kmph)

   (a) 60 (b) 80
   (c) 75 (d) 70

3. A car covers a distance of 420 km at a constant speed. If its speed is 10 kmph less, it would have taken one hour more to travel the same distance. Find the speed of the car.

   (a) 50 kmph (b) 75 kmph
   (c) 60 kmph (d) 70 kmph

4. Ram and Shyam started from A and B respectively. They travelled towards each other with speeds of 30 kmph and 20 kmph respectively. After their meeting, Shyam took 5 hours more to reach A than the time Ram took to reach B. Find the distance between A and B. (in km)

   (a) 250 (b) 300
   (c) 350 (d) 400

5. When speed is increased by 6 kmph, time taken to cover certain distance is decreased by 4 hours. If speed decreased by 4 kmph, then time taken to cover the same distance increases by 4 hours. Find the distance.

   (a) 480 km (b) 360 km
   (c) 240 km (d) Cannot be determined

6. A boy is late to his school by 20 minutes, if he travels at a speed of 4 kmph. If he increases his speed to 6 kmph, he is still late to his school by 10 minutes. At what speed should he travel to reach the school on time?

   (a) 8 kmph (b) 10 kmph
   (c) 12 kmph (d) 9 kmph

7. Two friends Rohan and Mohan start cycling from a place at speeds of 20 kmph and 17 kmph respectively in the same direction at 9 o'clock in the morning. By how many kilometres would Mohan be behind Rohan by 5 pm?

(a) 18 (b) 20

(c) 24 (d) 22

8. If a man increases his speed by 25%, he would take 10 minutes less to reach his destination. If he increases his speed by $11\frac{1}{9}\%$, find the number of minutes he saves to reach his destination.

(a) 4 (b) 5

(c) 6 (d) 8

9. If a person increased his speed by 2 kmph, he would take 2 hours less to reach his destination. If he increased his speed by 6 kmph, he would take 4 hours less to reach his destination. Find the distance (in km) he has to travel to reach his destination.

(a) 24 (b) 36

(c) 48 (d) 54

10. A train takes 50 seconds to cross a motorcyclist travelling at 36 kmph. It would take a minute to cross a stationary pole, if its speed was increased by 5 m/sec. Find its length (in m).

(a) 1000 (b) 1250

(c) 1500 (d) 1750

11. Uday and Aryan started from P and Q respectively towards each other with speeds of 50 kmph and 75 kmph respectively. After meeting, Uday took $3\frac{1}{3}$ hrs more to reach Q than the time Aryan took to reach P. Find the distance between A and B (in km).

(a) 400 (b) 600

(c) 500 (d) 550

12. A train takes 20 seconds to overtake a cyclist travelling at 9 kmph. It takes 30 seconds to overtake another cyclist travelling at 18 kmph. Find the length of the train. (in m)

(a) 75 (b) 100

(c) 125 (d) 150

13. A person travels from A to B at a speed of 60 kmph for 80 minutes and travels from B to C at a constant speed for 120 minutes. If his average speed from A to C is 48 kmph, then find the constant speed from B to C.

(a) 40 kmph (b) 50 kmph

(c) 70 kmph (d) 80 kmph

14. Trains A and B have lengths of 300 m and 200 m. They take 50 seconds to cross each other when travelling in the same direction. They take 10 seconds to cross each other when travelling in opposite directions. Find the speed of the faster train. (in m/sec)

(a) 35 (b) 40

(c) 45 (d) 30

15. Two cars are 80 km apart. If they travelled in the same direction, they would take 4 hours to meet. If they travelled in opposite directions, they would take one hour to meet. Find the speed of the slower car. (in kmph)

(a) 30 (b) 20

(c) 10 (d) 115

16. A log floating in a river took 12 hours to travel from A to B. Find the time (in hours) taken by a boat to make a round trip journey between A and B, if its speed in still water is five times the speed of the river.

(a) 6 (b) 4

(c) 7 (d) 5

17. A train leaves Hyderabad at 6 am, travelling at uniform speed reaches Chennai at 2 pm. Another train leaves Chennai at 7 am, travelling at a uniform speed reaches Hyderabad by 3 pm. When do the two trains meet?

(a) 10 am (b) 10:30 am

(c) 11 am (d) 11:30 am

18. Two cars are 120 km apart. If they travelled in the same direction, they would take 2 hours to meet. If they travelled in opposite directions, they would take $\frac{3}{2}$ hours to meet. Find the speed (in kmph) of the faster car.

(a) 70 (b) 80

(c) 60 (d) 50

19. An escalator moves up at the rate of 6 ft/second and its length is 20 ft. If a person walks up on the moving escalator at the rate of 2 ft/second, how much time does he take to cover the entire length?

(a) 1.5 seconds (b) 2 seconds
(c) 2.5 seconds (d) 3 seconds

20. Train A can cross a 180 m long platform in 90 seconds. Train B has a speed which is twice that of A. A's length is 90% of that of B. B can cross a 200 m long platform in x seconds. Find x.

(a) 40 (b) 45
(c) 50 (d) 60

21. Train A started from station P at 60 kmph at 9:00 am. Train B started from the same station at 80 kmph at 11:00 am. Both the trains travelled in the same direction and met at station Q. Find the time of their meeting.

(a) 4:00 pm (b) 6:00 pm
(c) 7:00 pm (d) 5:00 pm

22. In a 100 m race, Ashok is given by Bala a start of 20 m. Ashok beats Bala by 10 m. Which of the following represents the ratio of the speeds of Ashok and Bala?

(a) 6 : 5 (b) 8 : 9
(c) 5 : 6 (d) 2 : 1

23. In a 100 m race, Alok gives Bala a start of 10 m and beats him by 10 m or 2 seconds. Find Alok's speed (in m/sec).

(a) $6\frac{1}{4}$ (b) $5\frac{5}{9}$
(c) $6\frac{2}{3}$ (d) $5\frac{1}{3}$

24. In a 100 m race, Ravi beats Ramu by 4 m or by 2 seconds. Find the approximate speed of Ravi.

(a) 2.08 m/sec (b) 3.08 m/sec
(c) 3.18 m/sec (d) 2.18 m/sec

25. An escalator has 50 steps. Ajay starts walking up on it at 3 steps/second. If the escalator moves up at 2 steps/second, then find the time (in seconds) he would take to reach its top.

(a) 8 (b) 10
(c) 12.5 (d) 16

26. A train, running at a uniform speed, crosses a platform in 45 seconds and another longer platform in 1 minute. What is the ratio of the length of the longer platform to that of the smaller platform?

(a) 3 : 4 (b) 4 : 3
(c) 1 : 2 (d) Cannot be determined

27. If a person travels three equal distances at speeds of 15 kmph, 30 kmph and 20 kmph respectively, then the average speed during his whole journey is ______ kmph.

(a) 22.5 (b) 2.5
(c) 25 (d) 20

28. Raj travelled along a square plot ABCD. He travelled along AB, BC, CD and DA at speeds of 1 kmph, 2 kmph, 1 kmph and 2 kmph respectively using the route A $\rightarrow$ D $\rightarrow$ C $\rightarrow$ B $\rightarrow$ A. His average speed from A to C is $\frac{4}{3}$ kmph. Find his average speed (in kmph) from D to B.

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$
(c) $\frac{8}{3}$ (d) None of these

29. In a 100 m race, Ram gives Shyam a start of 10 m and is beaten by Shyam by 10 m. What is the ratio of the speeds of Ram and Shyam?

(a) 9 : 11 (b) 5 : 4
(c) 1 : 1 (d) 9 : 10

Level 2

30. In a 200 m race, Ajay gives Vijay a start of 10 m and beats him by 10 m. If Ajay started 16 m before the start line and Vijay started at the start line, Ajay would

(a) be beaten by Vijay by 0.8 m.
(b) have beaten Vijay by 5.6 m.
(c) have finished simultaneously with Vijay.
(d) None of these

PRACTICE QUESTIONS

31. A boat can travel at a speed of 8 kmph upstream and 10 kmph downstream. If it travels a distance of 40 km upstream and 50 km downstream, then the average speed for the entire journey is _________.

(a) 8 kmph (b) 10 kmph
(c) 12 kmph (d) 9 kmph

32. A train takes 60 seconds to cross a 480 m long platform. If the speed of the train had been 5 m/sec less, it would have taken 48 seconds to cross a stationary pole. Find the speed of the train in m/sec.

(a) 20 (b) 22
(c) 25 (d) 28

33. Mahesh and Nitin started from P and Q respectively towards each other. They meet after 5 hours. After meeting, Mahesh increased his speed by 1 kmph and Nitin decreased his speed by 1 kmph. Both proceeded to their destinations at their new speeds and reached them simultaneously. Thrice the initial speed of Nitin was 12 kmph more than the initial speed of Mahesh. Find the initial speed of Mahesh. (in kmph)

(a) 3.5 (b) 6.5
(c) 5.5 (d) 4.5

34. Suraj and Chand started from P and Q towards Q and P respectively with speeds of 10 m/sec and 15 m/sec respectively. The distance between P and Q is 75 m. On reaching their destinations, they proceeded towards their starting points with their speeds unchanged. Find the total distance Suraj would have travelled before meeting Chand for the second time (in m).

(a) 180 (b) 90
(c) 105 (d) 45

35. In a race, Prakash beats Pramod by 20 m. If the length of the race was 50 m less, Prakash would have beaten Pramod by 15 m. Find the length of the race (in m).

(a) 100 (b) 200
(c) 150 (d) 250

36. Ramesh and Suresh started from A and B towards B and A and with speeds of 3 m/sec and 5 m/sec respectively. The distance between A and B is 8 m. On reaching their destinations, they turn back towards their starting points with their speeds unchanged. Find the total distance Ramesh would have travelled before meeting Suresh for the second time. (in m)

(a) 9 (b) 10
(c) 11 (d) 12

37. An escalator is moving downwards. Ganga takes 270 steps to reach the top of the escalator from its bottom. Malik takes 108 steps to reach the bottom from the top of the escalator. Time taken by Malik to reach the bottom is same as the time in which Ganga takes 216 steps. How many steps are there from the bottom to the top of the escalator?

(a) 162 (b) 180
(c) 240 (d) 250

38. In a race on a track of a certain length, Raja beats Ramesh by 20 m. When Raja was 10 m ahead of the mid-point of the track Ramesh was 2 m behind it. Find the length of the track. (in m)

(a) 150 (b) 200
(c) 250 (d) 100

39. Saurav and Gaurav started from two towns A and B and travelled towards each other simultaneously. They met after 4 h. After meeting Saurav took 6 hours less to reach B than what Gaurav took to reach A. Find the ratio of the speeds of Saurav and Gaurav.

(a) 3 : 2 (b) 2 : 3
(c) 4 : 3 (d) 2 : 1

40. In a 500 m race, if A gives B a 50 m start, B wins by 5 seconds but if A gives B a 5 second start, A wins by 30 m. Find the time that B takes to run 500 m race.

(a) 240 seconds (b) 125 seconds
(c) 250 seconds (d) 120 seconds

41. A boat covers a round trip in a river in a certain time. If its speed in still water is doubled and the speed of the stream tripled, it would take the same time for the round trip journey. Find the ratio of the speed of the boat in still water and the speed of the stream.

(a) $\sqrt{3} : \sqrt{2}$ (b) $\sqrt{5} : \sqrt{2}$
(c) $\sqrt{7} : \sqrt{2}$ (d) $3 : \sqrt{2}$

42. Train X leaves Hyderabad at 5 pm and reaches Bangalore by 5 am. Train Y leaves Bangalore at 6 pm and reaches Hyderabad at 6 am. Both trains travel their entire journeys at uniform speeds. Find their meeting time.

(a) 12 pm (b) 11:30 pm
(c) 10:30 pm (d) 1 pm

43. For a boat, ratio of speed downstream to speed upstream is 9 : 5. What is the ratio of the speed of boat in still water to the speed of stream?

(a) 6 : 5 (b) 7 : 2
(c) 8 : 3 (d) 9 : 4

44. Speed of sound in air is 330 m/sec and in steel is 5960 m/sec. If sound travels 132 km in air, then how many km can it travel in steel in the same time?

(a) 2384 (b) 2469
(c) 2218 (d) 2513

45. A person travelled at a speed of 20 kmph for 3 hours and an equal distance at a speed of 15 kmph. What is the average speed of the person (in kmph)?

(a) $17\frac{1}{7}$ (b) $17\frac{1}{2}$
(c) $16\frac{1}{3}$ (d) $16\frac{1}{2}$

46. For a boat, speed downstream and speed upstream are 18 kmph and 14 kmph respectively. Find the speed of the boat in still water (in kmph).

(a) 14 (b) 15
(c) 16 (d) 17

Level 3

47. Two trains of length 270 m and 300 m travelling at 54 kmph and 36 kmph respectively, enter a two track tunnel 430 m long simultaneously on different tracks and from the opposite directions. After they cross each other, in how much time will the tunnel be free of the trains?

(a) 25 second (b) 30 second
(c) 33 second (d) 41 second

48. In a 1000 m race, A gives B a head start of 100 m and still he beats B by 100 m or 20 second. Find the speed of A.

(a) 15 m/sec (b) 6.25 m/sec
(c) 10 m/sec (d) 12.5 m/sec

49. An escalator is moving downwards. A takes 140 steps to reach the top from the bottom of an escalator. B takes 60 steps to reach the bottom from the top of the escalator. Time taken by B to reach the bottom is same as the time during which A takes 20 steps. How many steps are there from the bottom to the top of the escalator?

(a) 70 (b) 60
(c) 90 (d) 80

50. Ramesh and Satish started simultaneously from two towns P and Q and travelled towards each other. They met after 2 hours. After meeting, Ramesh took 3 hours less to reach Q than what Satish took to reach P. Find the ratio of Ramesh's speed and Satish's speed.

(a) 3 : 1 (b) 2 : 1
(c) 3 : 2 (d) 4 : 1

51. Rohan travelled from P to Q at a speed of 80 kmph for 60 minutes and travelled from Q to R at a certain speed for 60 minutes. His average speed of travel from P to R was 84 kmph. Find his travel speed from Q to R. (in kmph)

(a) 96 (b) 88
(c) 90 (d) 100

52. A car covered a distance of 600 km at a constant speed. If its speed was 50 kmph more, it would have taken 2 hours less to cover the same distance. Find its speed. (in kmph)

(a) 125 (b) 75
(c) 200 (d) 100

53. Two cars are 360 km apart. If they started in the same direction, then they will take 9 hours to meet. If they started simultaneously in opposite directions to each other, then they will take 3 hours to meet. Find the speed of the faster car. (in kmph)

(a) 70 (b) 80
(c) 65 (d) 75

54. A boy walks from home to school. One day, he walked 20% slower than his usual speed and reached 9 minutes late. Find the time taken to reach the school with usual speed. (in minutes)

(a) 30 (b) 36

(c) 45 (d) 60

55. Train P has a length 25% more than train Q. It is thrice as fast as Q. P would take 30 seconds to cross a 175 m long platform. Find the time that Q would take to cross a 140 m long platform. (in seconds)

(a) 48 (b) 90

(c) 72 (d) 63

56. If Hari increases his speed by 5 kmph, he would reach his destination 1 hour early. If he decreases his speed by 2.5 kmph, he will reach his destination 48 minutes late. Find the distance to be travelled by him. (in km)

(a) 45 (b) 60

(c) 75 (d) 90

57. Ram, Shyam and Tarun ran a race. The average of the speeds of the three when doubled would equal the sum of the average of the speeds of Ram and Shyam and the average of the speeds of Ram and Tarun. The speeds of the three are distinct. The winner cannot be

(a) Ram (b) Shyam

(c) Tarun (d) Cannot be determined

58. Ramesh and Suresh started simultaneously from P and Q respectively. Each of them started towards the starting point of the other. The speeds of Ramesh and Suresh throughout their journeys were 50 kmph and 40 kmph respectively. After meeting, Suresh took $(x + 9)$ hours to reach P while Ramesh took x hours to reach Q. Find PQ. (in km)

(a) 2160 (b) 1800

(c) 2070 (d) 2340

59. Ravi travelled from P to Q at 12 kmph. He then travelled from Q to R at 32 kmph and then from R to S at 10.8 kmph. 2PQ = 3QR = 4RS. Find Ravi's average speed for his entire journey. (in kmph)

(a) $\frac{64}{5}$ (b) $\frac{72}{5}$

(c) $\frac{54}{5}$ (d) $\frac{66}{5}$

60. Anil and Sunil started simultaneously from X and Y respectively. Each person started towards the starting point of the other. XY = 18 km. The speeds of Anil and Sunil throughout their journeys were 5 kmph and 4 kmph respectively. On reaching X or Y, each turned back immediately to resume the journey to his starting point. Find the total distance travelled by Anil before meeting Sunil for the second time. (in m)

(a) 30 (b) 27

(c) 24 (d) 33

61. Ganesh and Harish started simultaneously from A and B respectively travelling towards each other. They met after 6 hours. After meeting, Ganesh took $y + 5$ hours to reach B while Harish took y hours to reach A. Find the ratio of the speeds of Ganesh and Harish.

(a) 3 : 2 (b) 2 : 3

(c) 1 : 2 (d) 1 : 3

62. In a race, Ramu beats Somu by 10 m. When Ramu was 4.5 m ahead of the mid-point of the race track, Somu was 1 m behind it. Find the length of the race. (in m)

(a) 90 (b) 108

(c) 120 (d) 75

63. In a 1200 m race, Rohan beats Sohan by 40 seconds and Sohan beats Mohan by 80 seconds. In the same race, Rohan beats Mohan by 400 m. Find the time taken by Rohan to run the race. (in seconds)

(a) 480 (b) 240

(c) 360 (d) 300

64. A log was floating in a river. It took 36 hours to travel from a point P in the river to point Q in it. A boat has its speed in still water equal to 9 times the speed of the river. Find the time it would take to cover a round trip journey between P and Q. (in hours)

(a) 7.5 (b) 7.8

(c) 8.1 (d) 8.4

65. Two trains of lengths 360 m and 540 m have speeds of 97.2 kmph and 108 kmph respectively. They entered a 450 m long tunnel simultaneously. Find the time taken for the tunnel to be free of traffic. (in seconds)

(a) 33 (b) 36

(c) 30 (d) 39

66. Anil travelled from P to Q at f kmph. He returned to Q at r kmph. His average speed for the entire-journey was $\frac{f+r}{2}$ kmph. PQ = 240 km. His total travel time was 6 hours. Find f.

(a) 120 (b) 90

(c) 80 (d) 60

67. A boat covers a round trip journey between two points A and B in a river in T hours. If its speed in still water triples and the speed of the river doubles, it would take $\frac{9}{32}T$ hours for the same journey. Find the ratio of its speed in still water to the speed of the river.

(a) 3 : 1 (b) 3 : 2

(c) 2 : 1 (d) 4 : 3

68. Amish and Bala started simultaneously from X and Y respectively towards each other. They met after 4 hours. After meeting, Amish decreased his speed by 10 kmph while Bala increased his speed by 10 kmph. Each of them proceeded towards the starting point of the other. Both reached their destinations simultaneously. XY = 200 km. Find the initial speed of Amish (in km)

(a) 40 (b) 35

(c) 25 (d) 30

69. In a race, Ram beats Shyam by 30 m in the same race, Shyam beats Tarun by 60 m while Ram beats Tarun by 84 m. Find the length of the race. (in m)

(a) 360 (b) 270

(c) 300 (d) 450

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 6 km
2. 145
3. False
4. False
5. 240 km
6. False
7. $17\frac{1}{7}$ kmph
8. 30 second
9. $\frac{18(x+y)}{5(p+q)}$ second
10. 125 m
11. 50 : 41
12. $(x + 2y)$ kmph
13. 16 kmph
14. 800 m
15. Yes
16. 200 km
17. 7 : 2
18. 2384 km
19. True
20. 12 minutes
21. 1500 m
22. $29\frac{1}{31}$ kmph
23. 108 kmph
24. 270 m
25. 16 km
26. 28 kmph
27. 100 kmh
28. 1 hour
29. 75 kmph
30. 20 hours

Shot Answer Type Questions

31. 4050
32. 30
33. 60 second
34. 30
35. 14
36. 4 : 5
37. $222\frac{2}{9}$ m
38. 7.5 kmph
39. 135
40. 15 kmph
41. 1000 m
42. 48 km
43. 600 km
44. 140 minutes
45. 100 km

Essay Type Questions

46. 14
47. Ram
48. 45
49. 30
50. 4 : 3

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (c)
11. (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)
21. (d)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (b)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (c)	

Level 2

30. (b)	**31.** (d)	**32.** (a)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (d)
40. (c)	**41.** (c)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (c)			

Level 3

47. (c)	**48.** (b)	**49.** (a)	**50.** (b)	**51.** (b)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (b)
57. (a)	**58.** (b)	**59.** (b)	**60.** (a)	**61.** (b)	**62.** (a)	**63.** (b)	**64.** (c)	**65.** (a)	**66.** (c)
67. (c)	**68.** (d)	**69.** (c)							

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. He has to travel half of the distance in $\frac{1}{4}$ of the time.
2. Let the speed be x kmph, and the difference between the times taken $= \frac{13}{12}$ hours.
3. Let the speed be x kmph and difference in the times taken in two cases be 1 hour.
4. Let the distance be d km and difference between their total time taken is 5 hours.
5. (i) Speed × Time = Distance.
   (ii) Let the distance, speed and time be d, s and t respectively.
   (iii) From the given data, $d = st$, $(s + 6)(t - 4) = d$ and $(s - 4)(t + 4) = d$.
   (iv) Solve the above equations and find d.
6. Let the distance be d km and difference in the times taken in two cases be 10 minutes.
7. Find the relative speed.
8. Find the ratio of speeds in both the cases.
9. Let usual speed be x kmph and usual time be y hours and frame the equations.
10. (i) Length of the train = Time × Relative speed.
    (ii) Let the length of train be L m and its speed be s m/sec.
    (iii) Solve $\frac{L}{s+10} = 50$ and $\frac{L}{s+5} = 60$ for L.
11. Let the total distance be k km.
12. Find the length of the train with respect to speed and time in two cases.
13. Use the formula to find average speed.
14. (i) Use relative speed concept.
    (ii) Let the speeds of the faster and the slower trains be f m/sec and s m/sec respectively.
    (iii) Now, solve $\frac{300+200}{f-s} = 50$ and $\frac{300+200}{f+s} = 10$.
15. Use the concept of relative speed.
16. (i) Let the speed of the river be x kmph, then $AB = 12x$ km.
    (ii) Let the distance between A and B be d km.
    (iii) Speed of river $= \frac{d}{12}$ and Speed of boat $= \frac{5d}{12}$.
    (iv) Time taken for round journey
    $$= \frac{d}{\frac{5d}{12} - \frac{d}{12}} + \frac{d}{\frac{5d}{12} + \frac{d}{12}}.$$
17. Both trains take 8 hours each to reach their respective destinations.
18. Use the concept of relative speed.
19. Time taken = Total distance/Relative speed.
20. Form the equations using the given data.
21. Distance that A would have travelled by 11:00 am is 120 km.
22. Ashok has to run a distance of 80 m and Bala has to run a distance of 90 m.
23. Speed of Bala $= \frac{10}{2} = 5$ m/s.
24. Speed of Ramu $= \frac{4}{2} = 2$ m/s.
25. Use the concept of relative speed.
26. (i) The length or the speed of the train are not given.
    (ii) Let the length of the train, the smaller platform and the longer platform be l, l_1 and l_2 respectively.
    (iii) Speed of the train $= \frac{(l + l_1)}{45} = \frac{(l + l_2)}{60}$.
    (iv) As the length of the train is not given, check whether we get $l_2 : l_1$ or not.
27. (i) If a person covers three equal distances, then average speed $= \frac{3xyz}{xy + yz + zx}$, where x, y, z are the speeds.

(ii) Let the distance in each case be d km.

(iii) Total distance travelled = $3d$ km and the total time taken $=\left(\frac{d}{15}+\frac{d}{30}+\frac{d}{20}\right)$ hours.

(iv) Use, average speed $=\frac{\text{Total distance travelled}}{\text{Total time taken}}$.

28. (i) Average speed from D to B = Average speed from A to C.

(ii) Average speed from A to C is equal to the average speed from D to B.

29. Shyam has to run 90 m and Ram has to run 90 m.

Level 2

30. (i) The ratio of the speeds of Ajay and Vijay $=\frac{200}{200-20}=\frac{10}{9}$.

(ii) As Ajay gives Vijay a start of 10 m and beats him by 10 m, Vijay finished 180 m when Ajay finished 200 m.

(iii) Find the distance covered by Vijay when Ajay covered 216 m.

(iv) Consider, Vijay is 16 m ahead of Ajay.

31. Use the formula to find average speed.

32. (i) Find the length of the train in both the cases.

(ii) Let the length and speed of the train be l m and s m/sec.

(iii) $\frac{l+480}{s}=60$ and $\frac{l}{(s-5)}=48$. Solve for s.

33. (i) $\frac{5m}{n-1}=\frac{5n}{m+1}$, where m and n are speeds of Mahesh and Nithin.

(ii) In 5 hours Mahesh covers a distance of $5m$ km and Nitin covers a distance of $5n$ km.

(iii) Now solve, $\frac{5n}{m+1}=\frac{5m}{n-1}$ and $3n = m + 12$.

34. (i) Suraj and Chand travel a distance of (3×75) m when they meet for the second time.

(ii) They have to cover a distance of (3×75) m to meet for the second time.

(iii) Time taken $=\frac{(3\times 75)}{\text{Relative speed}}$. (say t)

(iv) Distance travelled by Suraj = Speed $\times$ t.

35. (i) Find the ratio of speeds of Prakash and Pramod.

(ii) Let the length of the track be x m.

(iii) If Prakash covers x m, then Pramod covers $(x - 20)$ m.

(iv) If Prakash covers $(x - 50)$ m, then Pramod covers $(x - 50 - 15)$ m.

(v) Solve $\frac{(x-20)}{x}=\frac{(x-65)}{(x-50)}$, for x.

36. (i) When they meet for the second time, the total distance covered by the two = 8 + 8 + 8 = 24 m.

(ii) They have to cover a distance of (3×8) m to meet for the second time.

(iii) Time taken $=\frac{(3\times 8)}{\text{Relative speed}}$ (say t)

(iv) Distance travelled by Suraj = Speed $\times$ t.

37. (i) Let the escalator move x steps while Malik takes 160 steps.

(ii) The ratio of speeds of Ganga and Malik = 216 : 108 = 2 : 1.

(iii) Let the speeds of Ganga and Malik be $2x$ steps/second and x steps/second. Let the speed of the escalator be e steps/second

(iv) Ganga's effective speed = $(2x - e)$ steps/second. Malik effective speed = $(x - e)$ steps/second.

38. (i) Find the ratio of the speeds of Raja and Ramesh.

(ii) Let the length of the track be $2x$ m.

(iii) For $2x$ m, Raja beats Ramesh by 20 m. For x + 10 m, Raja beats Ramesh by (10 + 2) m.

(iv) Solve $\frac{20}{2x}=\frac{12}{x+10}$, for $2x$.

39. (i) Let the speeds of Sourav and Gaurav be s m/sec and g m/sec.

(ii) In 4 hours, Sourva travels $4s$ m and Gourav travels $4g$ m.

(iii) $\frac{4s}{g}-\frac{4g}{s}=6$.

HINTS AND EXPLANATION

(iv) Let $\frac{s}{g} = a$ and solve for a.

40. If A gives B, a 50 m start, then the time B takes to run 450 m is 5 seconds less the time A takes to run 500 m. If A gives B, a 5 second start, it is 5 seconds more.

41. (i) Distance covered in both the cases is same.

(ii) Let the speeds of the boat in still water and that of the stream be b m/sec and s m/sec respectively.

(iii) $\frac{d}{b+s} = \frac{d}{b-s} = \frac{d}{2b+3s} + \frac{d}{2b-3s}$.

Simplify this equation to get $b : s$.

42. Let the distance between Hyderabad and Bangalore be d km.

Travel times of X and Y are 12 hours each.

Speed of X = $\frac{d}{12}$ kmph and speed of Y = $\frac{d}{12}$ kmph.

At 6 pm, X would have covered $\frac{d}{12}$ km while Y would start its journey. Distance between them at 6 pm $\left(d - \frac{d}{12}\right)$ km = $\frac{11d}{12}$ km.

They would meet in another $\dfrac{\frac{11d}{12}}{\frac{d}{12} + \frac{d}{12}}$ hours

= 5.5 hours.

∴ Meeting time = 11:30 pm.

43. Let the speed in still water and speed of the stream be x kmph and y kmph respectively.

$$\frac{x+y}{x-y} = \frac{9}{5}$$

$5x + 5y = 9x - 9y$

$\Rightarrow 4x = 14y \Rightarrow x : y = 7 : 2$.

44. Distance covered in steel $= 132 \times \frac{5960}{330} = 2384$ km.

45. Total distance covered = 2(20 × 3) = 120 km.

Total time taken $= 3 + \frac{60}{15} = 7$ hours.

Average speed $= \frac{120}{7}$ kmph $= 17\frac{1}{7}$ kmph.

46. Speed of the boat in still water $= \frac{18+14}{2} = 16$ kmph.

Level 3

47. (i) Find the time taken by each train to cross the tunnel.

(ii) Find the time taken in which the tunnel is free of the two trains, i.e., $\frac{(270+300+430)}{\text{Relative Speed}} = t$. (say)

(iii) Now find the difference between time taken by the slower train to cross the tunnel and t.

48. (i) B takes 20 second to cover 100 m.

(ii) Speed of B $= \frac{100}{20}$ m/s.

(iii) Find the time taken (say t) by B to run 800 m.

(iv) Time taken by A $= (t - 20)$s

(v) Speed of A $= \frac{100}{(t-20)}$ m/s.

49. (i) The ratio of the speeds of A and B = 20 : 60 = 1 : 3. Let the speeds of A and B be x steps/second and $3x$ steps/second. Let the speed of the escalator be e steps/second.

(ii) A's effective speed $= (x - e)$ steps/second.

B's effective speed $= (3x + e)$ steps/second.

(iii) Time taken by A to travel 140 steps $= \frac{140}{x}$ second. Time taken by B to travel 60 steps $= \frac{60}{3x}$ second.

(iv) Number of steps of escalator $(d) = \frac{60}{3x}$

$$(3x + e) = \frac{140}{x}(x - e).$$

(v) Use the relation between x and e and then proceed.

50. (i) $\frac{2s}{r} = \frac{2r}{s} - 3$, where s and r are speeds of Ramesh and Satish.

(ii) Let the speeds of Ramesh and Suresh be r m/sec and s m/sec respectively.

(iii) In two hours, Ramesh travels $2r$ m and Suresh travels $2s$ m.

HINTS AND EXPLANATION

(iv) $\therefore \frac{2r}{s} - \frac{2s}{r} = 3$

(v) Let $\frac{r}{s} = a$, then solve for a.

51. $PQ = (80)\left(\frac{60}{60}\right)\text{km} = 80\,\text{km}$

$PR = (84)\left(\frac{60+60}{60}\right)\text{km} = 168\,\text{km}$

QR = PR – PQ = 88 km

$\therefore$ Travel speed $= \frac{88}{\left(\frac{60}{60}\right)}\text{kmph} = 88\,\text{kmph}$

52. Let its speed be s kmph

Time taken $= \frac{600}{s}$ hours

$\frac{600}{s} - \frac{600}{s+50} = 2$

$\frac{600(s+50-s)}{s(s+50)} = 2$

$30000 = 2(s^2 + 50s)$

$s^2 + 50s - 15000 = 0$

$(s + 150)(s - 100) = 0$

$s > 0$

$s = 100$.

53. Let the speeds of the faster and the slower cars be f kmph and s kmph respectively.

$360 = 9(f - s) = 3(f + s)$

$f - s = 40$ and $f + s = 120$

Adding these and then simplifying, $f = 80$.

54. Let the distance be d km

Let his usual speed be u kmph.

Let his usual time be t hours.

$\frac{d}{u} = t$

$\frac{d}{\left(\frac{4}{5}\right)u} = t + \frac{9}{60} \Rightarrow \frac{t}{\frac{4}{5}} = t + \frac{9}{60}$

$\Rightarrow \frac{t}{4} = \frac{9}{60} \Rightarrow t\frac{36}{60}$ hours or 36 minutes

55. Let the length of Q be l m.

Let its speed be q m/sec

Length of $P = l\left(1 + \frac{25}{100}\right)\text{m} = \frac{5}{4}l\,\text{m}$

Its speed = $3q$ m/sec

$\frac{\frac{5}{4}l + 175}{3q} = 30$

$\frac{\frac{5}{4}(l + 140)}{3q} = 30$

$\frac{l + 140}{q} = 72$

Required time $= \frac{l+140}{q}$ seconds

= 72 seconds.

56. Let the usual speed of Hari be u kmph.

Let his usual time be t hours.

Distance (in km) = $(u + 5)(t - 1)$

$= (u - 2.5)\left(t + \frac{48}{60}\right) = ut$

$(u + 5)(t - 1) = ut$

$\therefore ut + 5t - u - 5 = ut$

$5t - 5 = u$ (1)

$(u - 2.5)\left(t + \frac{48}{60}\right) = ut$

$ut - 2.5t + \frac{4}{5}u - 2 = ut$

$\frac{4}{5}u = 2.5t + 2$ (2)

$4t - 4 = 2.5t + 2$ (From Eqs. (1) and (2))

$1.5t = 6$

$t = 4$

$\therefore u = 15$

$\therefore ut = 60$

57. Let the speeds of Ram, Shyam and Tarun be r m/sec, s m/sec and t m/sec respectively.

$2\left(\frac{r+s+t}{3}\right) = \frac{r+s}{2} + \frac{r+t}{2}$

$$2\left(\frac{r+s+t}{3}\right) = \frac{2r+s+t}{2}$$

$4r + 4s + 4t = 6r + 3s + 3t\ s + t = 2r$

$\therefore r = \dfrac{s+t}{2}$ (1)

Given: r, s and t are distinct.

(1) $\Rightarrow$ r is the average of s and t

$\therefore$ r must lie between s and t.

$\therefore$ Ram cannot be the winner.

58. Let the meeting point be M.

Let the time taken to meet be t hours.

PM = (50)(t) km = 40(x + 9) km

$\therefore$ $50t = 40(x + 9)$ (1)

MQ = (40)(t) km = 50(x) kmph

$\therefore$ $40t = 50x$ (2)

(1) $\Rightarrow x + 9 = \dfrac{5}{4}t$

i.e., $x = \dfrac{5}{4}t - 9$

(2) $\Rightarrow x = \dfrac{4}{5}t$

$\therefore x = \dfrac{5}{4}t - 9 = \dfrac{4}{5}t$

$\dfrac{9}{20}t = 9$

$t = 20$

PQ = PM + MQ

$= 90t = 1800$ km.

59. Times taken by him to travel, PQ, QR and RS were $\dfrac{PQ}{12}$ hours, $\dfrac{QR}{32}$ hours and $\dfrac{RS}{10.8}$ hours respectively.

2PQ = 3QR = 4RS

$\therefore \text{QR} = \dfrac{2}{3}\text{PQ}$ and $\text{RS} = \dfrac{\text{PQ}}{2}$

Total travel time (in hours)

$$= \frac{PQ}{12} + \frac{\frac{2}{3}PQ}{32} + \frac{\frac{PQ}{2}}{10.8}$$

$$= PQ\left(\frac{1}{12} + \frac{1}{48} + \frac{1}{21.6}\right) = \frac{65PQ}{432}$$

$$\text{Total distance} = \left(PQ + \frac{2}{3}PQ + \frac{PQ}{2}\right)\text{km}$$

$$= \frac{13}{6}PQ\ \text{km}$$

$$\text{Average speed} = \frac{\frac{13}{6}PQ}{\frac{65PQ}{432}}\text{kmph} = \frac{72}{5}\text{kmph.}$$

60. Total distance travelled by them when they met for the second time = 3(XY) = 54 m

Total time taken when they met for the second time $= \dfrac{54}{5+4} = 6$ seconds.

Total distance travelled by Anil (6 second) (5 m/sec) = 30 m

61. Let the speeds of Ganesh and Harish be g kmph and h kmph respectively. Let the meeting point be M.

$AM = 6g$ km and $BM = 6h$ km

$y = \dfrac{AM}{h}$ and $y + 5 = \dfrac{BM}{g}$

$\therefore y = \dfrac{6g}{h}$ and $y + 5 = \dfrac{6h}{g}$

$$y + 5 - y = 6\left(\frac{h}{g} - \frac{g}{h}\right)$$

Let $\dfrac{h}{g}$ be x

$$0 = 6\left(x - \frac{1}{x}\right) - 5$$

$0 = 6x^2 - 5x - 6$

$0 = (3x + 2)(2x - 3)$

$x > 0$

$\therefore x - \dfrac{3}{2}$

$\therefore \dfrac{g}{h} = \dfrac{2}{3}.$

62. Let the length of the track be L m.

Let the speeds of Ramu and Somu be r m/sec and s m/sec respectively.

Let the time taken by Ramu to run the race be t seconds.

Distance run by Somu when Ramu finishes = $(L - 10)$ m

$$\therefore t = \frac{L}{r} = \frac{L-10}{s}$$

$$\therefore \frac{r}{s} = \frac{L}{L-10} \qquad (1)$$

When Ramu had run $\left(\frac{L}{2} + 4.5\right)$ m, Somu had run $\left(\frac{L}{2} - 1\right)$ m

$$\therefore \frac{r}{s} = \frac{\frac{L}{2} + 4.5}{\frac{L}{2} - 1}$$ ($\because$ this is obtained in a manner similar to the way (1) is obtained)

$$\frac{r}{s} = \frac{\frac{L}{2} + 4.5}{\frac{L}{2} - 1} = \frac{L}{L-10}$$

$$\frac{L^2}{2} - 0.5L - 45 = \frac{L^2}{2} - L$$

$0.5L = 45$

$L = 90.$

63. Let the time taken by Rohan to run the race be t seconds.

Time taken by Sohan to run the race = $(t + 40)$ seconds.

Time taken by Mohan to run the race = $(t + 40 + 80)$ seconds = $(t + 120)$ seconds.

Distance run by Mohan when Rohan finishes 1200 m = $(1200 - 400)$ m = 800 m

$$\therefore \text{The speed of Mohan} = \frac{800}{t} \text{ m/s}$$

$$= \frac{1200}{t+120} \text{ m/s}$$

$$\frac{800}{t} = \frac{1200}{t+120}$$

$2t + 240 = 3t \Rightarrow t = 240.$

64. Let the speed of the boat in still water be x kmph.

Let the speed of the river be y kmph.

$\therefore PQ = 36y$

$x = 9y$

Total time taken by the boat

$$= \left(\frac{PQ}{9y + y} + \frac{PQ}{9y - y}\right) \text{hours}$$

$$= \left(\frac{36y}{10y} + \frac{36y}{8y}\right) \text{hours} = (3.6 + 4.5) \text{ hours}$$

= 8.1 hours.

65. Time taken for the tunnel to be free of traffic is the maximum of the times taken by the trains to cross the tunnel.

Time taken by the 360 m long train to cross tunnel

$$= \frac{360 + 450}{(97.2)\left(\frac{5}{18}\right)} \text{ seconds} = \frac{810}{27} \text{ seconds} = 30 \text{ seconds}$$

Time taken by the 540 m long train to cross the tunnel

$$= \frac{540 + 450}{(108)\left(\frac{5}{18}\right)} = \frac{990}{30} \text{ seconds} = 33 \text{ seconds}$$

The required time = 33 seconds.

66. Average speed $= \frac{2fr}{f + r}$ kmph

Given:

Average speed $= \frac{f + r}{2}$ kmph

$$\frac{2fr}{f+r} = \frac{f+r}{2}$$

$4fr = (f + r)^2$

$0 = f^2 + r^2 + 2fr - 4fr$

$\therefore 0 = (f - r)^2$

$\therefore f - r = 0$

$\therefore f = r$

$\therefore$ Average speed $= f$ kmph

$$\therefore f = \frac{2(240)}{6} = 80.$$

67. Let the speed of the boat in still water be x kmph. Let the speed of the river be y kmph.

Let $AB = d$ km

$$\frac{d}{x+y}+\frac{d}{x-y}=T$$

$$\frac{d(2x)}{x^2-y^2}=T \quad (1)$$

$$\frac{d}{3x+2y}+\frac{d}{3x-2y}=\frac{9}{35}T$$

$$\frac{d(6x)}{9x^2-4y^2}=\frac{9}{32}T \quad (2)$$

$$\frac{(2)}{(1)}=\frac{3(x^2-y^2)}{9x^2-4y^2}=\frac{9}{32}$$

$96(x^2-y^2)=81x^2-36y^2$

$15x^2=60y^2$

$x^2=4y^2$

$x, y>0$

$\therefore\ x=2y$

$\Rightarrow x:y=2:1.$

68. Let the initial speeds of Amish and Bala be a kmph and b kmph respectively.

$$\frac{200}{a+b}=4$$

$\therefore a+b=50$ (1)

After meeting, the speeds of Amish and Bala were $(a-10)$ kmph and $(b+10)$ kmph respectively. Distances covered by Amish and Bala after meeting were $4b$ km and $4a$ km respectively. They reached simultaneously

$$\therefore\ \frac{4b}{a-10}=\frac{4a}{b+10}$$

$$\frac{b}{a-10}=\frac{a}{b+10} \quad (2)$$

$b^2+10b=a^2-10a$

$a^2-b^2=10(a+\text{b})$

$a-b=10$ ($\because a+b \# 0$) (2)

From Eqs. (1) and (2),

$a=30.$

69. Let the length of the race be l.

Let the speeds of Ram, Shyam and Tarun be r m/sec, s m/sec and t m/sec respectively.

When Ram finishes, Shyam would have run $(l-30)$ m.

Suppose Ram takes x seconds to run the race.

$$x=\frac{l}{r}=\frac{l-30}{s}$$

$$\therefore\ \frac{r}{s}=\frac{l}{l-30}$$

Similarly $\frac{s}{t}=\frac{l}{l-60}$ and $\frac{r}{t}=\frac{l}{l-84}$

$$\frac{r}{t}=\left(\frac{r}{s}\right)\left(\frac{s}{t}\right)=\frac{l^2}{(l-30)(l-60)}$$

$$\frac{l}{l-84}=\frac{l^2}{(l-30)(l-60)}$$

$l(l-30)\ (l-60)=l^2(l-84)$

$l(l^2-90l+1800)=l^3-84l^2$

$l^3-90l^2+1800\ l=l^3-84l^2$

$1800l-6l^2=0$

$6l(300-l)=0$

$\because l>0$

$\therefore\ 6l-0$

$\therefore\ 300-l=0$

$\therefore\ l=300.$

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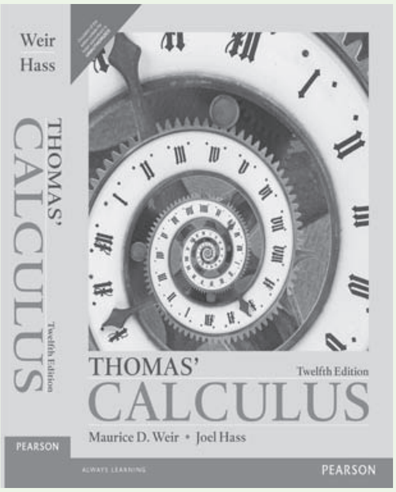

Thomas' Calculus, 12/e

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About the Book

Calculus hasn't changed, but students have. Today's students have been raised on immediacy and the desire for relevance, and they come to calculus with varied mathematical backgrounds. Thomas' Calculus, Twelfth Edition, helps students successfully generalize and apply the key ideas of calculus through clear and precise explanations, clean design, thoughtfully chosen examples and superior exercise sets. This book offers the right mix of basic, conceptual and challenging exercises, along with meaningful applications. This significant revision features more examples, more mid-level exercises, more figures, improved conceptual flow and the best in technology for learning and teaching.

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- Complete and precise multivariable coverage enhances the connections of multivariable ideas with their single-variable analogues studied earlier in the book.

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Joel Hass received his Ph.D from the University of California Berkeley. He is currently a professor of mathematics at the University of California, Davis. He has coauthored six widely used calculus texts as well as two calculus study guides. He is currently on the editorial board of *Geometriae Dedicata* and Media-Enhanced Mathematics. He has been a member of the Institute for Advanced Study at Princeton University and of the Mathematical Sciences Research Institute and he was a Sloan Research Fellow. Hass's current areas of research include the geometry of proteins, three dimensional manifolds, applied math and computational complexity. In his free time, Hass enjoys kayaking.

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George B. Thomas, Jr. (late) of the Massachusetts Institute of Technology, was a professor of mathematics for 38 years; he served as the executive officer of the department for 10 years and as graduate registration officer for five years. Thomas held a spot on the board of governors of the Mathematical Association of America and on the executive committee of the mathematics division of the American Society for Engineering Education. His book, Calculus and Analytic Geometry, was first published in 1951 and has since gone through multiple revisions. The text is now in its twelfth edition and continues to guide students through their calculus courses. He also co-authored monographs on mathematics, including the text of Probability and Statistics.